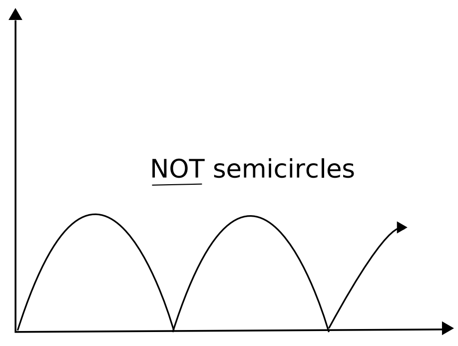
8.022 Lecture Notes Class 23 - 10/25/2006



$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Cycloidal Motion

$$\vec{E} = E\hat{y} \quad \vec{B} = B\hat{z}$$

$$\begin{aligned} \vec{F} = m\tfrac{d\vec{v}}{dt} &= q(\vec{E} + \vec{v} \times \vec{B}) \\ = \tfrac{d\vec{v}}{dt} &= q(E\hat{y} + \vec{v} \times B\hat{z}) \end{aligned}$$

$$\begin{cases} \frac{mdv_2}{dt} = & q(0+0) \Rightarrow v_2 = 0 \quad \text{Since } \vec{E} \text{ is in } \hat{y} \text{ and the cross prod contains } \hat{z} \\ \frac{mdv_x}{dt} = & q(Bv_y) \qquad \natural \\ m\frac{dv_y}{dt} = & q(E + B(-v_x)) \qquad \bigstar \end{cases}$$

Differentiate [$\natural$] with respect to time , get $\frac{dv_y}{dt}$ equation, substitute into equation [$\bigstar$]

Get:

$$\frac{d^2 v_x}{dt^2} + \frac{q^2 B^2}{m^2} v_x = \frac{q^2 BE}{m^2}$$

have $\omega = \frac{qB}{m}$, so

$$\frac{d^2 v_x}{dt^2} + \omega^2 v_x = \frac{\omega^2}{B} E$$

$$\Rightarrow \begin{cases} v_x(t) = & \frac{E}{B} + c_1 \cos(\omega t) + c_2 \sin(\omega t) \\ v_y(t) = & -c_1 \sin(\omega t) + c_2 \cos(\omega t) \end{cases}$$

$$v_x(0) = v_y(0) = 0 \Rightarrow c_1 = -\frac{E}{B}, c_2 = 0$$

$$\begin{cases} v_x(t) = & \frac{E}{B}(1 - \cos \omega t) \\ v_y(t) = & \frac{E}{B} \sin \omega t \end{cases}$$

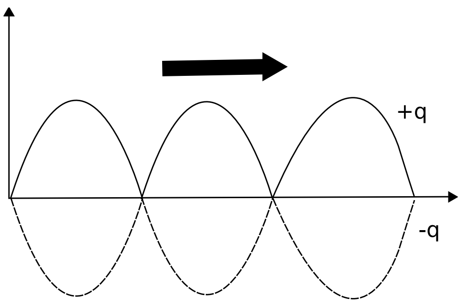
From ICS , $x(0) = 0, y(0) = 0$

$$\int dt \Longleftrightarrow \begin{cases} x(t) = \frac{E}{B\omega}(\omega t - \sin \omega t) \\ y(t) = \frac{E}{B\omega}(1 - \cos \omega t) \end{cases}$$

$$\begin{cases} \sin \omega t \approx \omega t + (\omega t)^3 \\ \cos \omega t \approx 1 + (\omega t)^2 \end{cases}$$

$$x(t) = \frac{E}{B} t - \frac{E}{\omega B} \sin \omega t$$

average motion in x is independent of q and m



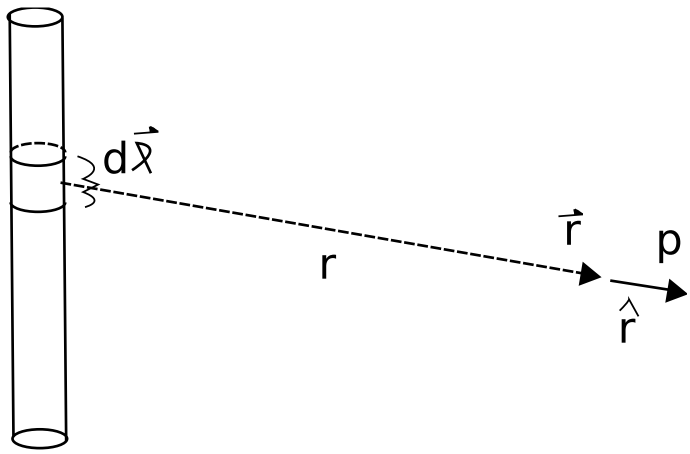
$$\vec{v_0} = \frac{E}{B}\hat{x} = \frac{\vec{E} \times \vec{B}}{|\vec{B}|^2}$$

Magnetostatics

Biot-Savart Law

- Source of $\vec{B} - fields$ (force from current)
  - steady-state currents
  - anything else ?

Infinitesimal Current Element



$$d\vec{B} = \frac{\mu_0}{4\pi} = \frac{(Id\vec{l}) \times \hat{r}}{r^2}$$

$$c = \frac{1}{\epsilon_0 \mu_0}$$

$2.99792485 \times 10^8 m/s$ $\quad\quad$ $\mu_0 = 4\pi \times 10^{-7} T \cdot M/A$

$$\begin{aligned}\vec{B}(\vec{x}) = & \int d\vec{B} \\ = & \frac{\mu_0}{4\pi} \int \frac{Id\vec{l} \times \hat{r}}{r^2}\end{aligned}$$