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8.02 Electricity and Magnetism, Spring 2002
Transcript – Lecture 22

When I expose material to an external magnetic field, then we learned last time that the field inside that material is modified.

And we expressed that in terms of an equation, that the field inside the material is kappa of M, which is called the relative permeability, times the external field, and I will refer to that all the time as the vacuum field.

And when we have diamagnetic material, kappa M is just a hair smaller than one; with paramagnetic material, it is a hair larger than one; but when we have ferromagnetic material it can be huge.

It can be thousands, 10 thousands, and even higher.

Now in the case of para- and ferromagnetic material, the kappa of M is the result of the fact that the intrinsic dipoles of the atoms and the molecules are going to be aligned by the external field.

And today I want to raise the question, how large can the magnetic dipole moment of a single atom be?

And then comes the logical question, how strong can we actually, then, have a field inside ferromagnetic material?

That means if we were able to align all the dipole moments of all the atoms, what is the maximum that we can achieve?

To calculate the magnetic dipole moment of an atom, you have to do some quantum mechanics, and that's beyond the scope of this course.

And so I will derive it in a classical way, and then at the very end I will add a little pepper and salt, which is quantum mechanics, just to make the result right.

But it can be done in a classical way, and it can give you a very good, good idea.

If I have a hydrogen atom, which has a proton at the center, has a charge plus e -- e is the charge of the electron but this is the plus charge.

And let this have an orbit R, circular orbit.

And the electron here e, I'll give it the minus sign to make sure that you know that it's negative.

Say the electron goes around in this direction.

This is the velocity of the electron.

That means that of course the current around the proton would then be in this direction.

If an electron goes like this, the current goes like that, that's just by convention.

The mass of an electron -- you should know that by now -- is approximately 9.1 times 10 to the -31 kilograms.

The charge of the electron, 1.6 times 10 to the -19 coulombs.

And the radius of the orbit in a hydrogen atom -- it's often called the Bohr radius, by the way -- is approximately 5 times 10 to the -11 meters.

We're going to need these numbers.

That's why I write them down for you.

If you look at this current running around the proton, it's really a current which the current, say, goes in this direction.

And here is that proton -- trying to make you see three dimensionally.

Then it creates a magnetic field in this direction, and so the magnetic dipole moment mu is up.

And the magnitude of that magnetic dipole moment, as we learned last time, is simply this current I times the area A of this current loop.

Now the area A is trivial to calculate.

That's pi R square, R being the radius of the orbit.

And so A, that's the easiest, is pi R squared.

And if I use my 5 times 10 to the minus 11, then I find that this area is 8 times 10 to the -21 square meters.

So that's easy.

But now comes the question, what is I?

What is the current?

So now we have to do a little bit more work.

And we have to combine our knowledge of, uh 8.02 with our knowledge of 8.01.

If this electron goes around, the reason why it goes around is that the proton and the electron attract each other.

And so there is a force in this direction.

And we know that force, that's the Coulomb force.

It's an electric force.

That force is this charge times this charge, so that's e squared, divided by our famous 4 pi epsilon 0, and then we have to divide it by the radius squared.

So that's Coulomb's Law.

But from 8.01, from Newtonian mechanics, we know that this is what we call the centripetal force that holds it in orbit, so to speak, and that is M V squared, M being the mass of the electron, V being the speed of the electron, and V squared divided by R.

And so this allows me to calculate as a first step, before we get into the current, what the velocity of this electron is.

It's phenomenal.

It's an incredible speed.

So V then becomes -- I lose one R -- so I get the square root, I get an e squared upstairs here, my M goes downstairs, I have 4 pi epsilon 0, and I have here an R.

And I know all these numbers.

I know what e is, I know what capital R is, I know what 4 pi epsilon 0 is -- one over 4 pi epsilon 0 is the famous 9 to the power -- 9 times 10 to the power 9.

And so I can calculate what V is.

And if I stick in the numbers and if I did not make a mistake, then I find about 2.3 times 10 to the 6 meters per second.

