Massachusetts Institute of Technology
Department of Physics
Physics 8.022 - Fall 2002

Assignment #2
Field Lines, Electric Flux, Gauss' Law
Work, Energy of Fields, Potential

**Reading** *Purcell* Chapter 1 and 2.

Problem Set #2
Work on **all** problems. Not all problems receive equal points. Total points for this set is 100. Notice that some problems have **OPTIONAL** questions. THESE ARE NOT REQUIRED, it is only for your entertainment.

- **(10 points) [1]** Electric Field from Potential.
  Derive the electric field described by the following potential that is written in cylindrical coordinates: $V(\rho, \phi, z) = K(\rho^2 sin\phi cos\phi + z^2 tan\phi)$. What are the units of K in SI and CGS? Express the electric field in cylindrical, cartesian and polar coordinates.

- **(15 points) [2]** *Soda Centaurus*.
  An extraterrestrial spaceship from *Mondus Lontanus* lands at MIT's campus and chief scientist *Aurelius Studiolanum* hands to 8.022 students a print out depicting a structure known to his people as *Soda Centaurus* that they hit upon their voyage to Earth.
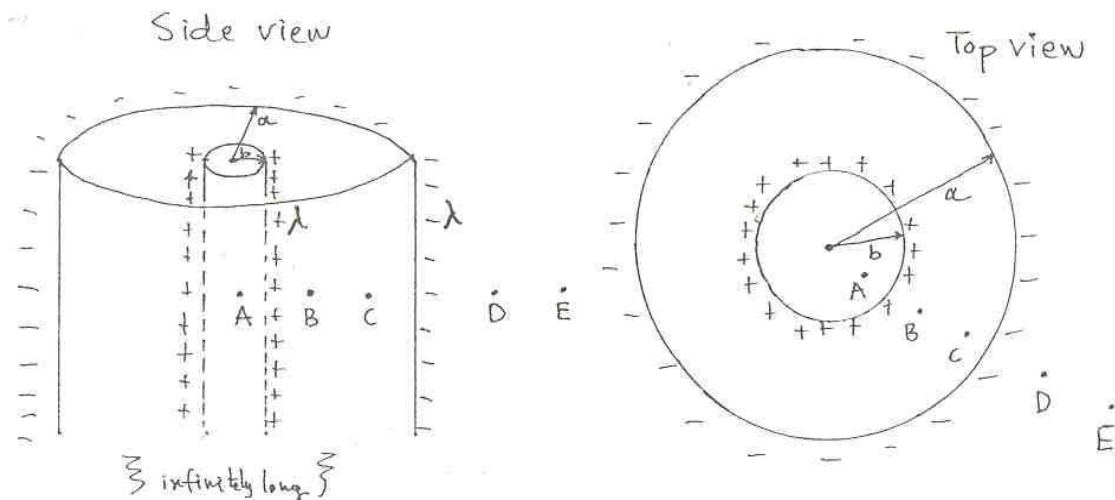
Soda Centaurus, *Aurelius* says, is a very simple structure: imagine two thin-walled, infinitely long, concentric cylinders (soda cans) of radii a and b each carrying equal and opposite charges of constant line density λ uniformily distributed around the cans.

Using an electric probe of an infinitesimally small charge aboard their spacecraft, *Aurelius* measured the electric field magnitude **E** on the five points shown on the plot above (A,B,C,D,E).

  o On the basis of the magnitude of **E**, rank the five points from the smallest to the largest.
  o Draw a Gaussian surface of your choice for each of the following cases that encloses the geometric point C (no electric charge resides on C) and has (I) positive net electric flux, (II) zero net electric flux, (III) negative net electric flux.

  o During their trip within *Soda Centaurus*, *Aurelius*- contrary to his commander's instructions- had secretly carried into his pocket an electron e^-. What is the work done (if any?) on e^- by the electric field (if any?) in Soda Centaurus as the spacecraft traveled from the outer cylinder (radius a) to the inner one (radius b)? Do we need to know the spaceship's trajectory in order to calculate the work?

- **(10 points) [3]** *Purcell* Problem 1.30 (p.36): Energy of concentric charged balls.

- **(10 points) [4]** *Purcell* Problem 1.8 (p.35): Potential energy of a 1-dim crystal.

- **(15 points) [5]** *Purcell* Problem 1.16 (p.36): A hollow sphere.
  Optional: Find the electric field **E** *anywhere* inside the cavity. Will this change if instead we had carved out a sphere of radius a/4 centered in the same location (a/2)?

- **(10 points) [6]** *Purcell* Problem 2.8 (p.80): Charged Cylinder.

- **(15 points) [7]** *Purcell* Problem 2.10 (p.80): Potential from continuous charge distribution (finite rod).
  Optional: Imagine a variation in this problem where the z=0 is not at the center of the rod but in one of its ends. Find V for a point P'_1 on the z and P'_2 on the x axis. Can you determine **E** (watch out, vector!) at P'_1 and P'_2 just from V you have just (re)calculated? How about finding V at these points if the rod was NOT uniformly charged but, say, $\lambda = kz$ where k is a constant?

- **(15 points) [8]** Electric Dipole.
  Two equal and opposite charges q are fixed at the ends of a rigid rod of length d. This is called an electric dipole. The dipole moment **p** is defined by **p**= q**d** where **d** is the vector from the negative to the positive charge. The dipole is placed in a uniform electric field **E**. Show that there is a total torque on the dipole given by $\vec{\tau}$ =**p** $\times$ **E** and that the energy of the dipole is U = **-pE**.

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