Class 10: Outline

Hour 1:

DC Circuits

Hour 2:

Kirchhoff's Loop Rules

Last Time:
Capacitors & Dielectrics

Capacitors & Dielectrics

Capacitance

$$C = \frac{Q}{|\Delta V|}$$

To calculate:

1) Put on arbitrary ±Q
2) Calculate E
3) Calculate ΔV

Energy

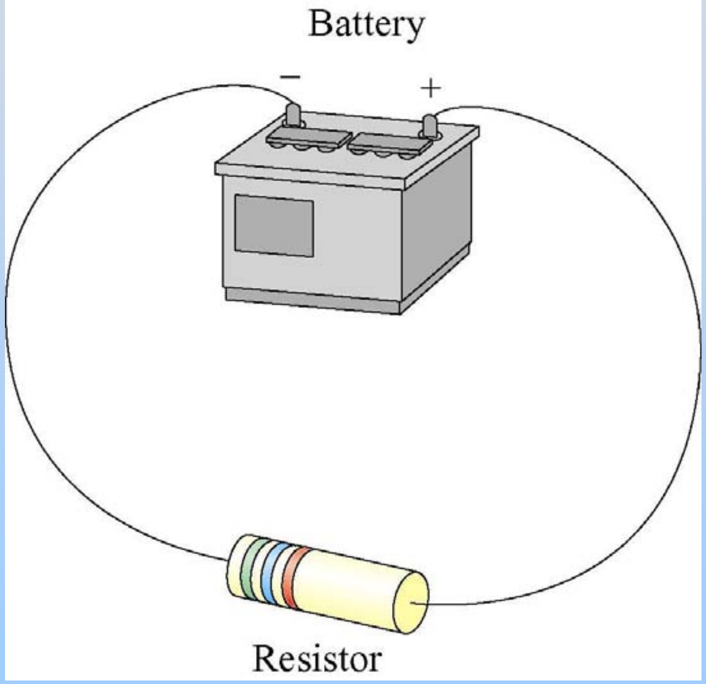
$$U = \frac{Q^2}{2C} = \frac{1}{2} Q|\Delta V| = \frac{1}{2} C|\Delta V|^2 = \iiint u_E \, d^3 r = \iiint \frac{\varepsilon_o E^2}{2} d^3 r$$

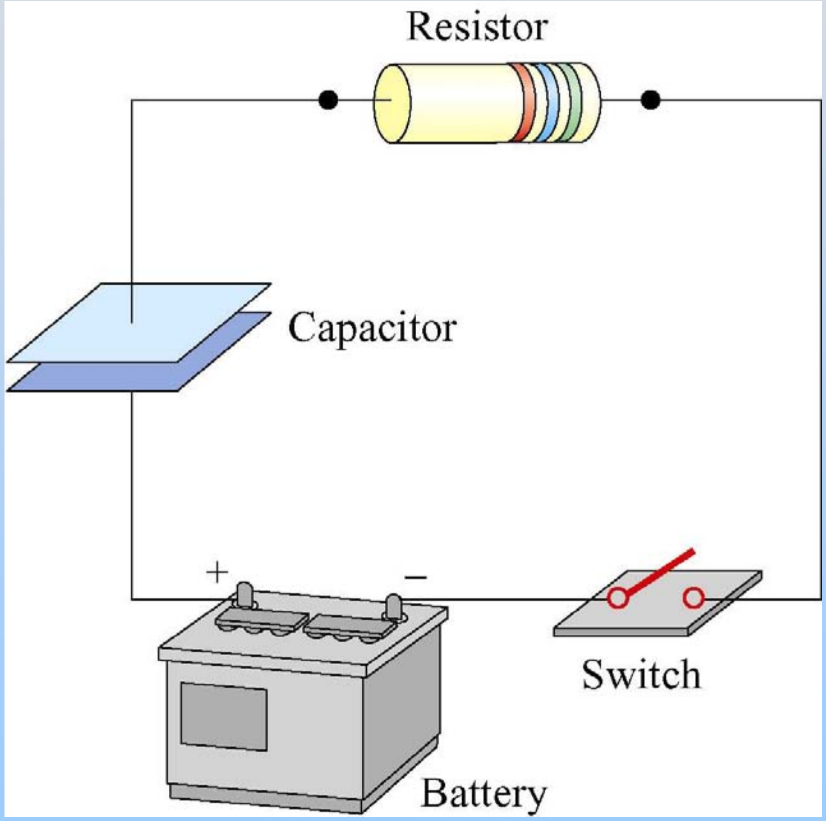
Dielectrics

$$\oint\!\!\oint_S \kappa \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{inside}^{free}}{\varepsilon_0} \Rightarrow C_{\text{Filled with Dielectric}} = \kappa C_0$$

This Time:
DC Circuits

Examples of Circuits





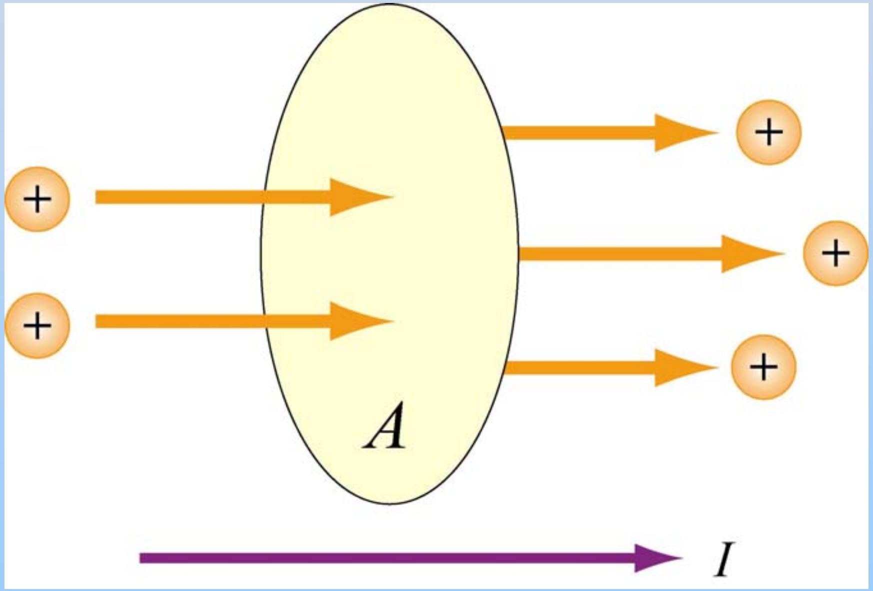
Current: Flow Of Charge

Average current I_{av}: Charge ΔQ flowing across area A in time Δt

$$I_{av} = \frac{\Delta Q}{\Delta t}$$

Instantaneous current: differential limit of I_{av}

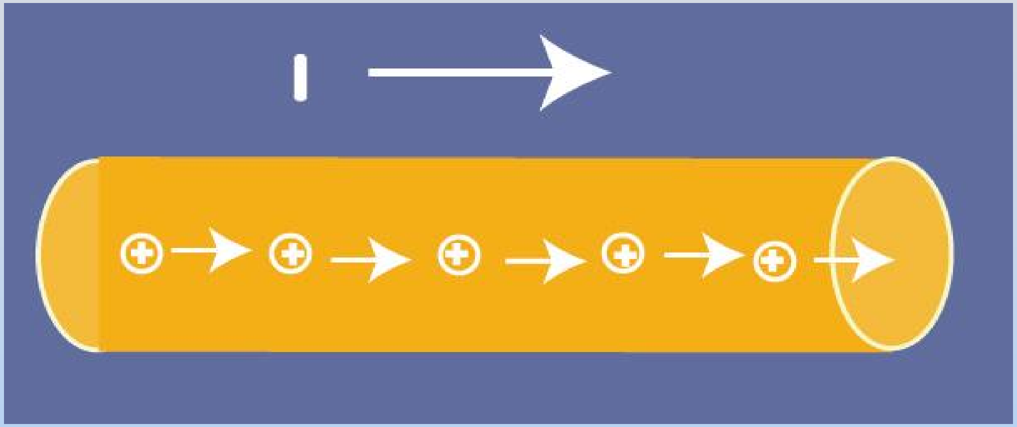
$$I = \frac{dQ}{dt}$$



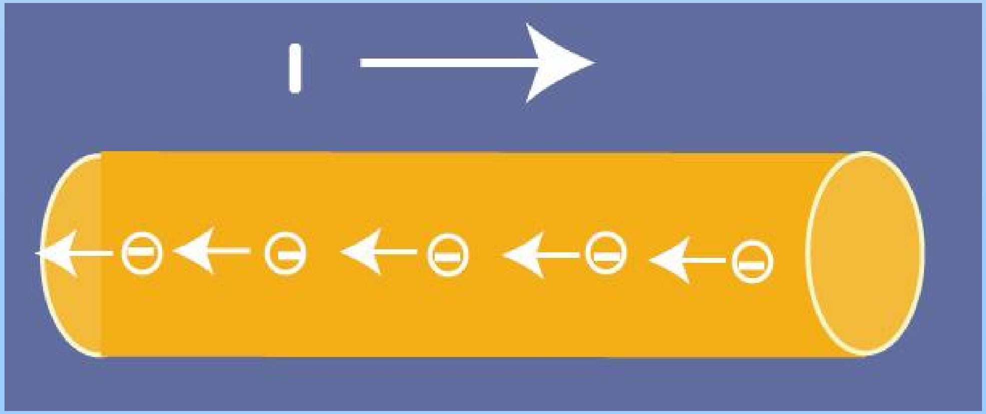
Units of Current: Coulombs/second = Ampere