It's an immensely high speed, 5 million miles per hour.

If this were a straight line, you would make it to the moon in three minutes.

5 million miles per hour this electron goes around the proton.

Now I have to go to the current.

I have to find out what the current is.

So the question that I'm going to ask now is how long does it take for this electron to go around.

Well, that time, capital T, is of course the circumference of my circle divided by the speed of the electron.

Trivial.

Even the high school students in my audience will understand that one, I hope.

And so I know what 2 pi R is, because I know R and I know V and so I can calculate that time, just by sticking in the numbers.

And I find that it is about 1.14 times 10 to the -16 seconds.

Just imagine how small that time is.

You cannot even -- we cannot even imagine what it's like.

It goes 10 to the 16 times per second around, because it has this huge speed.

The 1.14 times 10 to the -16 really should have been 1.4 times 10 to the -16.

Of course, it doesn't make much difference, but in case you substitute in the numbers, it is 1.4 times 10 to the -16.

Now, we still haven't found the current, but we're almost there.

Because when you look here, there is this electron going by, and every 1.14 10 to the -16 seconds, that electron goes by.

So the current I, that's the definition of current, is the charge per unit time.

And so every capital T seconds, the charge e goes by, and so this is per definition the current.

And so this current, then, that you have, which is simply due to the electron going around the proton, is about 1.1 times 10 to the -3 amperes.

And that is mind-boggling.

A milliampere.

One electron going around a proton represents a current of a milliampere.

And now of course I have the magnetic moment mu, that is I times A.

We already calculated A, and now we also have the current I, and so we now get that mu is approximately 9.3, if you put in all the decimals correctly, times 10 to the -24, and the unit is of course amperes square meters.

This is area, and this is current.

This A has nothing to do with that A, hey.

This is amperes.

Be careful.

And this is square meters.

But these are the units.

And this has a name.

This is called the Bohr magneton.

Bohr magneton.

What we cannot understand with our knowledge now, but you can if you ever take quantum mechanics, that the magnetic moment of all electrons in orbit can only be a multiple of this number, nothing in between.

Quantum mechanics, the word says it is quantization.

It's not in between.

It's either or.

It includes even 0, which is even harder to understand, that it can even be 0.

In addition to a dipole moment due to the electron going around the proton, the electron itself is a charge which spins about its own axis, and that also means that a charge is going around on the spinning scale of the electron.

And that magnetic dipole moment is always this value.

And so the net magnetic dipole moment of an atom or a molecule is now the vectorial sum of all these dipole moments, all these electrons going around, means orbital dipole moments, and you have to add the spin dipoles.

Some of these pair each other out.

One electron would have its dipole moment in this direction and the other in this direction, and then the vectorial sum is 0.

The net result is that most atoms and molecules have dipole moments which are either one Bohr magneton or 2 Bohr magnetons.

That is very common.

And that's what I will need today to discuss with you how strong a field we can create if we align all those magnetic dipoles.

The magnetic field that is produced inside a material when I expose it to an external field, that magnetic field B is the vacuum field that I can create with a solenoid -- we will discuss that further today -- plus the field which I will call B prime which is that magnetic field that is the result of the fact that we're going to align these dipoles.

The external field wants to align these dipoles, and the degree of success depends on the strength of the external field and of course on the temperature.

If the temperature is low, it's easier to align them, because there is less thermal agitation.

If, and that's a big if -- today you will see why it's a big if -- if B prime is linearly proportional to B vacuum, if that is the case -- today you will see that there are situations where that's not the case -- then I can write down that B prime equals chi of M -- we called that last lecture the magnetic susceptibility -- times B vacuum.

The linear proportionality constant.

If I can do that, then of course B is also proportional to B vacuum because now I can write down that B is one plus chi M times B vacuum.

And that, for that we write kappa of M times B vacuum, which is the equation that I started out with today.

And so that is only a meaningful equation if the sum of the alignment of all these dipoles can be written as being linearly proportional with the external field.

And this is what I want to explore today in more detail.

