MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Problem Solving 8: Circuits

OBJECTIVES

1. To gain intuition for the behavior of DC circuits with both resistors and capacitors or inductors. In this particular problem solving you will be working with an RC circuit. You should carefully consider what would change if the capacitor were replaced with an inductor.

2. To calculate the time dependent currents in such circuits

REFERENCE: Chapter 7, 8.02 Course Notes.

An RC circuit consists of both resistors and capacitors, and typically a battery to get the current flowing. Capacitors, when uncharged, act like pieces of wire ("shorts") as they have no voltage drop across them. However, once charge has flowed on to them for a while, they "charge up," eventually reaching a potential equal and opposite that trying to charge them and effectively preventing the further flow of current.

This problem solving consists of two parts. In the first you will answer a series of short questions developing your intuition for the behavior of these circuits on short and long time scales. In the second part you will work through a quantitative problem.

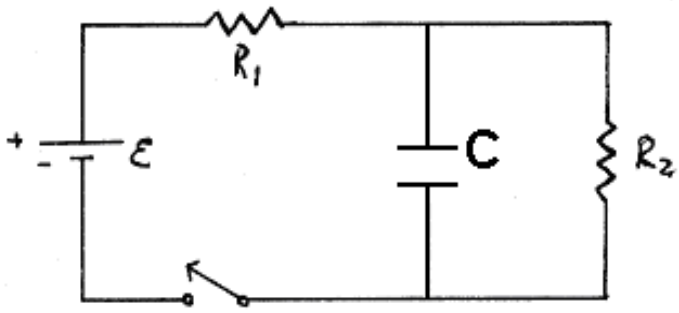


Figure 1: RC Circuit
An RC circuit consists of two resistors, R_1 and R_2, a capacitor C, a battery ε, and a switch. The switch has been open for a very long time before it is closed at time t=0.

***Write your answer to this and all following questions on the tear-sheet at the end*!**
What is/are…
Question 1: the current I_C (through the capacitor) at $t=0^+$ (just after switch is closed)?

Question 2: the currents I_1 and I_2 (through R_1 and R_2 respectively) at $t=0^+$?

Question 3: the current I_C (through the capacitor) at $t=\infty$?

Question 4: the currents I_1 and I_2 (through R_1 and R_2 respectively) at $t=\infty$?

At intermediate time *t* assume there is a charge *q* on the capacitor.

Question 6: Using Kirchhoff's Loop Rules, obtain a differential equation for the charge *q* on the capacitor, assuming $R_1=R_2=R$ (in other words, the only current in the equation should be the current through the capacitor, which can be rewritten in terms of d*q*/dt).

Question 7: What is the time constant for charging the capacitor?

Question 8: Write an equation for the time dependence of the charge on the capacitor

After a long time *T* the switch is opened.
What is/are…
Question 9: the current I_C (through the capacitor) at $t=T^+$ (just after switch is opened)?

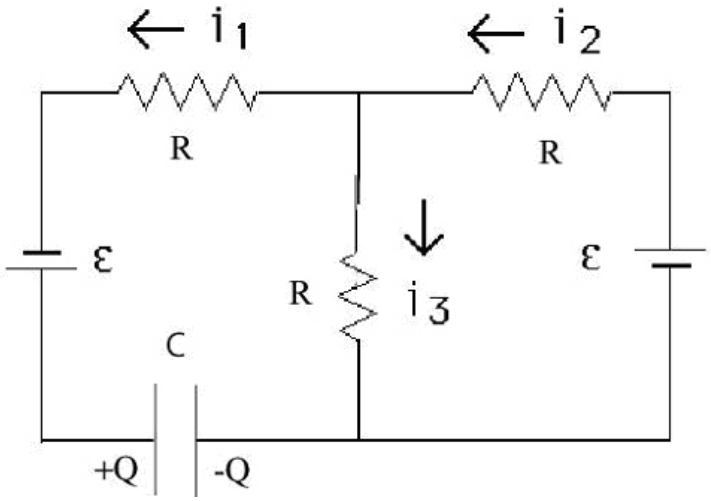
Question 10: the currents I_1 and I_2 (through R_1 and R_2 respectively) at $t=T^+$?

Question 11: Using Kirchhoff's Loop Rules, obtain a differential equation for the charge *q* on the capacitor after the switch has been opened, assuming $R_1=R_2=R$ (in other words, the only current in the equation should be the current through the capacitor, which can be rewritten in terms of d*q*/dt).

Question 12: What is the time constant for discharging the capacitor?

Question 13: Write an equation for the time dependence of the charge on the capacitor after time T.

Sample Exam Question (If time, try to do it by yourself, closed notes)



(a) From Kirchoff's first rule, what is the relation between i_1, i_2, and i_3?

(b) What does the loop theorem (Kirchhoff's second rule) yield if we traverse the left loop of the above circuit *moving counterclockwise*, in terms of the quantities shown, with the directions of the currents as shown?

(c) What does the loop theorem (Kirchhoff's second rule) yield if we traverse the right loop of the above circuit *moving counterclockwise*, in terms of the quantities shown, with the directions of the currents as shown?

(d) After a very long time, $t >> RC$, what is the current i_1?

(e) After a very long time, $t >> RC$, what are the currents i_2 and i_3?

(f) After a very long time, $t >> RC$, what is the voltage across the capacitor in terms of the quantities given? (Hint: use your results from part (b)-(e)).

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Tear off this page and turn it in at the end of class !!!!

Note:
Writing in the name of a student who is not present is a COD offense.

Problem Solving 9: Circuits

Group _____ (**e.g. L02 6A Please Fill Out)**

Names _____

_____

_____

Question 1: What is the current I_C (through the capacitor) at $t=0^+$ (just after switch is closed)?

Question 2: What are the currents I_1 and I_2 (through R_1 and R_2 respectively) at $t=0^+$?

Question 3: What is the current I_C (through the capacitor) at $t=\infty$?

Question 4: What are the currents I_1 and I_2 (through R_1 and R_2 respectively) at $t=\infty$?

Question 6: Using Kirchhoff's Loop Rules, obtain a differential equation for the charge q on the capacitor, assuming $R_1=R_2=R$ (in other words, the only current in the equation should be the current through the capacitor, which can be rewritten in terms of dq/dt).

Question 7: What is the time constant for charging the capacitor?

Question 8: Write an equation for the time dependence of the charge on the capacitor

Question 9: What is the current I_C (through the capacitor) at $t=T^+$ (just after switch is opened)?

Question 10: What are the currents I_1 and I_2 (through R_1 and R_2 respectively) at $t=T^+$?

Question 11: Using Kirchhoff's Loop Rules, obtain a differential equation for the charge *q* on the capacitor after the switch has been opened, assuming $R_1=R_2=R$ (in other words, the only current in the equation should be the current through the capacitor, which can be rewritten in terms of dq/dt).

Question 12: What is the time constant for discharging the capacitor?

Question 13: Write an equation for the time dependence of the charge on the capacitor after time T.