Assignment #6
**Spacetime in Special Relativity**
and review on Current, Resistance, Ohm's Law
EMF, Circuits, Kirchhoff's Rules, RC Circuits

Reading *Purcell* Chapter 4 and Appendix I (pages 451-458), handout on Special Relativity by Prof. Fisher.

Problem Set #6
Work on **all** problems. Not all problems receive equal points. Total points for this set is 100.

- **(20 points) [1]** RC circuit: qualitative analysis.
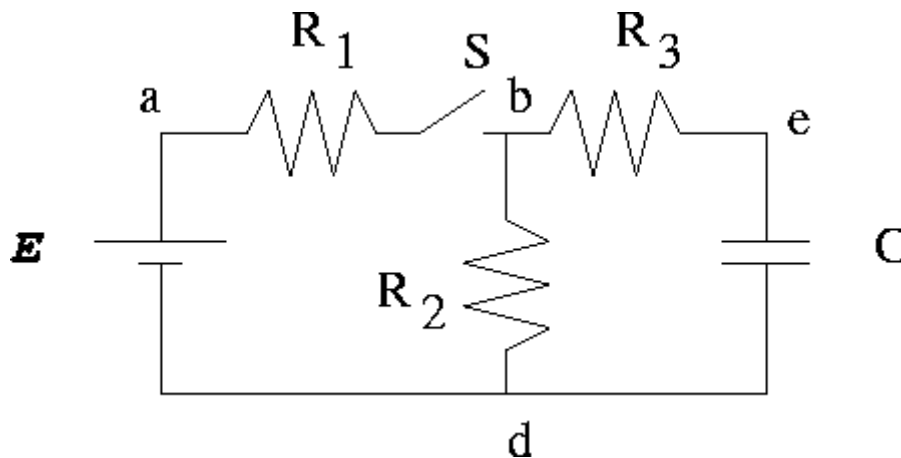
Three resistors (R_1, R_2 and R_3), a capacitor C and an *Emf E* are connected as shown in the figure below. Let us denote $V_{R_1} = V_a - V_b$, $V_{R_2} = V_b - V_d$, $V_{R_3} = V_b - V_e$ and $V_C = V_e - V_d$. The capacitor is assumed uncharged at t=0 when we ``close'' the switch S.

  o Perform a qualitative analysis of the circuit and derive the values for $V_{R_1}, V_{R_2}, V_{R_3}, V_C$ as well as of the currents $I_{R_1}, I_{R_2}, I_{R_3}$ flowing through the resistors for a time *immediately after the switch S is closed,* i.e. $t \rightarrow 0$

  o Perform the same qualitative analysis of the circuit after infinite time elapses (i.e., $t \rightarrow \infty$).

- o Plot the values of the above 7 quantities you have calculated at $t \to 0$ and $t \to \infty$ and estimate their behaviour for the times t in between 0 and ∞.
- o Following infinite time since we closed the switch S, we now ``open'' the switch S. Perform the same qualitative analysis and derive the values for $V_{R_1}, V_{R_2}, V_{R_3}, V_C, I_{R_1}, I_{R_2}, I_{R_3}$ after infinite time has elapsed since the switch S was opened. Extend the previous plot in order to show the behaviour of these 7 quantities after the switch S is open.
- o We are now ready to lay out the system of equations that can yield the exact functional form (with respect to t) of $V_{R_1}, V_{R_2}, V_{R_3}, V_C, I_{R_1}, I_{R_2}, I_{R_3}$. Using Kirchhoff's rules, write down the system of equations that can yield these functions before and after the switch S was opened. Identify the number of unknowns and the number of variables in each case. **Do not** solve the system of equations.

