Massachusetts Institute of Technology
Department of Physics
8.022 Fall 2004
Assignment 2: Electric Potential
Due date: Thursday, September 23th

1. Purcell 2.4: $\vec{E}$ and ρ from ϕ.

2. Purcell 2.8: Cylindrical charge distribution.

3. Purcell 2.14: Laplace's equation.

4. Purcell 2.29: Two charged nonconducting sphere shells.

5. Energy of a radial charge distributions. A spherically symmetric charge distribution has charge density

$$\rho = \rho_0 \frac{r}{a}, r < a \tag{1}$$
$$= 0, r \geq a. \tag{2}$$

(a) Find the electric field $\vec{E}$ everywhere.
(b) Find the electrostatic potential ϕ everywhere.
(c) Determine the energy needed to assemble the charge distribution using 2 different approaches.

6. Electrostatic potentials.
(a) Find the electric field $\vec{E}$ from the electrostatic potential

$$\phi = \frac{\alpha z}{r} \tag{3}$$

where α is a constant and r is the distance from the origin.
(b) An electrostatic potential has the form

$$= -2\pi al(x + l/4), x < -l/2 \tag{4}$$
$$\phi = 2\pi ax^2, -l/2 < x < l/2 \tag{5}$$
$$= 2\pi al(x - l/4), l/2 < x \tag{6}$$

where a and l are constants. Find the charge distribution which gives this potential.
(c) Give the electric field of the charge distribution you found in part(b).

7. Electric field, potential and flux.

A hollow spherical shell carries charge density

$$\rho = \frac{k}{r^2} \tag{7}$$

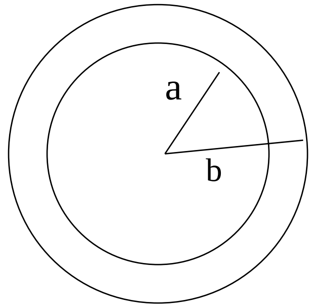

Figure 1: Concentric sphere shell with inner radius a, outer radius b.

in the region $a \leq r \leq b$.(figure 1)
(a) Find the electric field $\vec{E}$ everywhere in space.
(b) Find the potential ϕ everywhere in space.
(c) Calculate the flux
i) through the concentric sphere with radius $r_1 > b$.
ii) through the concentric sphere with radius $a < r_2 < b$.
iii) through the concentric sphere with radius $r_3 < a$.
iv) through a nonconcentric sphere with $r_4 = 2b$, centered at any arbitrary point on the outer surface of the shell.

8. Divergence in different coordinate systems.

We have learnt how to calculate $\mathbf{div}\vec{F} = \lim_{\Delta_V \to 0} \frac{\oint \vec{F} \cdot d\vec{A}}{V}$ in Cartesian coordinates. Now we will now work out the divergence for simple functions in cylindrical and spherical coordinates.

Cylindrical: consider a function $\vec{F} = F(\rho)\hat{\rho}$, where ρ is the radius of a cylinder. Consider a cylindrical shell with inner radius ρ, outer radius $\rho + \Delta\rho$, and height h. Take the normal on the inside to point in, that of the outside to point out. (figure 2)

(a) what is the total flux through this shell?
(b) Divide by the volume of the shell, take the limit $\Delta\rho \to 0$. What is $\mathbf{div}\vec{F}$?

Spherical: consider $\vec{F} = F(r)\hat{r}$, where r is spherical radius. Consider a spherical shell with inner radius r and outer radius $r + \Delta r$. Take the normal on the inside to point in, that of the outside to point out.(figure 3)

(c) What is the total flux through this shell?
(b) Divide by the volume of the shell, take the limit $\Delta r \to 0$. What is $\mathbf{div}\vec{F}$?

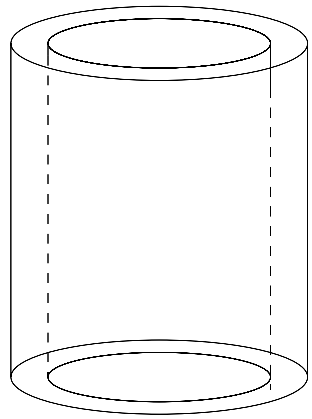

Figure 2: A cylinder with inner radius ρ, outer radius $\rho + \Delta\rho$

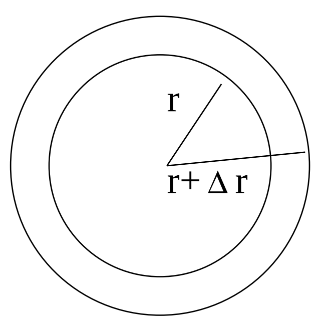

Figure 3: Concentric sphere shell with inner radius r, outer radius r+Δr