8.022 Lecture Notes Class 29 - 11/9/2006

Ampere's Law in Magnetized Materials

$\vec{M}$ Magnetization

$$\vec{M} = \vec{M}(\vec{J_B}, \vec{k_A})$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{M}(\vec{r'}) \times (\vec{r'} - \vec{r})}{|\vec{r} - \vec{r'}|^3} d\tau'$$

$$\nabla' \frac{1}{|\vec{r} - \vec{r'}|} = \frac{(\vec{r'} - \vec{r})}{|\vec{r} - \vec{r'}|^3}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \left[\int \frac{1}{|\vec{r} - \vec{r'}|} [\vec{\nabla}' \times \vec{M}(\vec{r'})] d\tau' + \oint \frac{1}{|\vec{r} - \vec{r'}|} [\vec{M}(\vec{r'}) \times d\vec{a'} \right]$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \left[\int \frac{1}{|\vec{r} - \vec{r'}|} \vec{J_{\text{bound}}} d\tau' + \oint \frac{1}{|\vec{r} - \vec{r'}|} \vec{k_{\text{bound}}} \cdot da' \right]$$

So : $\vec{J_B} = \vec{\nabla} \times \vec{M}$; $\vec{k_B} = \vec{M} \times \hat{n}$ (bound current exists when there is magnetization)

Must have something else :

$$\vec{J} = \vec{J_B} + \vec{J_{\text{free}}}$$

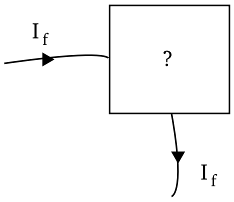
$$\frac{1}{\mu_0}(\vec{\nabla} \times \vec{B}) = \vec{J} = \vec{J_f} + \vec{\nabla} \times \vec{M}$$

$$\nabla \times (\frac{1}{\mu_0}\vec{B} - \vec{M}) = \vec{J_f}$$

Let $\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$; $\vec{\nabla} \times \vec{H} = \vec{J_f}$; $\oint \vec{H} \cdot dl = I_{\text{freeenclosed}}$

$$\vec{B} = \vec{B}(I_f + I_B)$$

• In electrostatics, $\vec{E}$ is more useful than $\vec{D}$ ($\vec{E}$ does not depend on material) . Opposite in magnetism.



$$\vec{H} = \vec{H}(I_f)$$

$$\vec{\nabla} \cdot \vec{H} = -\vec{\nabla} \cdot \vec{M}$$

$$\begin{aligned} H = & \ \tfrac{1}{\mu_0} B - M \\ (\vec{\nabla} \cdot \frac{1}{\mu_0} \vec{B}) = & \ \vec{\nabla}(\vec{H} + \vec{M}) \\ 0 = & \ \vec{\nabla} H + \vec{\nabla} \cdot \vec{M} \end{aligned}$$

$$\vec{M} = \chi_M \vec{H}$$

χ_M is the magnetic susceptibility.

$$\begin{aligned} \vec{B} = & \ \mu_0(\vec{H} + \vec{M}) \\ = & \ \mu_0(\vec{H} + \chi_M \vec{H}) \\ = & \ \mu_0 \vec{H}(1 + \chi_M) \\ \vec{B} = & \ \mu \cdot \vec{H} \end{aligned}$$