- **(20 points) [2]** Another RC circuit.
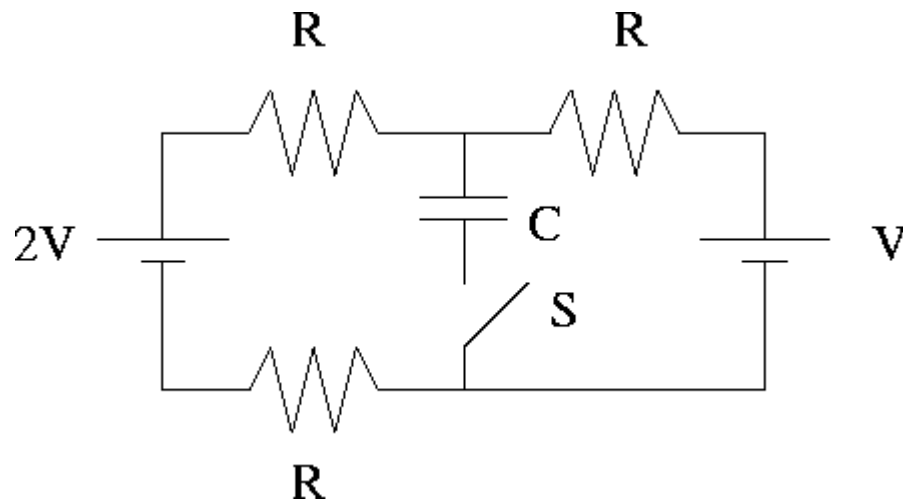
Three resistors (each of R), a capacitor C and two batteries each of V and $2V$ are connected as shown in the figure below. The switch S is initially open and a steady-state current is flowing through the circuit.
- o Find the current flowing through the circuit.
- o Now, suppose that the switch S is closed at $t=0$ and that the capacitor is originally uncharged. Find the charge and voltage on the capacitor as a function of time. Find the energy stored in it when it is fully charged (after infinite time).
- o Now, suppose that we wait until the capacitor is completely charged and at that point we replace the two batteries with two resistanceless wires and we reset our clocks to zero. Find the charge on the capacitor as a function of time. Compute the total energy dissipated in the resistors and compare it to the energy stored in the capacitor when it was completely charged (you have calculated this in the previous item).

- **(15 points) [3]** Invariance of intervals.
  A quantity that is left unchanged by the Lorentz transformations is called a **Lorentz invariant.**
  Consider two events described in the *laboratory frame* with event coordinates x_1, y_1, z_1, t_1 and x_2, y_2, z_2, t_2. Show that Δs where $\Delta s^2 = (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta x)^2$ is a **Lorentz**

invariant. Here Δt is the time interval between the two events and $\sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta x)^2}$ is the spatial distance between them in the same inertial system.

- **(15 points) [4]** Lorentz transformations.
  Two events occur at the same place in the laboratory and are separated in time by 3 seconds. What is the spatial distance between these two events in a moving frame with respect to which the events are separated in time by 5 seconds? What is the relative speed of the moving and laboratory frames?

- **(15 points) [5]** Low velocity Lorentz transformations.
  Show that for small enough $\beta = \frac{v}{c}$ the Lorentz transformations reduce to the Galilean transformations. Use Taylor expansions for γ to show this and compute the next term in the expansion to find v such that this term contributes 10% to the x transformation. For what value of v does the value of γ differ from unity by 1%?

- **(15 points) [6]** Transforming angles.
  A meter stick lies at rest in an inertial reference frame (``moving frame'') that moves with a constant velocity $\vec{v} = v\hat{i}$ with respect to the laboratory inertial reference frame (``lab frame''). The meter stick makes an angle ϕ' with the x' axis in the moving frame. What angle ϕ does the same meter stick make with the x axis of the lab frame? *Assume* now that the directions of electric field lines around a point charge transform in the same way as the directions of meter sticks that lie along these lines. Draw *qualitatively* the electric field lines due to an isolated positive point charge at rest in the ``moving frame'' as seen in (a) the ``moving frame'' and (b) the ``lab frame''.
  What can you say about the force acting on a test charge q at distance D perpendicular to $\vec{v}$ as measured by an observer in each of the frames? Will it be the same of different?

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