Direction of The Current

Direction of current is direction of flow of pos. charge



or, opposite direction of flow of negative charge

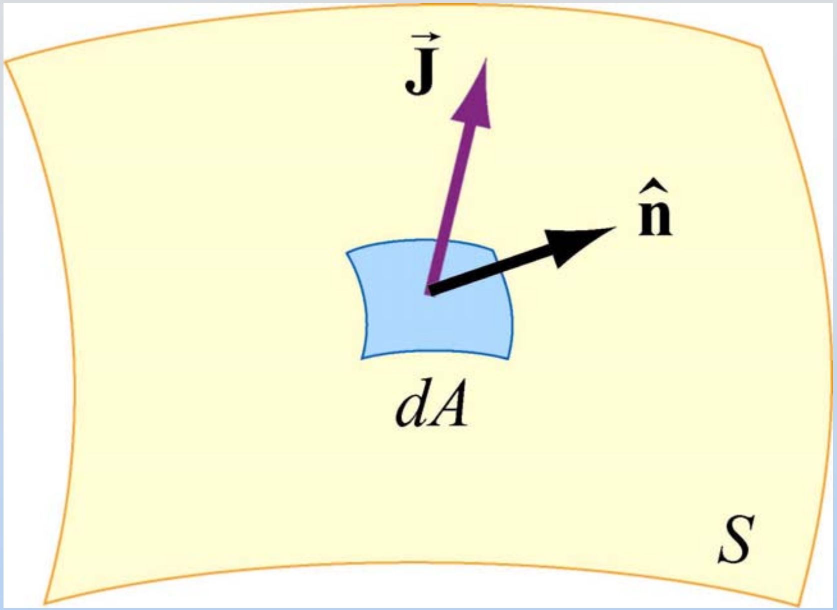


Current Density J

J: current/unit area

$$\vec{\mathbf{J}} \equiv \frac{I}{A}\hat{\mathbf{I}}$$

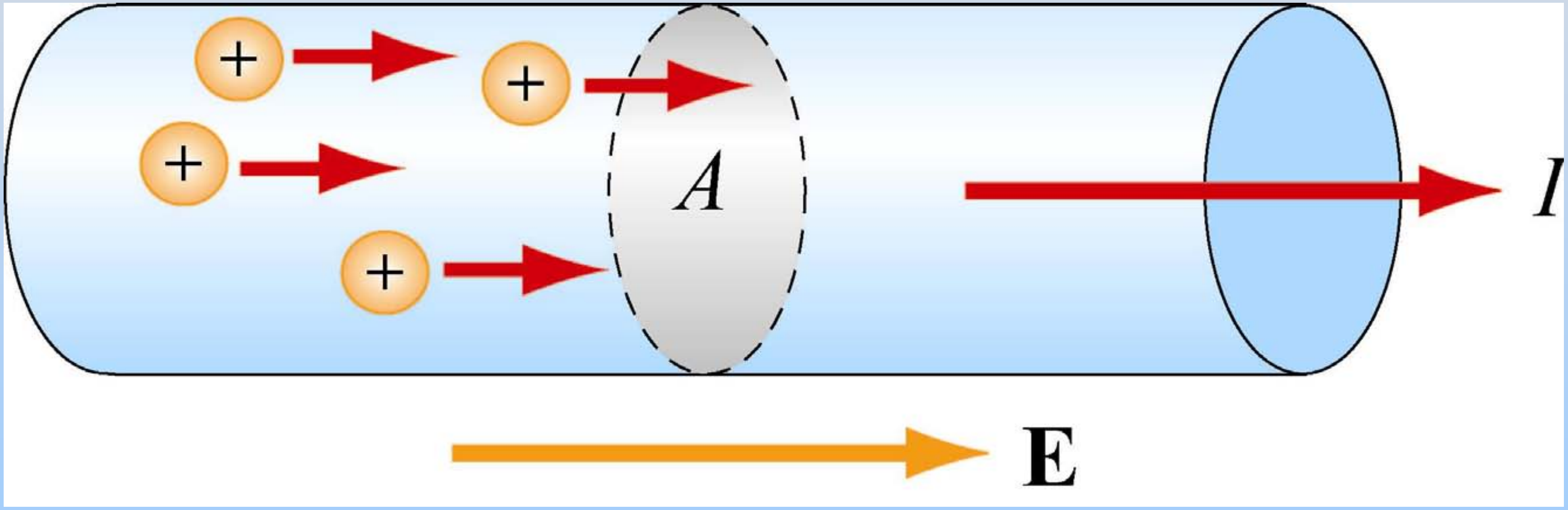
$\hat{\mathbf{I}}$ points in direction of current



$$I = \int_S \vec{\mathbf{J}} \cdot \hat{\mathbf{n}}\, dA = \int_S \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}}$$

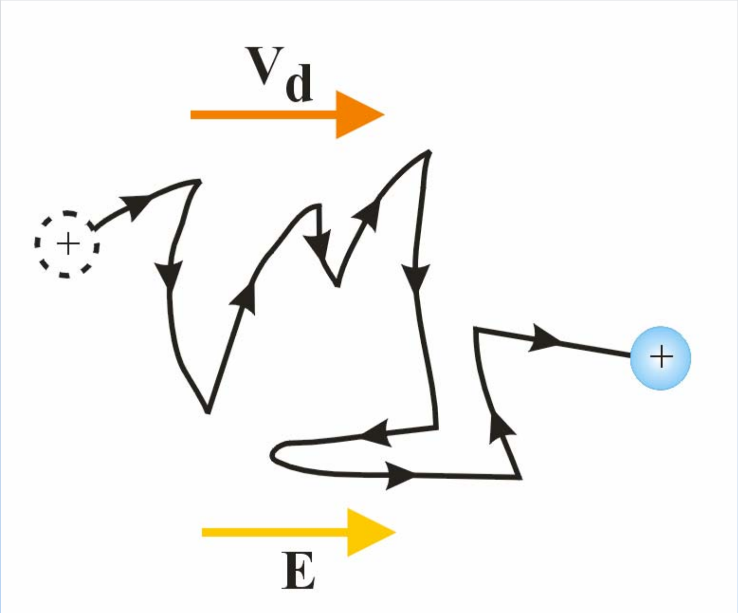
Why Does Current Flow?

If an electric field is set up in a conductor, charge will move (making a current in direction of E)



Note that when current is flowing, the conductor is not an equipotential surface (and $E_{inside} \neq 0$)!

Microscopic Picture



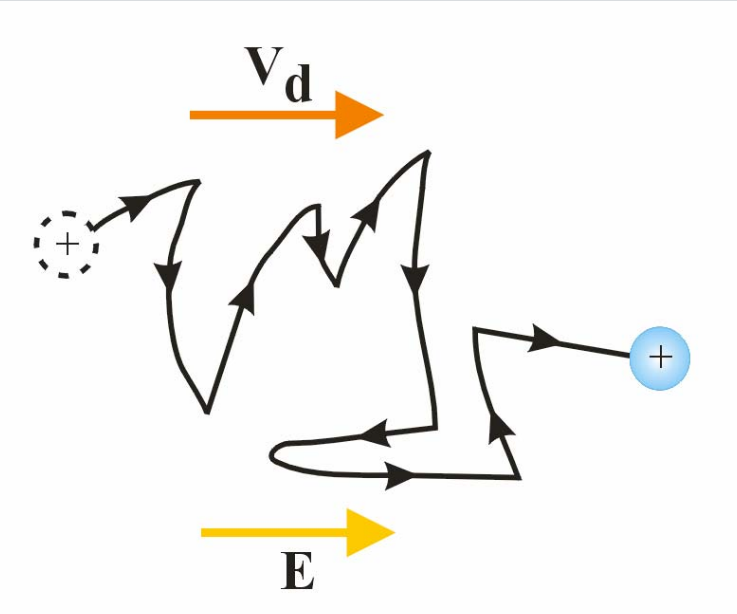
Drift speed is velocity forced by applied electric field in the presence of collisions.

It is typically 4×10^{-5} m/sec, or 0.04 mm/second!

To go one meter at this speed takes about 10 hours!

How Can This Be?