With paramagnetic material, there is never any worry that the linearity doesn't hold.

But with ferromagnetic material, that is not the case, because with ferromagnetic material, it is relatively easy to align these dipoles, because they already group in domains, as we discussed last time, and the domains flip in unison.

And so with ferromagnetic material, as you will see today, we can actually go into what we call saturation, that all the dipoles are aligned in the same direction.

And now the question is, how strong would that field be?

I'm going to make a rough calculation that gives you a pretty good feeling for the numbers.

It depends on what material you have.

I will choose a material whereby the magnetic dipole moment is 2 Bohr magnetons, so this is -- I told you it's either one or two or three, I pick one for which it is two.

And I have them all aligned.

So I take the situation that they're all aligned.

So here is the current going around the nucleus, here's another one, here's another one, here's another one.

This is a solid material, so these atoms or these molecules are nicely packed.

And here we see all these currents going around, and all these magnetic dipole moments are nicely aligned.

And so these magnetic fields are supporting each other.

And the question now is, what is the magnetic field inside here?

Well, that's an easy calculation, because this really looks like a solenoid, like you have windings, and you have a current going around.

And you remember, or should remember, that if we have a solenoid and we run a current through a solenoid that the magnetic field in the solenoid is mu 0 -- this mu 0 is not this mu, this mu 0 is the same one that -- oh no, it's no- we don't have it on the blackboard.

You notice the famous 4 pi times 10 to the -7.

And then we have the current I, and then we have N, if that's the number of windings that we have in the solenoid, and then we have L, which is the length of the solenoid.

So this is the number of windings of the solenoid per unit length, the number of windings per meter.

So if we could figure out, for this arrangement, what this quantity is, then we're in business.

I take a material, which is not unreasonable, whereby the number density of atoms, I call that capital N, written in the subscript way, is about 10 to the 29.

So this is atoms or molecules, whatever may be the case, per cubic meter.

That's not unreasonable.

And now I have to somehow manipulate, massage the mathematics, so that I get in here this magnetic moment, this Bohr magneton.

The 2 Bohr magnetons.

And there are several ways of doing that.

I have chosen one way, and that's the following.

I take here a length of 1 meter.

So this is a solenoid, and I take only 1 meter.

Could have taken 3 or 5 meters, makes no difference.

I take 1 meter.

And each one of these loops here has an area A.

So we would agree, I hope, that the area -- that the volume, the volume of this solenoid -- this has a length 1 meter -- that that volume is A square meters times 1.

And so the volume is A cubic meters.

A times 1 meter is A cubic meters.

But the number of atoms per cubic meter is 10 to the 29th, and so the number of atoms that I have in this solenoid per meter is this A times that N.

So this is the number of windings, if I call this one winding, the number of windings per meter.

Or you can think of it the number of atoms per meter, the way they're lined up.

And so now I am in business, because this now is my N divided by L.

And so I can write now mu 0 times the current I times that area A times N, which is 10 to the 29.

But look now.

Now you see why I did it this way, because I times A is the magnetic dipole moment of my atoms.

And that was 2 Bohr magnetons.

And so this now also equals mu 0 times twice mu Bohr times N.

And I'm finished, because I know what mu 0 is, and I know what moo- mu Bohr is -- we calculated that, it's still here, this number -- and I know what my capital subscript N is, 10 to the 29 atoms per cubic meter.

And so I just shove those numbers in my equation, and I find that this is approximately 2.3 tesla for the numbers that I have chosen.

It's not for all materials this way, because I have adopted 2 Bohr magnetons for the magnetic dipole moment of each atom, and I have adopted a density of 10 to the 29 atoms per cubic meter.

And for that situation, you get 2.3 tesla.

Now I want to use this number and understand what's going to happen in ferromagnetic material.

I take ferromagnetic material and I expose it to an external field, I call it a vacuum field.

So I stick it in a solenoid and I can choose the current through my solenoid.

And I'm going to plot for you the vacuum field.

This vacuum field is linearly proportional to the current through my solenoid.

This is an actual physical solenoid.

