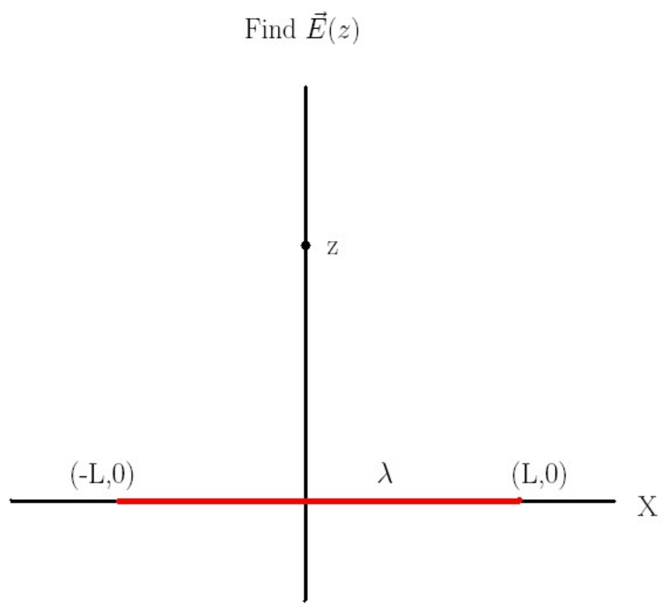
8.022 - Class 3 - 9/11/2006

October 20, 2007



$$\vec{E}(z) = \frac{1}{4\pi\epsilon_0}\int_{-L}^{L}\frac{\lambda}{(x^2+z^2)^{3/2}}(-x\hat{i}+z\hat{j})dx \qquad u = x^2+z^2$$

$$du = 2xdx$$

$$= \frac{1}{4\pi\epsilon_0}\left(\frac{\lambda}{(x^2+z^2)^{1/2}}\right|_{-L}^{L} + \dots \qquad x = z\tan\theta$$

$$dx = z\sec^2\theta d\theta$$

Dimensionless Integrals

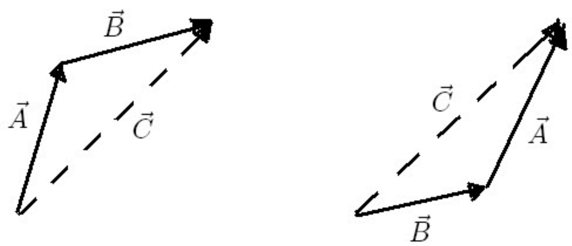
$$L \qquad \int_0^L \frac{z}{(z^2+l^2)^{3/2}} dl$$

$$\frac{L}{L^3} \quad \int_0^1 \frac{L(\frac{z}{L})}{(L^2 \cdot \frac{z^2}{L^2} + L^2 \cdot \frac{l^2}{L^2})^{3/2}} d(\frac{l}{L})$$

<u>Vector</u>

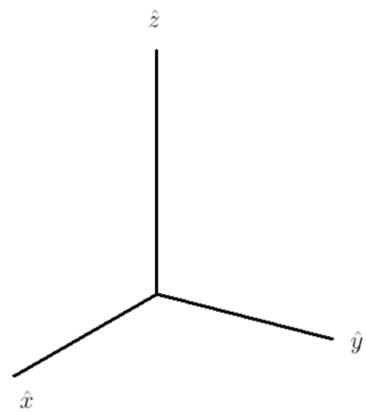
- a sequence of numbers $(a_1, ...)$
- magnitude and direction $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A} = \vec{C}$

$$\vec{A} + \vec{B} \quad = \quad \vec{C} \quad = \quad \vec{B} + \vec{A}$$



$$\vec{A} \times \vec{B} = |A||B| \sin(\theta) \hat{n}$$

$$\begin{aligned} \vec{A} \times (\vec{B} + \vec{C}) = &\quad (\vec{A} \times \vec{B}) + (\vec{A} \times \vec{C}) \\ \vec{A} \times \vec{B} = &\quad -\vec{B} \times \vec{A} \\ \vec{A} \times \vec{A} = &\quad 0 \end{aligned}$$



$$\begin{aligned}\vec{A} &= A_x\hat{x} + A_y\hat{y} + A_z\hat{z} \\ &= A_1\vec{e_1} + A_2\vec{e_2} + A_3\vec{e_3} \\ &= \sum_{i=1}^{3} A_i\vec{e_i}\end{aligned}$$

$\Leftrightarrow A_i\vec{e_i}$ (Einstein summation convention)

$$\begin{aligned} \vec{A}+\vec{B} = & \sum_i A_i\vec{e_i} + \sum_i B_i\vec{e_i} \\ = & \sum_i (A_i+B_i)\vec{e_i} \end{aligned}$$

$$a\vec{A} = a\sum_i A_i\vec{e_i} = \sum_i (aA_i)\vec{e_i}$$

$$\vec{A}\cdot\vec{B} = (\sum_i A_i\vec{e_i})\cdot(\sum_j B_j\vec{e_j}) = \sum_i A_iB_i\vec{e_i}$$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

ϵ_{ijk} - antisymmetric tensor ; $i =$ indices for A , $j =$ indices for

$\epsilon_{123} = 1 \Leftarrow 123, 231, 312$
$\epsilon_{213} = -1 \Leftarrow 132, 321, 213$
else $\epsilon = 0$.
$\vec{A}\times\vec{B} = \epsilon_{ijk}A_iB_j$
$\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B})$
Prove using epsilon notation - by definition $\epsilon_{ijk} = \epsilon_{jki} = \epsilon_{kij}$

$$\begin{aligned} \vec{A}\cdot(\vec{B}\times\vec{C}) = & \vec{A}\cdot(\epsilon_{ijk}\cdot B_iC_j) \\ = & A_k\epsilon_{ijk}B_iC_j \\ = & \epsilon_{ijk}B_iC_jA_k \\ = & c_j\epsilon_{kij}B_iA_kC_j \\ = & \vec{C}\cdot(\epsilon_{kij}A_kB_i) \\ = & \vec{C}\cdot(\vec{A}\times\vec{B}) \end{aligned}$$

not exactly correct notation