Conductivity and Resistivity



Ability of current to flow depends on density of charges & rate of scattering

Two quantities summarize this:

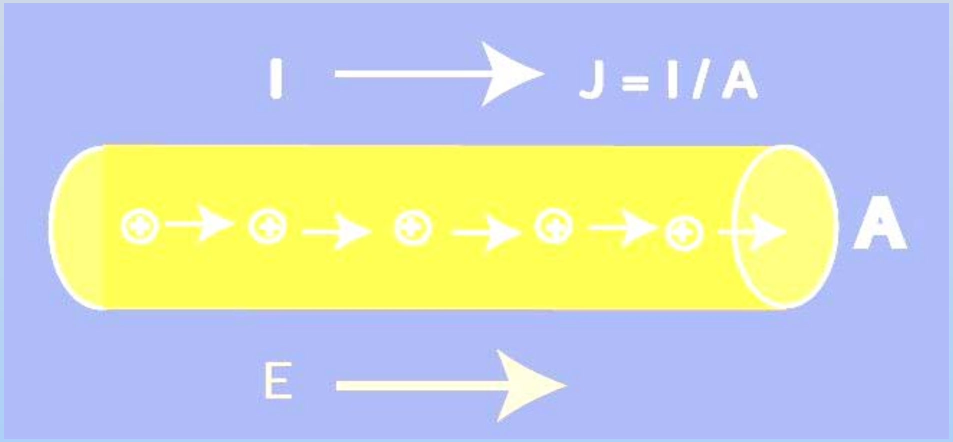
σ: conductivity

ρ: resistivity

Microscopic Ohm's Law

$$\vec{\mathbf{E}} = \rho \vec{\mathbf{J}} \quad \text{or} \quad \vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}$$

$$\rho \equiv \frac{1}{\sigma}$$



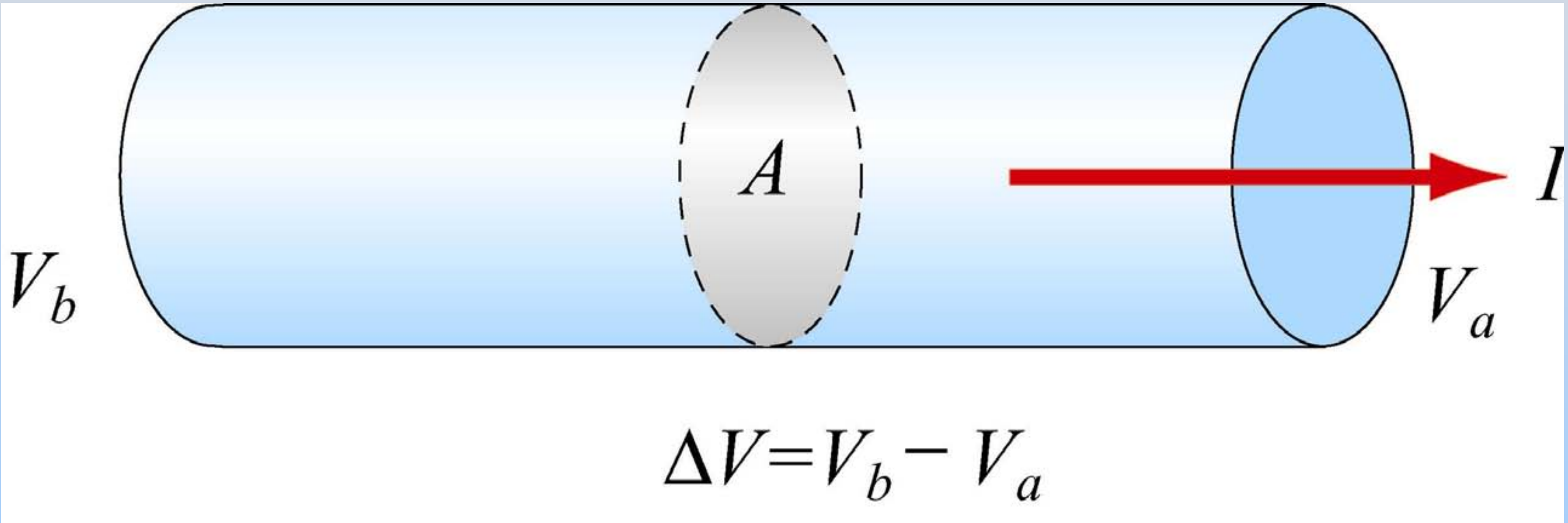
ρ and σ depend only on the microscopic properties of the material, not on its shape

Demonstrations: Temperature Effects on ρ

PRS Questions: Resistance?

Why Does Current Flow?

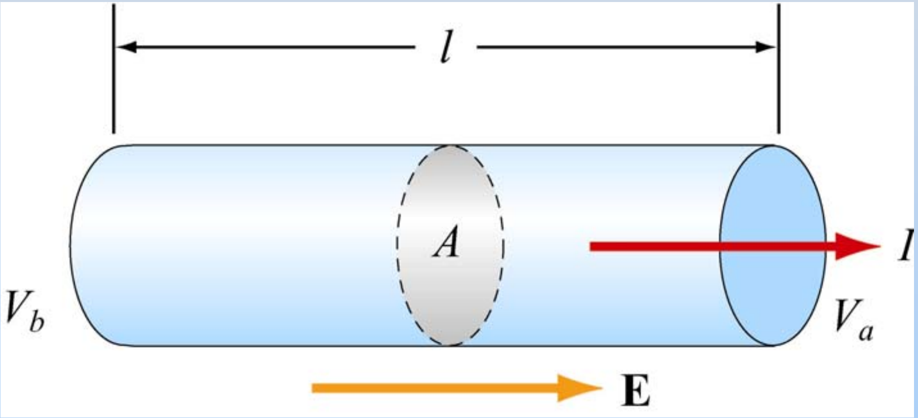
Instead of thinking of Electric Field, think of potential difference across the conductor



Ohm's Law

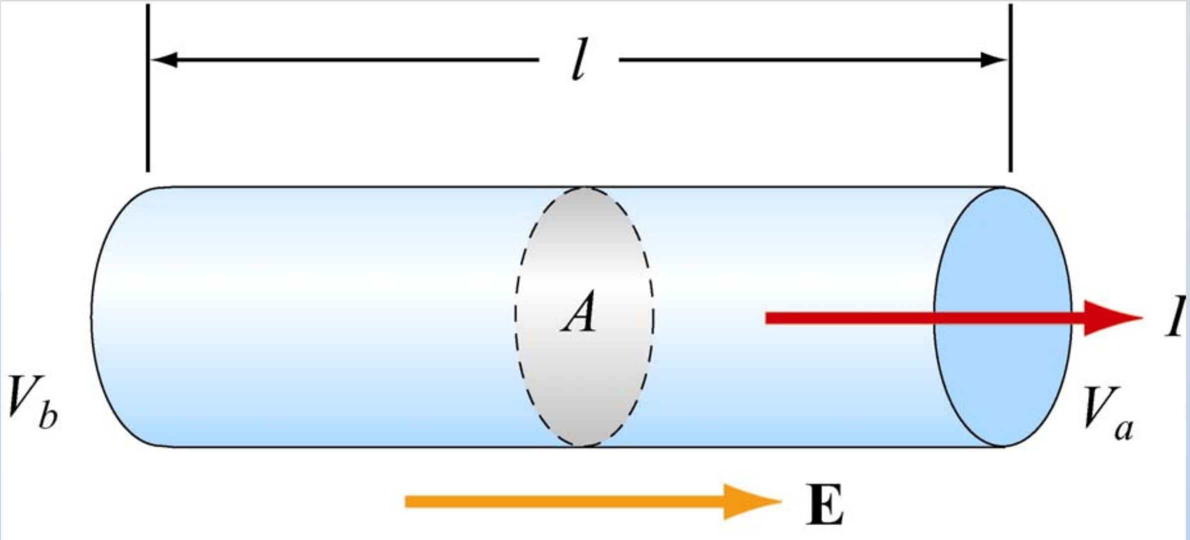
What is relationship between ΔV and current?

$$\Delta V = V_b - V_a = -\int_a^b \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}} = E\ell$$



$$\left.\begin{aligned} J &= \frac{E}{\rho} = \frac{\Delta V / \ell}{\rho} \\ J &= \frac{I}{A} \end{aligned}\right\} \Rightarrow \Delta V = I\left(\frac{\rho\ell}{A}\right) \equiv IR$$

