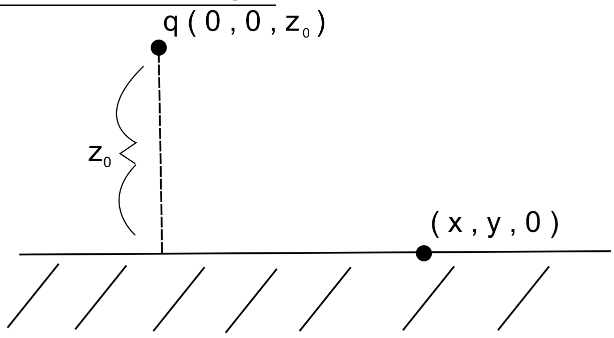
8.022 Lecture Notes Class 14 - 10/02/2006

Method of Images



Find $V(x, y, z) \quad z \geq 0$

$V(x, y, 0) = c$ on conducting plane, Let $c = 0$

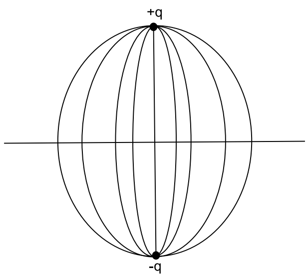
$$\nabla^2 V = \frac{1}{\epsilon_0}\rho \qquad \text{Solve Poisson's Equation}$$

$$\rho = \delta(x)\delta(y)\delta(z - z_0)q$$

Images

Use the first uniqueness theorem - find simpler situation with "same" stuff, then the solutions will be the same .

Displacement (Dont care about stuff below)



At $(x, y, 0)$,

$$
\begin{aligned}
V_q &= \tfrac{q}{4\pi\epsilon_0(x_1^2+y_1^2+z_0^2)^{1/2}} \\
V_{-q} &= -\tfrac{q}{4\pi\epsilon_0(x_1^2+y_1^2+(-z_0)^2)^{1/2}} \\
V &= 0
\end{aligned}
$$

We have proved that dipole configuration is same as in problem situation!

- Instead of Solving Poisson's, We solve V for dipole

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{\sqrt{x^2+y^2+(z-z_0)^2}} + \frac{-1}{\sqrt{x^2+y^2+(z+z_0)^2}}\right]$$

Works only for $z \geq 0$ (Within our V , above S)
Electric Field?

$$
\begin{aligned}
E &= -\nabla V \\
&= \tfrac{q}{4\pi\epsilon_0}\left[\tfrac{x\hat{x}+y\hat{y}+(z-z_0)\hat{z}}{\sqrt{(x^2+y^2+(z-z_0)^2)^3}} - \tfrac{x\hat{x}+yy+\hat{}(\hat{z}-z_0)}{\sqrt{(x^2+y^2+(z+z_0)^2)^3}}\right]
\end{aligned}
$$

Charge on a surface?

$$
\begin{aligned}
\vec{E} &= \tfrac{\sigma}{\epsilon_0}\hat{n} \\
\vec{E_\perp} &= \tfrac{\sigma}{\epsilon_0} \\
\sigma &= \epsilon_0 E_\perp
\end{aligned}
$$

Why image? Conductor (obeys $V = 0$) acts as a mirror
Integrate surface charge over surface

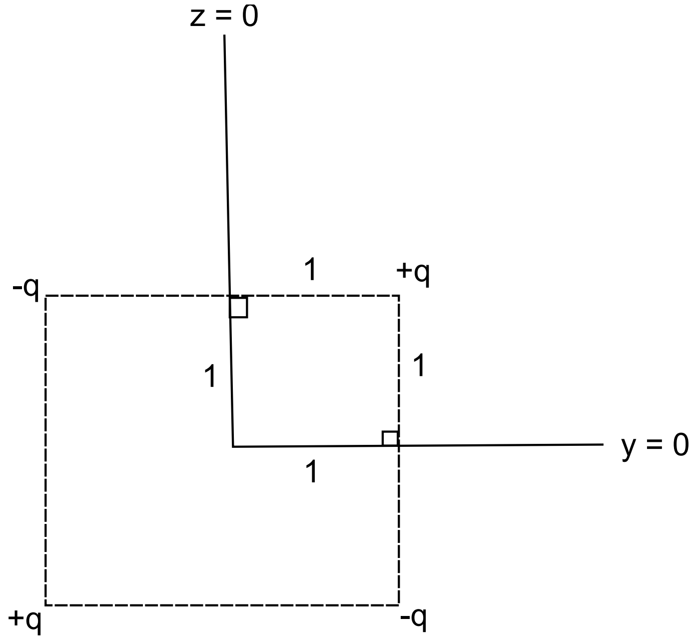
$$\int_0^\infty \int_0^{2\pi} \sigma(r) \cdot r d\phi dr$$
$$= \int_0^\infty 2\pi \frac{-qz_0}{2\pi(r^2+z_0^2)^{3/2}} dr$$
$$= \frac{qz_0}{\sqrt{r^2+z_0^2}}\Bigg|_0^\infty = -q \quad \text{(which works!)}$$

$$\vec{F} = -\frac{1}{4\pi\epsilon_0}\frac{q^2}{(2z_0)^2}\hat{z} \quad \text{(using dipole)}$$

$$W = \int_\infty^{z_0} \vec{F} \cdot dl = \frac{1}{4\pi\epsilon_0}\int_\infty^{z_0} \frac{q^2}{4z_0^2} dz$$
$$= \frac{q^2}{4\pi\epsilon_0}\left(-\frac{1}{4z}\right)\Bigg|_\infty^{z_0}$$
$$= \frac{q^2}{4\pi\epsilon_0} \cdot \frac{-1}{4z_0}$$
$$= -\frac{1}{4\pi\epsilon_0}\frac{q^2}{2(2z_0)}$$

half of the situation for dipole. Why?

energy αE^2, never had to build up E below plane, so $\frac{1}{2}$



Use Inversive Geometry (lots of geometric properties hold)