I have a wire, it goes around like this.

I don't have one here now.

And I run a current through there.

And that vacuum B field equals mu 0 times I times N divided by L, except that this N divided by L is now the number of windings of my current wire, and this is the length of my solenoid.

So don't confuse it with the one we had there, because that was on an atomic scale and this is on a macroscopic scale.

How many windings you have could be -- where you had a big one here in class, 2800 windings, and we had 60 centimeters long, and so that's what this number is all about.

So the moment that I know what the current is through my solenoid, I immediately know what my vacuum field is.

There's a one-to-one correspondence.

And now I'm going to measure inside the ferromagnetic material, which I stick in the solenoid.

I'm going to measure there the magnetic field.

And what's going to happen now?

Well, first of all I cannot at all plot this curve on a one-to-one scale, the reason being that kappa of M for ferromagnetic material is so large -- let us adopt for now a number of 1000, say.

Or it could be larger, even.

Let's say kappa of M is 1000.

That means that if the magnetic field in terms of a vector is this big, 1 centimeter in the length of the vector, that the field inside the ferromagnetic material is a thousand times higher.

If this is 1 centimeter in length, a thousand times higher is 10 meters.

So this is the length of the vector of the magnetic field inside the ferromagnetic material.

That's why I cannot do it to scale.

So when I draw this line here, keep in mind that if I did it to scale, if I did, but I can't, then the tangent of alpha would be kappa of M, which in this case would be 10 to the third.

And so this angle alpha is something like 89.9 something degrees.

So I cannot do it to scale.

Keep that in mind.

So in the beginning I get a nice linear curve, but now slowly I'm beginning to reach saturation, that all these dipoles are going to be aligned, and what you're going to see is that this curve bends over and bends over and bends over, and the magnetic field that you finally achieve here is the famous 2.3 tesla, which I calculated for that imaginary material, plus B vacuum.

The 2.3 is now the field which I will call B prime.

This is the field that is the result of the alignment of all those dipoles.

And so when I increase the vacuum field, this goes into saturation, and settles for 2.3 and can no longer increase, because I have aligned all these magnetic dipoles.

And so if your vacuum field is this strong, then this field is no longer thousand times stronger than the vacuum field.

You're no longer in the linear part.

So you could also think of it as kappa of M being smaller than a thousand.

Whichever way you prefer is fine.

But it's no longer proportional to the value of 1000.

If the temperature is lower of the material, it's easier to align them, and so you will achieve saturation earlier, and so your curve would go like this.

So the curve is also a function of temperature.

So this is if the temperature is low relative to this one.

So these curves depend on, on temperature as well.

The lower the temperature, the easier it is to align them.

If I reach this point here, when my B prime goes into saturation, I can only increase the field, the B field in the material, by increasing the vacuum field, because B prime is not going to go up again.

And so I can only get a higher field by increasing this current so that this B, this B vacuum goes up.

And that goes up very slowly, because this huge magnification factor of 1000 is gone now.

So the slow, it's very slow, the growth, and that's why you see that I drew it like this, that it increases very slowly.

But my plot is not to scale anyhow.

Now I want to discuss with you what happens with the material once I have driven it into saturation.

What happens if now I change the current and I make my vacuum field 0 again?

And now you get a very unusual behavior.

Let me do that here on the blackboard.

So I'll make a new drawing.

I could have continued with that one, but let me make a new one.

So I'm going to do the following experiment in my head.

I have a solenoid and I run a current through that solenoid.

And if the current is in clockwise direction, my vacuum field will be in this direction.

And when the current is in counterclockwise direction, I will assume that my vacuum field is in this direction.

So there is ferromagnetic material in here.

If the current flows in clockwise direction, the vacuum field is in this direction.

If I run it in counterclockwise direction, the vacuum field is in that direction.

And here is going to be my vacuum field.

It's easy for me to know what that is, because if I know the current through my solenoid, this equation will immediately tell me what the vacuum field is.

So I have never any problems with the vacuum field.

I stick a probe in here and I measure the magnetic field inside that material.