Ohm's Law

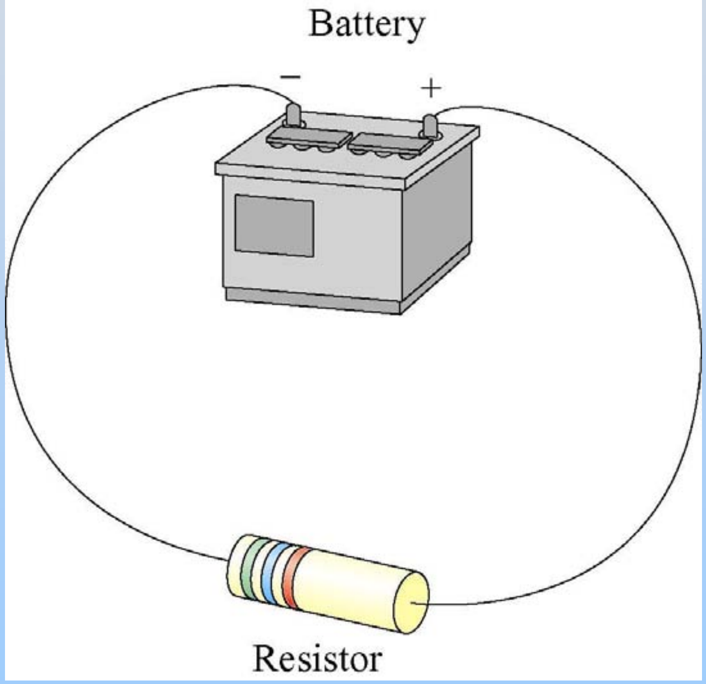


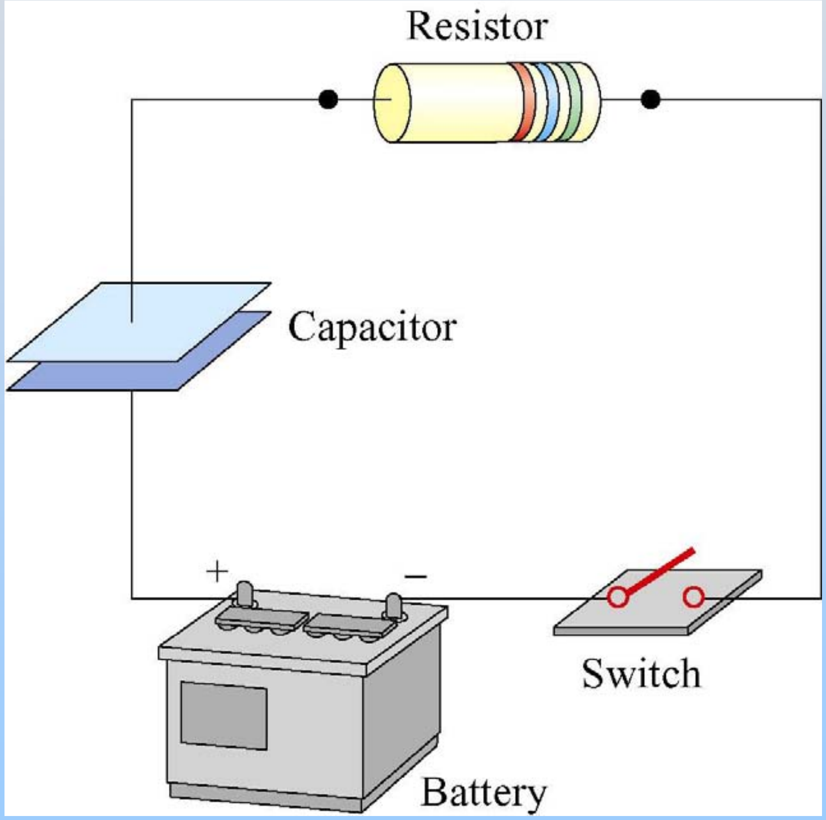
$$\Delta V = IR$$

$$R = \frac{\rho \ell}{A}$$

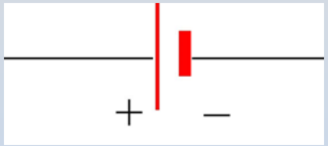
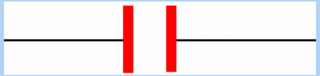
R has units of Ohms (Ω) = Volts/Amp

Examples of Circuits



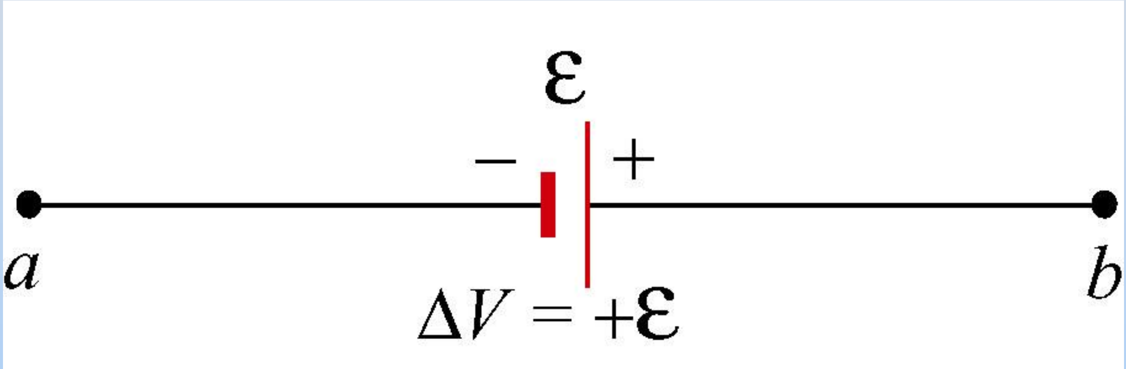


Symbols for Circuit Elements

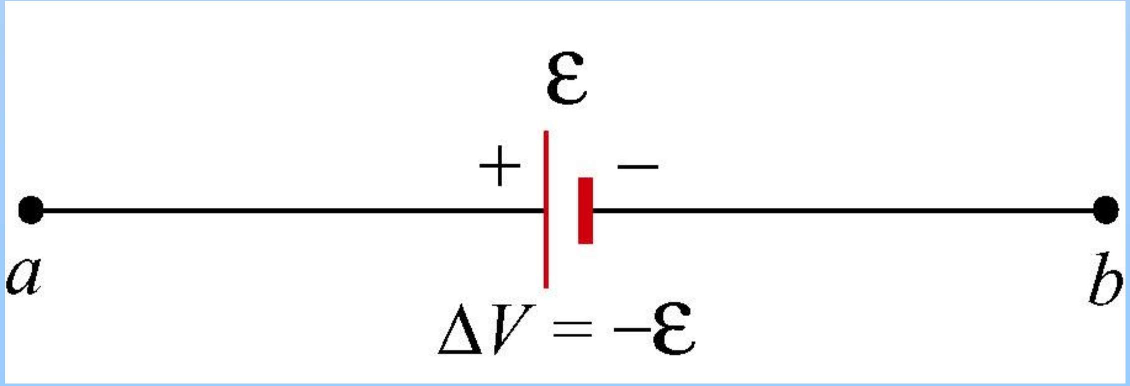
Battery	
Resistor	
Capacitor	
Switch	

Sign Conventions - Battery

Moving from the negative to positive terminal of a battery **increases** your potential



$$\Delta V = V_b - V_a$$

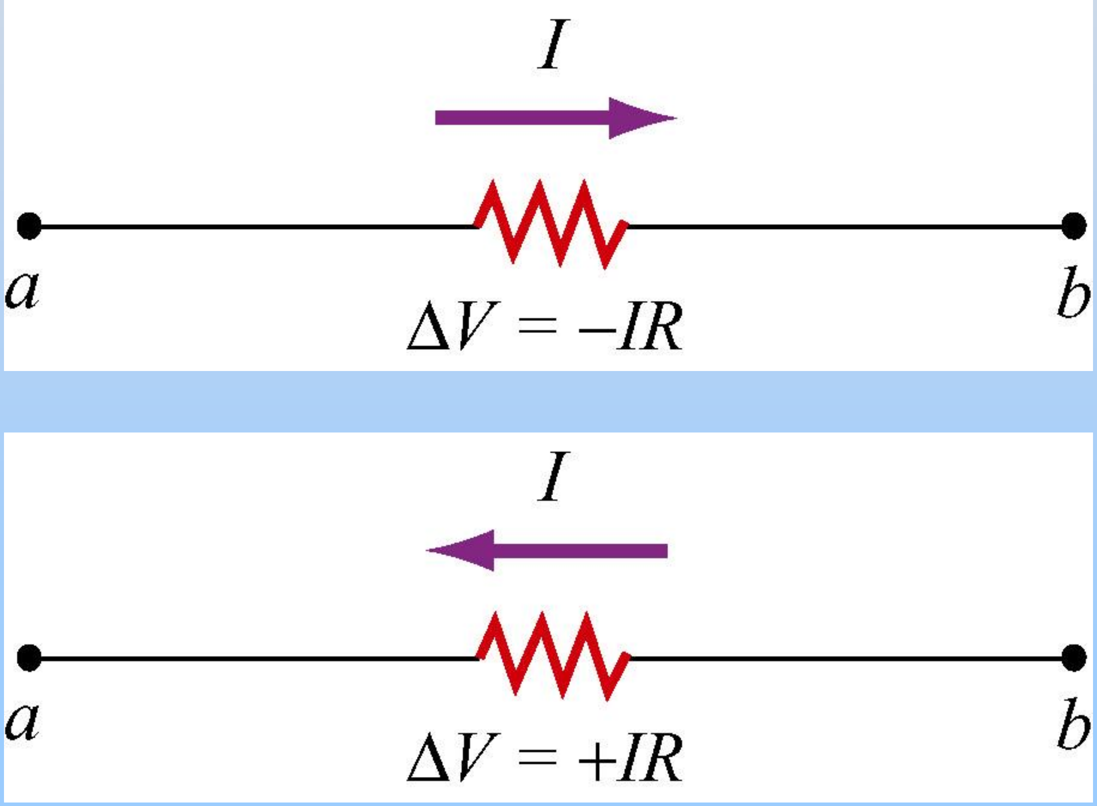


Think:

Ski Lift

Sign Conventions - Resistor

Moving across a resistor in the direction of current **decreases** your potential



