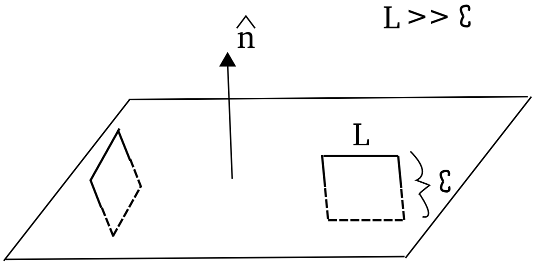
8.022 Lecture Notes Class 27 - 11/1/2006

Boundary Conditions on $\vec{B}$ at a surface



Look at the tangential B field

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \quad : \text{Magnetostatics}$$
$$\vec{\nabla} \times \vec{E} = 0 \quad : \text{Electrostatics}$$

$\vec{k}$ is surface current density

$$\vec{\nabla} \cdot \vec{B} = 0$$
$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

$$\oint B \cdot dl = LB_{y,a} - LB_{y,b} = \mu_0 \int J_x dydz = \mu_0 L \int J_x dz$$

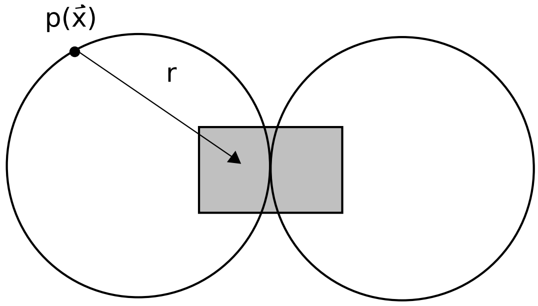
$$J_x = \delta(z) \cdot k_x = \mu_0 L k_x = L(\Delta B_y)$$

$$AB_y = \mu_0 \cdot k_x$$

Now turn loop 90°

$$AB_x = \mu_0 k_y$$
$$\vec{B_{xy}} = \mu_0 \hat{n} \times \hat{k}$$

Magnetic Dipole



$$\frac{1}{|\vec{x}-\vec{x'}|} = \frac{1}{r} - \frac{\hat{r}\cdot\hat{x'}}{r^2} + \vartheta(\frac{r^{12}}{r^3})$$

$$\int [d^3x' \vec{J}(\vec{x'})] = 0$$

True for any localized current distribution

$$J_i(\vec{x}) = \nabla\cdot(x\cdot\vec{J}) - x_i\vec{\nabla}\cdot\vec{J}$$

$$\int [d^3x\vec{\nabla}\cdot(x_i\vec{J}) = 0$$

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi r^2}\int \vec{J}(\vec{x})\hat{r}\cdot\vec{x'}d^3x'$$

$$= \frac{\mu_0}{4\pi}\int\frac{\vec{J}(\vec{x'})d^3x'}{|\vec{x}-\vec{x'}|} = \frac{\mu_0}{4\pi}\int\vec{J}(\vec{x})\frac{1}{r}d^3x' + ...\vec{M} = \frac{1}{2}\int\vec{x'}\times\vec{J}(\vec{x'})d^3x'$$

Index notation

$$\int\nabla\cdot(x_ix_j\vec{J})d^3x = \int(x_iJ_j + x_jJ_i)d^3x = 0$$

$\nabla \cdot \vec{J}$ divergence of localized charge distribution? convert to surface integral, expand to $\vec{J} = 0$, So entire thing is 0.
(so , $x_i J_j = -x_j J_i$)

$$\int (x_i J_j - x_j J_i) d^3x = \quad 2 \int x_i J_j d^3x$$
$$= \epsilon_{ijk} \int (\vec{x} \times \vec{J})_k d^3x$$

$$\int x_i J_j d^3x = \epsilon_{ijk} m_k$$

$$\int J_j(\vec{x'}) \hat{r} \cdot x'_i d^3x'$$

$$(\int J_j(\vec{x'} x'_i d^3x')) \hat{r}$$
$$= \quad \epsilon_{ijk} m_k \hat{r}_j = (\hat{m} \times \hat{r})_i$$
$$= \int \vec{J}(\vec{x'})(\hat{r} - \vec{x'}) d^3x' = \vec{m} \times \hat{r}$$

So,

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \vec{r}}{r^2} = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \vec{x}}{|\vec{x}|^3}$$

$$\vec{B} = \vec{\nabla} \times \vec{A} = \frac{\mu_0}{4\pi} [\frac{3\hat{r}(\vec{m} \cdot \hat{r}) - \hat{m}}{r^3}]$$