How we do that is not so easy, but we can do it.

There are a few things that I can't tell you.

This is one of them.

So here is the magnetic field inside the material.

All right, so there we start.

We do the same thing that we did here.

So we approach the saturation.

But now when I'm here, I am reducing the current and go back to 0.

Remember that when we are here, all these domains that we discussed last time have all flipped in the direction of the vacuum field.

So this field is enormous.

But now I make the current go back to 0.

And what happens now is I end up here, at this point P here.

The current now is 0.

There is no current going through the solenoid.

Notice the vacuum field is 0.

I can take the material out, the ferromagnetic material.

The material itself is now magnetic.

And you see there is a magnetic field inside it.

Why is that?

Because some of those domains remain aligned, they don't go back.

And so we have created permanent magnetism.

And so in the location, at the location P, we have B vacuum is 0, but B prime, which is the result of those aligned magnetic moments, is still in this direction.

Nothing is to scale here, of course.

And so you still have a magnetic field.

Now I reverse the current.

I go counterclockwise.

So I'm creating a magnetic field vacuum now in this direction.

And so now what will happen with this curve?

I come up here.

And look now here, at this location Q.

What do I have now?

I have something very bizarre.

I have now a situation whereby the vacuum field is in this direction but there is no magnetic field inside the material.

The magnetic field inside is 0.

So when we have point Q, so we have B vacuum is in this direction, but B inside, B prime -- oh no, it's not B prime, it's B.

It's the total field inside, is 0.

The reason being that B prime is still in this direction.

The reason being that the domains are still aligned in this direction, and so the vacuum field plus the B prime field, which has to be vectorially added, adds up with a net field 0.

Quite bizarre, isn't it?

Now I increase the current, but I keep going counterclockwise, and so the magnetic field of the vacuum remains in this direction.

I go into saturation again, in a similar way that I went into saturation here.

And now I stop here -- oh, I don't want to lose my brooch.

And now I stop here and I say to the current, go back to 0 again.

So my current now goes back to 0.

There we go.

And now I arrive here, point S.

And again, I have a situation that my vacuum field is 0.

I could take the material out of the solenoid, just walk around with it on the street.

It will be a permanent magnet.

But now the magnetic field inside this material in point P it was in this direction.

If I take it out here, then it is in this direction.

Now some domains stay aligned in this direction.

The reason was that I had counterclockwise current, and so those domains flipped over.

And they're not all willing to flip back again.

So I've also made a permanent magnet here.

Vacuum field 0, but B prime is now in the opposite direction.

And then if I continue now to go clockwise with current again and increase the current, I end up there.

And this is a bizarre curve.

We call this the hysteresis curve.

If you look at this curve, it's really amazing.

It's actually hard to, to digest this.

You, you have to give it a little bit of thought.

Because for one particular value of the current, for instance here, once I have a particular value of the current, I know that B vacuum is a given, I have two possibilities here for the magnetic field.

And for the current here, I have two possibilities for the magnetic field.

And so I cannot even know when I take this material and I expose it to an external field, I can't even calculate what the magnetic field inside will be.

It depends on the history of this material.

Look at this point here and at this point there.

If I asked you what is kappa of M, [pff] it's almost a ridiculous question.

Because what is kappa of M?

I have, um, I have a vacuum field, but I have no field inside.

So the field inside, which is B, is 0, and the vacuum field is not 0.

So you would have to answer, kappa M is 0.

That's the only thing you could say.

Quite bizarre, right.

But right here, there is a vacuum field but no field inside.

So kappa M is 0 here and kappa M is 0 here.

And remember, here it was 1000.

Look at the situation, take this point of the curve and this point of the curve.

Kappa M is less than 0, is negative, because here the vacuum field is in this direction, but B prime is in that direction, so they are in opposite directions, so the net field is in that direction, but B vacuum is in this direction.

And here it's also reversed.

So there's a bizarre situation that you are effectively having situations whereby kappa of M is 0 and kappa of M can also be negative for those s- points that I have there.