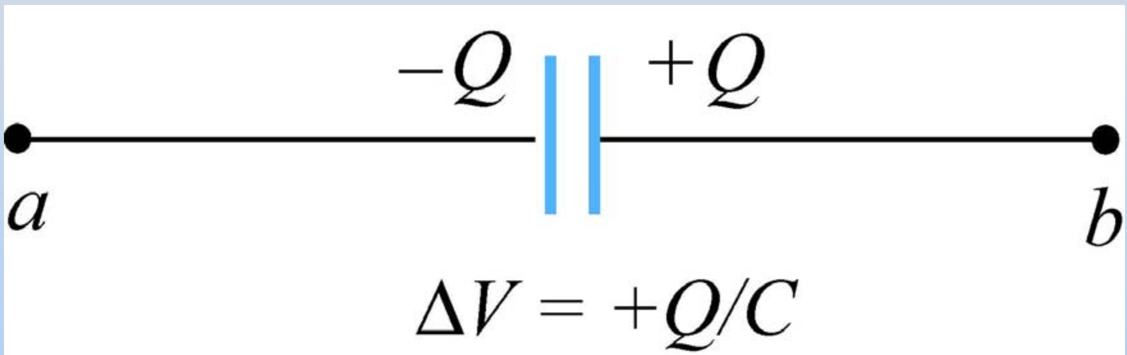
$$\Delta V = V_b - V_a$$

Think:

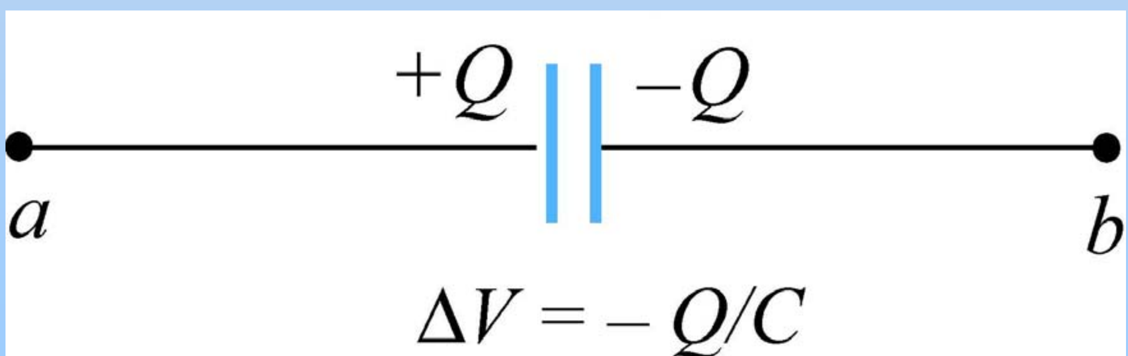
Ski Slope

Sign Conventions - Capacitor

Moving across a capacitor from the negatively to positively charged plate **increases** your potential



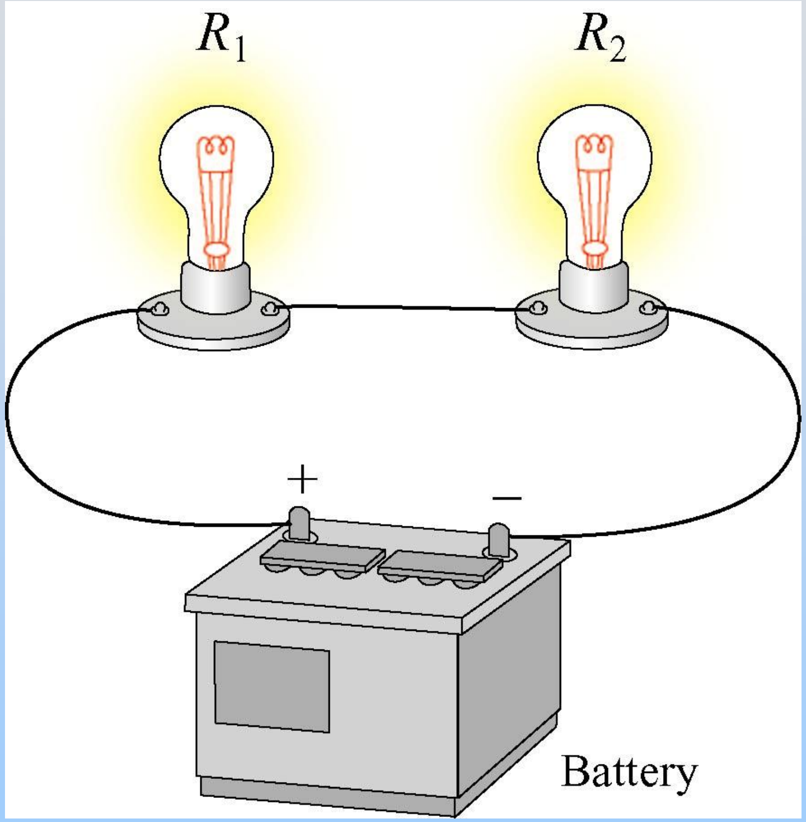
$$\Delta V = V_b - V_a$$

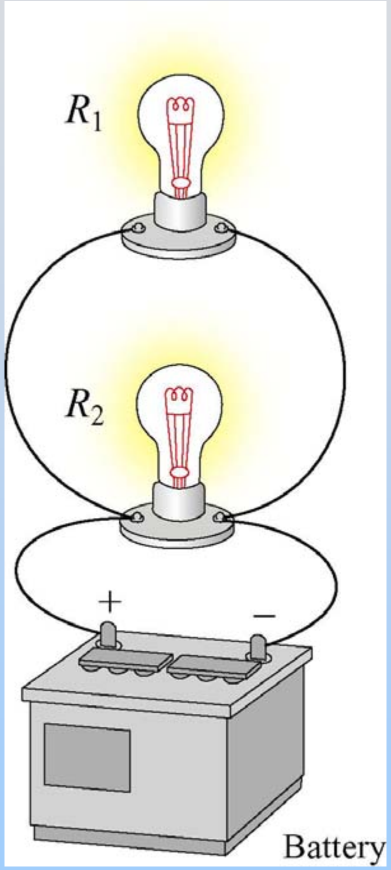


Think:

Ski Lodge

Series vs. Parallel

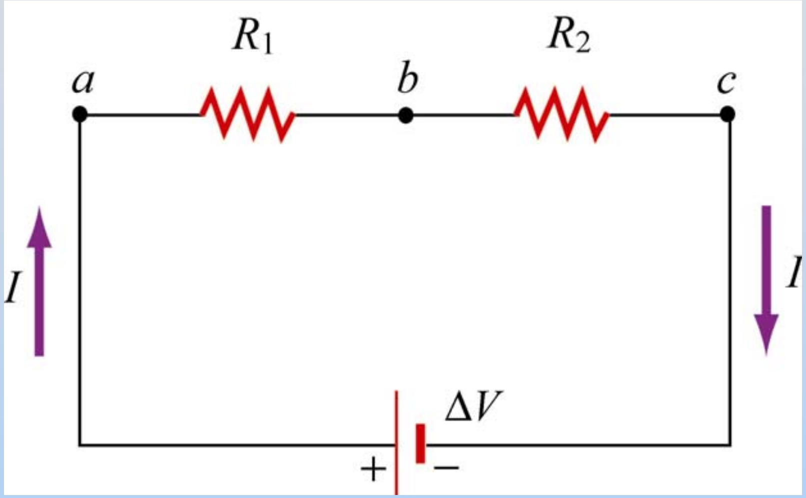


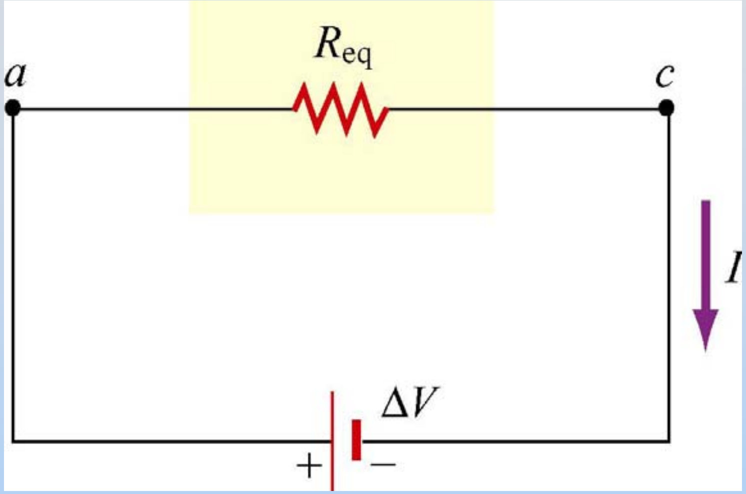


Series **Parallel**

Resistors In Series

The same current *I* must flow through both resistors



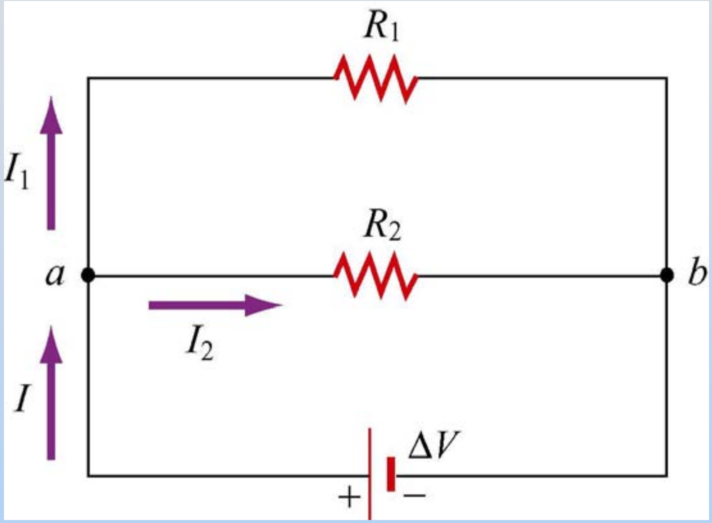


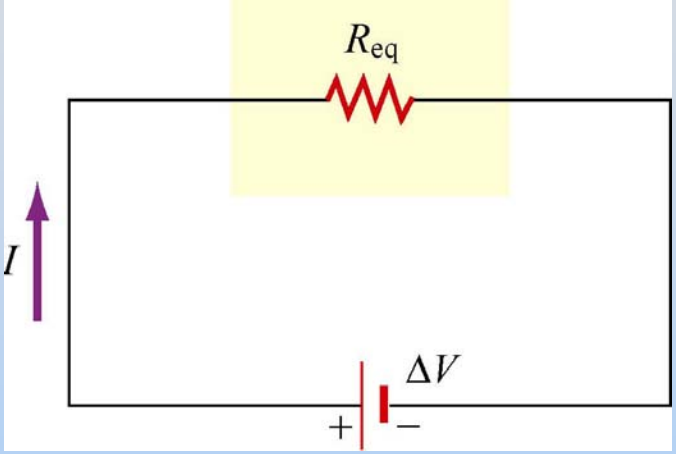
$$\Delta V = I\,R_1 + I\,R_2 = I(R_1 + R_2) = I\,R_{eq}$$

$$\boxed{R_{eq} = R_1 + R_2}$$

Resistors In Parallel

Voltage drop across the resistors must be the same





$$\Delta V = \Delta V_1 = \Delta V_2 = I_1 R_1 = I_2 R_2 = I R_{eq}$$

$$I = I_1 + I_2 = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \frac{\Delta V}{R_{eq}}$$

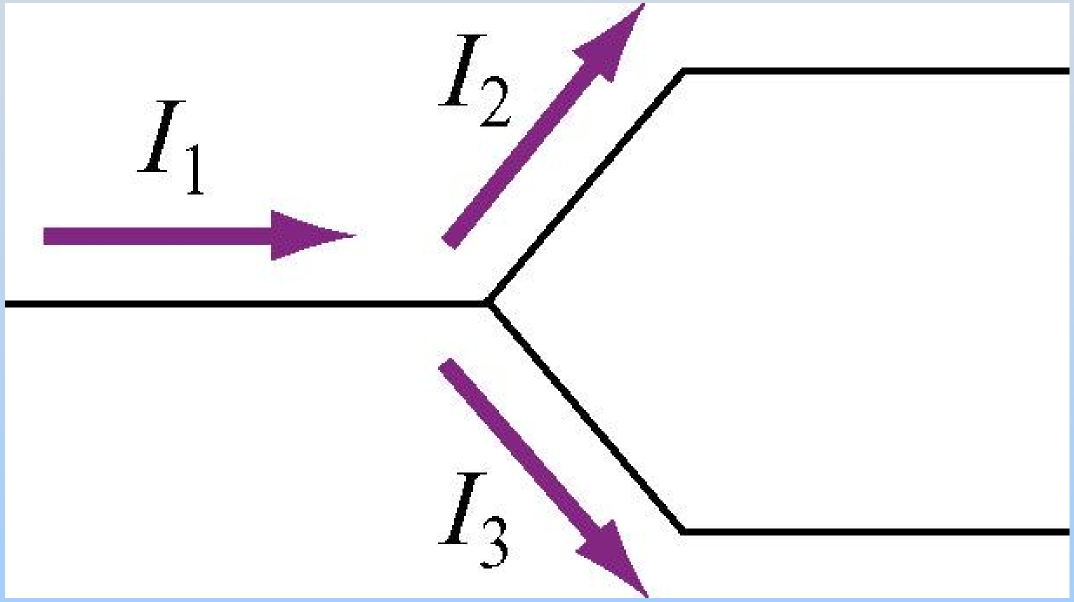
$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}}$$

PRS Questions: Light Bulbs

Kirchhoff's Loop Rules

Kirchhoff's Rules

1. Sum of currents entering any junction in a circuit must equal sum of currents leaving that junction.

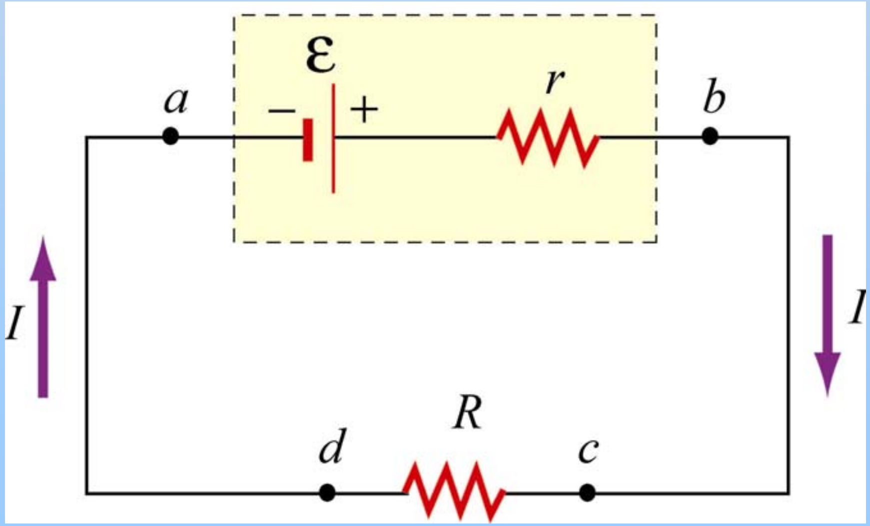


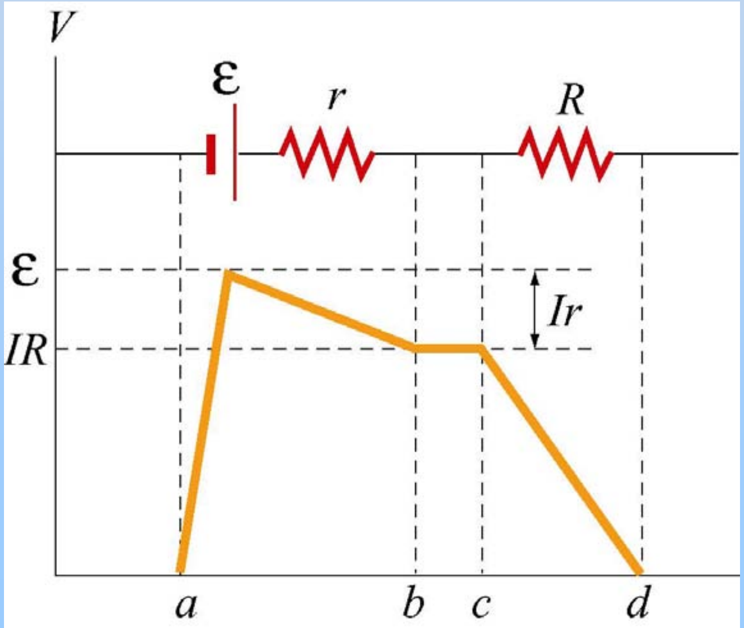
$$I_1 = I_2 + I_3$$

Kirchhoff's Rules

2. Sum of potential differences across all elements around any closed circuit loop must be zero.

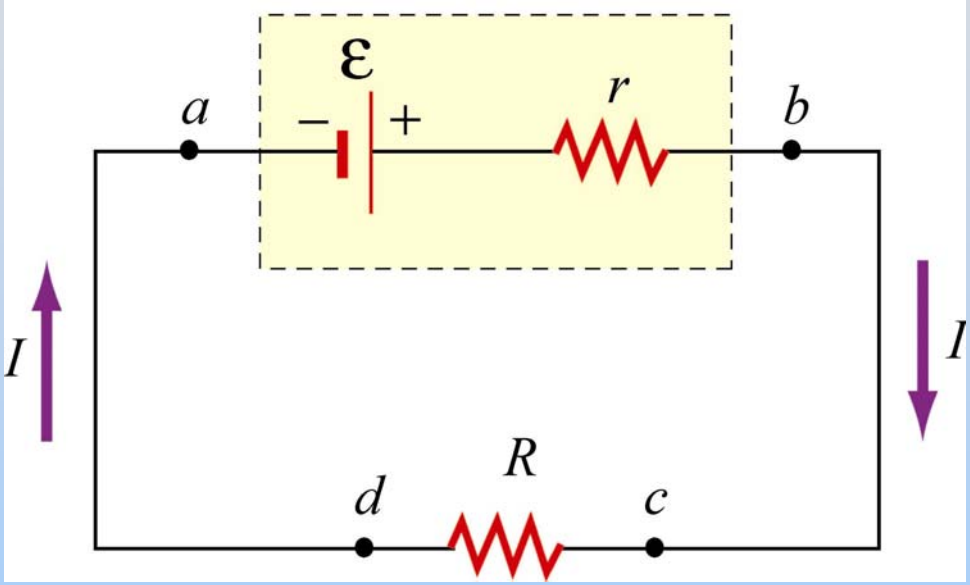
$$\Delta V = -\oint_{\substack{Closed\\Path}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = 0$$