I can show you this hysteresis curve, and I do it exactly the way that I explained to you, except that I will not be able to run this current very slowly up and down.

I do it with 60 hertz alternating current, just get it out of the, the wall.

And so I run through this solenoid n- 60 hertz alternating current.

That means we go through this curve very quickly back and forth.

Between this point maximum current and this point maximum current.

The current clockwise, counterclockwise, clockwise, counterclockwise, and we change that 60 times per second.

And then I will show you this curve, again, highly distorted.

I cannot plot it one-to-one, for the reasons that I explained to you.

And you will see then the hysteresis curve.

That's called the hysteresis curve.

And for that, I have to do several things.

And I always forget what I have to do, but just don't worry about it, I will find out.

My TV goes on.

This light goes off and this light goes off.

Look there on the screen, and you will see within seconds there is the hysteresis curve.

And as I said to you earlier, I cannot start here, unfortunately, because we switch it so fast back and forth that this part of the curve, by the way, which is called the virginal curve -- virginal, because it's once and never again.

Once you have reached that point, from that moment on, you always stick to this -- I know, you should know about that.

[laughter].

And so here you see a striking example of a hysteresis curve.

And so you can ask yourself now the question, can we make this material virginal again?

And then the answer is yes.

There are various ways you can do that.

One way is you could take the material out, so that means you take it out, for instance, when there is remnant magnetism here, so it's a magnet, and now you heat it up above the Curie point, as we did last time in my lecture, and then the domains completely fall apart, and then you cool it again below the Curie point, and then it is virginal material again.

And then you could start here again.

That's one way you could do it.

There is another way you could try to do it.

You take a hammer and you just bang on it.

So you take it out and you have permanent magnetism, either here or there, and you bang on it, and you hope for the best.

And maybe you can get it back to here.

There is another way, which we call demagnetization, and I think that's what happens when you steal a book in the library and you try to get away with it, and the alarm goes off.

Someone hasn't demagnetized the magnetic strip in the book.

You may have noticed that when you take it out, that someone under the table goes like this.

And what they are doing then is they are demagnetizing that strip.

And the way you do that is as follows.

Here the current goes back and forth between this value and this value.

That means B vacuum and I are directly coupled to each other.

So I think of this as being current.

And now I go up to here and then back and then here and here and here and here and here and here and here and here and here and I end up there again.

And I can show you that.

So again you have alternating current, but the amplitude of the current you decrease and decrease and decrease, and you're going to see that there.

And you see that I can turn this material -- see the hysteresis curve changes.

So the amplitude of the current is not as large, so the amplitude of the B vacuum is not as large, and I slowly go back.

And I can change a non-virgin back into a virgin.

And that's the way we do it as physicists.

Demagnetization.

Did I turn that off?

Yes, I did.

I have here a, a coil.

You can't see that there is a coil inside here, but there is.

I can, uh, power this coil, putting a current through it.

Ferromagnetic material, I don't know what kappa is, but, uh, at least a thousand.

I get an enormously strong field inside here.

And this field is so strong that this piece of ferromagnetic material will be attracted.

The field is non-uniform outside.

We discussed last time that it will go clunk, it will stick there.

So let's do that.

So I power this electromagnet -- you can't see that I did, you have to take my word for it, but you believe it now.

There it goes.

Oh boy.

The force is so large that I would ask two people to come and see whether they can pull it apart.

The force that the two are together is so large that you may not even be able to separate it.

Do we have two strong people?

One strong woman, one strong man.

You look very strong to me.

[laughter].

Come on.

It's not going to be a tug of war between the two of you.

That's not my plan.

Be very careful, because if you touch this, you get electrocuted.

So don't do that.

[laughter].

But make sure that everyone can see you.

Because there is a current running through this solenoid now, yeah?

OK.

Now just in case that you succeed, I don't want you to get hurt.

Student: OK.

So make sure that you secure yourself.

[laughter].

Because suppose the current stopped running all of a sudden.

It's possible, it's MIT [laughter] right, anything could happen.