Internal Resistance

Real batteries have an internal resistance, r, which is small but non-zero



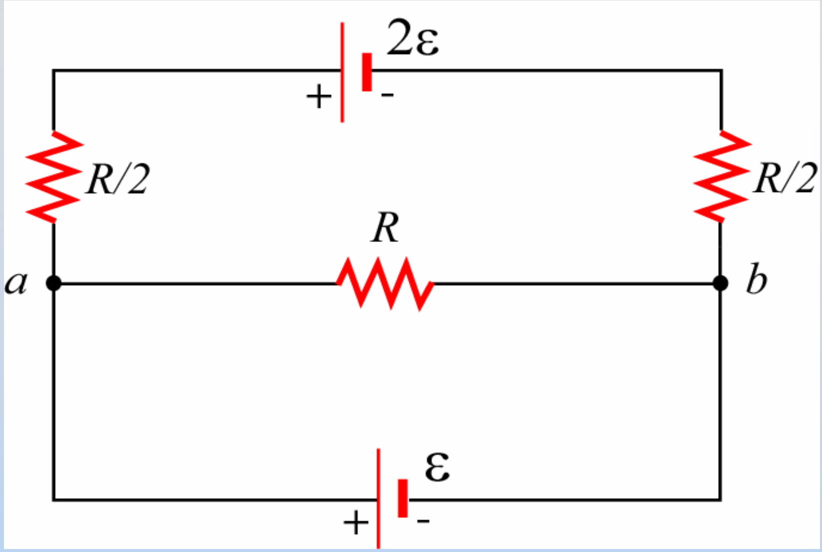
Terminal voltage: $\Delta V = V_b - V_a = \mathcal{E} - I\,r$

(Even if you short the leads you don’t get infinite current)

Steps of Solving Circuit Problem

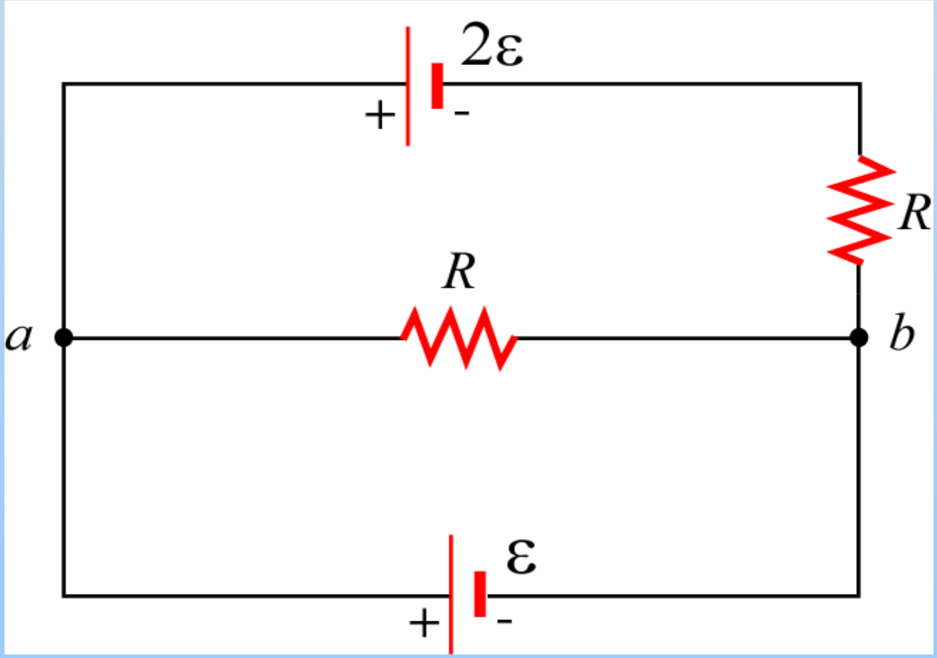
1. Straighten out circuit (make squares)
2. Simplify resistors in series/parallel
3. Assign current loops (arbitrary)
4. Write loop equations (1 per loop)
5. Solve

Example: Simple Circuit

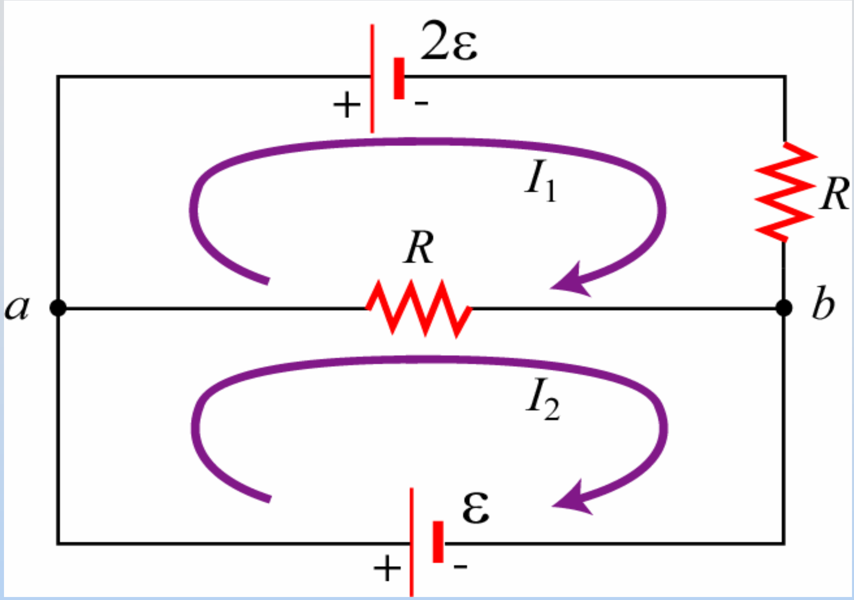


You can simplify resistors in series (but don't need to)

What is current through the bottom battery?



Example: Simple Circuit



Start at a in both loops

Walk in direction of current

$$-2\varepsilon - I_1 R - (I_1 - I_2) R = 0$$

$$-(I_2 - I_1) R + \varepsilon = 0$$

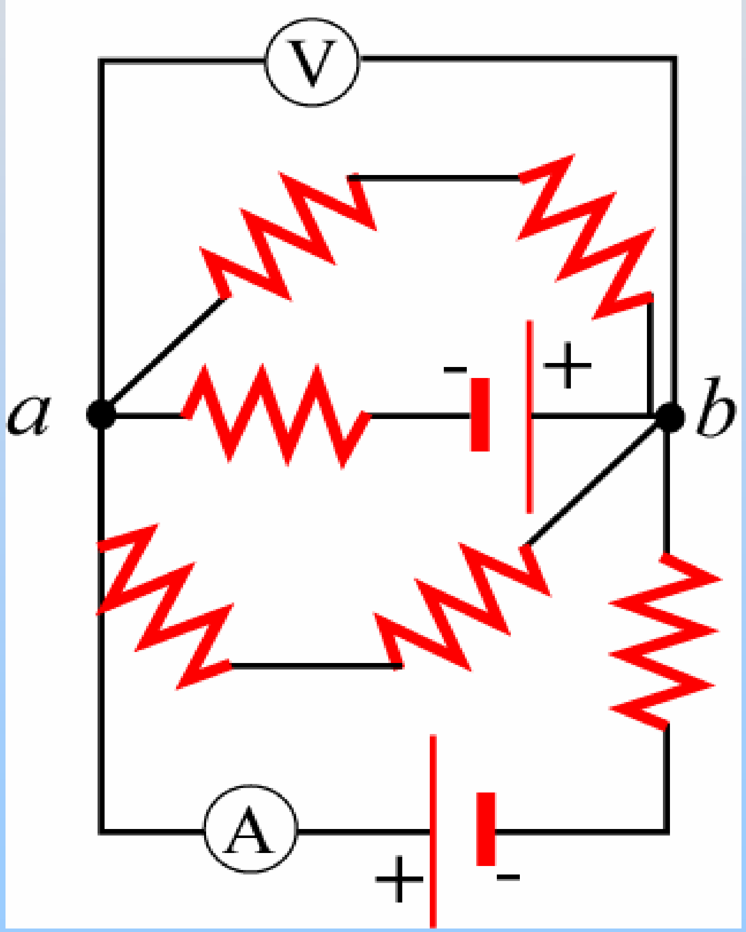
Add these: $-2\varepsilon - I_1 R + \varepsilon = 0 \rightarrow I_1 = \dfrac{-\varepsilon}{R}$

We wanted I_2: $(I_2 - I_1) R = \varepsilon \rightarrow I_2 = \dfrac{\varepsilon}{R} + I_1$

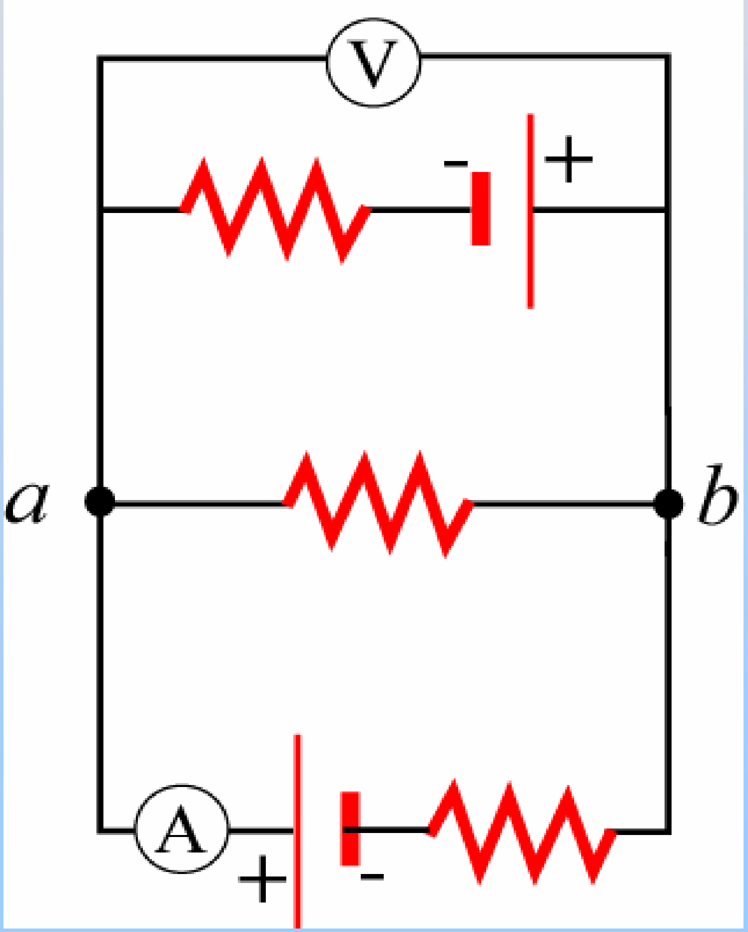
$$\boxed{I_2 = 0}$$

Group Problem: Circuit

Find meters' values. All resistors are *R*, batteries are $\mathcal{E}$



HARDER



EASIER

Power

Electrical Power

Power is change in energy per unit time

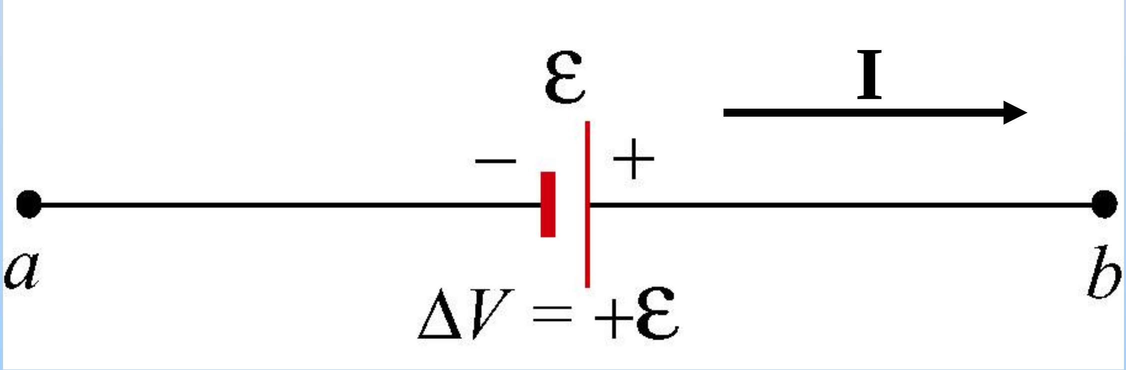
So power to move current through circuit elements:

$$P = \frac{d}{dt}U = \frac{d}{dt}\left(q\Delta V\right) = \frac{dq}{dt}\Delta V$$

$$\boxed{P = I\,\Delta V}$$

Power - Battery

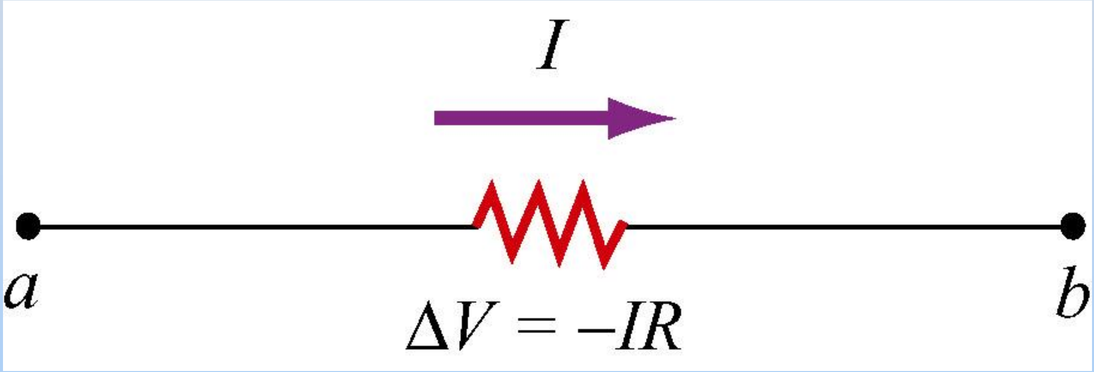
Moving from the negative to positive terminal of a battery **increases** your potential. If current flows in that direction the battery **supplies** power



$$P_{\text{supplied}} = I\,\Delta V = I\,\varepsilon$$

Power - Resistor

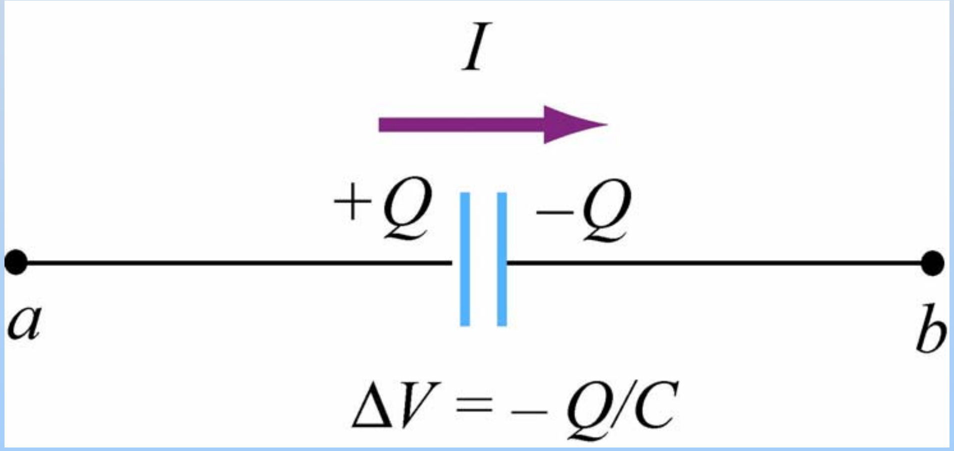
Moving across a resistor in the direction of current **decreases** your potential. Resistors always **dissipate** power



$$P_{\text{dissipated}} = I\,\Delta V = I^2 R = \frac{\Delta V^2}{R}$$

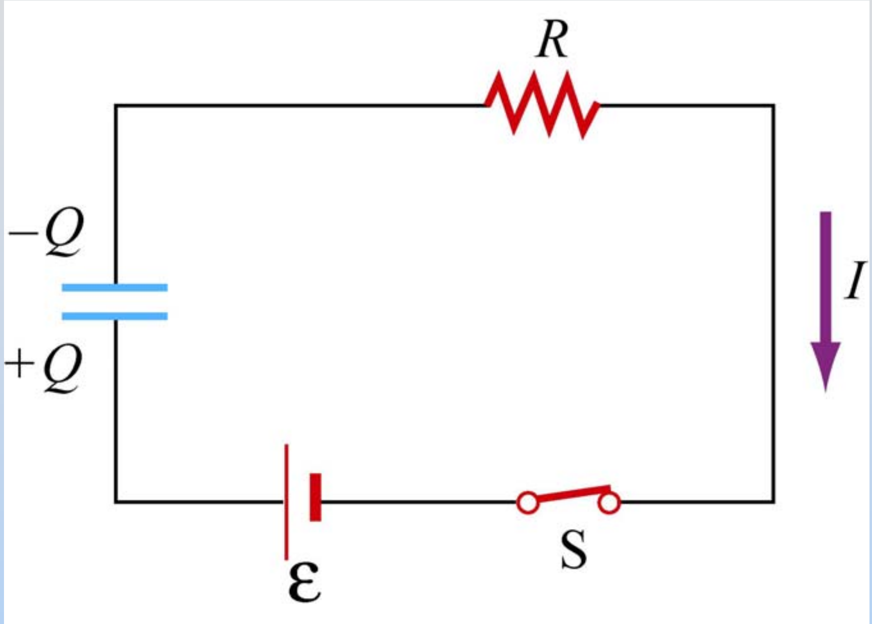
Power - Capacitor

Moving across a capacitor from the positive to negative plate **decreases** your potential. If current flows in that direction the capacitor **absorbs** power (stores charge)



$$P_{\text{absorbed}} = I\,\Delta V = \frac{dQ}{dt}\frac{Q}{C} = \frac{d}{dt}\frac{Q^2}{2C} = \frac{dU}{dt}$$

Energy Balance



$$\mathcal{E} - \frac{Q}{C} - IR = 0$$

Multiplying by *I*:

$$\mathcal{E}I = I^2R + \frac{Q}{C}\frac{dQ}{dt} = I^2R + \frac{d}{dt}\left(\frac{1}{2}\frac{Q^2}{C}\right)$$

(power delivered by battery) = (power dissipated through resistor)
+ (power absorbed by the capacitor)

PRS Questions: More Light Bulbs