Then, of course, it's no longer a strong magnet.

It's only a strong magnet as long as the current is running through the solenoid.

So you secure yourself too.

OK, three, two, one, zero, go.

[laughter].

Don't worry.

I knew that in advance.

But thank you very much.

[laughter].

Very kind of you.

[applause].

Now comes something that you will understand.

If I make the current go to 0, then the vacuum field goes to 0.

That means the field that is generated by the solenoid goes to 0.

I will do that now in front of your own eyes.

No more current, right?

Why is this still hanging there?

Yeah?

The solenoid [inaudible] magnetize the ferrous [inaudible].

You hear?

You've taken the vacuum field out, but the domains to a certain degree are still aligned, and so the whole thing is still a magnet.

Not as strong as magnet -- if I were to invite you to come now and take it apart, you could.

But it still takes sub- substantial force.

And I will show you how large that force is.

I'm going to load it down now.

There's now 1 kilogram hanging on it.

Ooh, let me make sure I secure that, otherwise I can get killed for a change.

OK, 3 kilograms is hanging on it now.

5 kilograms is hanging on it now.

7 kilograms is hanging on it now.

9 kilograms is hanging on it now.

Oh boy, we may never make it.

10 kilograms is -- there it goes.

[laughter].

10 kilograms.

Now the show is not over yet.

What is very interesting, and I want you to think about it, that if now I take these two pieces of -- [laughter].

If I take these two pieces of ferromagnetic material, that -- nothing.

Do you know why?

I dropped it on the floor.

[laughter].

That's really why.

I dropped it on the floor, and that is like banging it with a hammer, and then the domains go away.

Had I not dropped it on the floor [laughter], there would have been something left, but very little, which is interesting by itself.

The shock of the separation, when that happened, already makes many of the domains flip back, and there would be very little left.

Not enough to carry this weight anymore, but that's largely because I dropped it on the floor.

I did that purposely so you can see when you drop [laughter] things on the floor.

If I bring ferromagnetic material in the vicinity of a magnet, I change the magnetic field configuration, and that's very easy to understand now.

Suppose I have here a magnet, north pole, south pole, and the magnetic field, magnetic dipole field is sort of like this.

And now I bring in the vicinity here a piece of ferromagnetic material.

Could be a wrench.

What happens now is that this ferromagnetic material will see this vacuum field -- this is called the vacuum field, is an external field.

And so these domains in there trying to align a little bit.

Degree of success depends on how strong the field is, depends on the temperature, depends on the kappa M of that material.

But certainly this will become sort of a south pole, this will become a north pole.

That's the way that these dipoles are going to align themselves.

They are going to create themselves a field in this direction.

They're going to support that field.

And so the net result is that the field inside here becomes very strong.

And so what happens with these field lines, they go like this.

They're being sucked into this ferromagnetic material.

It's very hard to know exactly how they go.

And the field here will weaken.

And I'm going to demonstrate that to you.

This is actually very easy to demonstrate.

And the way I'm going to demonstrate that is as follows.

I have there a setup whereby we have a, a magnet and we have a nail and we have a string.

And the nail wants to go to the magnet.

The mail its- the nail itself is, uh, ferromagnetic.

So the nail would love to go in this direction, but it can't.

So it just sits there, hangs there in space.

First of all, what I want to show you is that if I bring paramagnetic material in the vicinity, that that magnetic field configuration here is not going to change at all.

Paramagnetic material has a kappa of M so close to one that nothing is going to happen.

But the moment that I bring ferromagnetic material, for instance, here, then you get a field configuration change, and if I do that just the right way, then the nail will fall.

In other words, there is not enough magnetic field here in order to hold the nail in that direction.

And I'm going to show that for you, to you there.

Need some power here, I believe, and we have to make it dark here.

I'm going to shadow project it for you.

And the shadow projection you will see coming up very shortly.

This is a carbon arc.

You have to give it a little bit of time to start.

There is the carbon arc.

So there you see the nail.

And here you see the magnet.

You see that?

That's exactly the way I drew the picture.

And here, you hav- I have here a piece of aluminum, which is paramagnetic.

I can bring that through the field here.

Nothing happens.

My hands, believe it or not, are definitely not ferromagnetic, so I can also bring my hands here.

Nothing.

Nothing.

So magnetic field is not disturbed in any way, in any serious way, either by paramagnetic material, aluminum, or my hands, which I think are also paramagnetic, but I'm not sure.

I'm not sure whether I'm diamagnetic or paramagnetic, but it doesn't make any difference, because in both cases there is no significant change of the magnetic field.

But now I have a wrench here.

Here, there's a wrench.

You see it?

[laughter].

OK.

And now I'll bring the wrench close to that magnet.

My major worry is that magnetic field is so strong that once the wrench go- there's no way I can get it off again.

So I get only one shot at it.

And there goes the nail.

So what I -- what you saw now in front of your own eyes is that I changed the magnetic field configuration in such a way that the field was not strong enough to pull in the, the nail.

Now comes an important question, a big moment in our life.

And that is, what now is the effect of magnetic material on Maxwell's equations?

And let's take a look at Maxwell's equations.

Here we have Maxwell's equations the way we know them.

[laughter].

And let's first look at number one.

That's Gauss's Law.

Gauss's Law says that the closed surface integral of E dot dA -- that's the electric flux through a closed surface -- E- is equal to all the charge inside divided by epsilon 0, but you have to allow for the kappa, for the electric, dielectric constant.

The kappa, by the way, always lowers the field inside the material when we deal with electric fields.

It never increases it like magnetic fields.

It always lowers it.

Kappa is normally a few -- except for water, it is 80, it's quite large, and there are some ridiculous substances whereby kappa can be as large as 300.

I think strontium titanate -- I just looked it up this morning -- has a ridiculous value of kappa of 300.

So that's Gauss's Law.

Nothing's going to change there as far as I can see.

And then we have the second one.

The closed surface integral of B dot dL equals 0.

Oh, it says an, it says an L.

That shouldn't even be an L.

Ho, I hope you caught that.

This is an A, of course.

How could I?

This is the closed surface integral of B dot dA is 0.

What this is telling me is that magnetic ma- magnetic monopoles don't exist.

At least we think they don't exist.

Don't think that people are not trying to find them.

And if you find a magnetic monopole, and if you put that inside a box, then the closed surface integral of the magnetic flux through that box would not be 0.

And so then this is not true.

But as far as we know, it's always true, because we don't think that magnetic poles- monopoles exist.

And so then we come to the Faraday's Law.

Faraday's Law runs our economy.

Faraday's Law tells you when you move conducting loops in magnetic fields that you create electricity.

This equation runs our economy.

And now we come -- none of these, by the way, require any adjustment in terms of kappa of M.

But now we come to Ampere's Law.

Ampere's Law, which was amended by Maxwell himself, tells me what the magnetic field is, and all these results are for vacuum.

But now we know that that's not true anymore.

So this has to be adjusted now by a factor of kappa of M, which is the relative permeability.

And kappa of M is perfectly kosher for paramagnetic and diamagnetic materials.

There's never any problem there.

So here it comes.

For diamagnetic materials, it's a little less than one.

For paramagnetic materials, a little larger than one.

But when we deal with ferromagnetic materials, you have to be very careful, because we have seen today this hysteresis phenomenon, that there are even situations whereby kappa of M is negative, whereby kappa M is 0, and whereby kappa M can be huge, can be 10 to the 3.

So there you have to be very, very careful when you apply this equation without thinking.

Maxwell's equations are so important that I'm sure you want to see more of them.

So you see them there again.

And maybe that's not enough.

[laughter].

Maybe you want to see even more of them.

So look at them.

Inhale them.

Let them penetrate your brains.

[laughter].

I don't care in which direction you look now.

It's hard not to see them.

Today is therefore very special, because today we have all four Maxwell's equations in place.

And this was one of the main objectives of 8.02.

So we have completed a long journey, and on April 5, we have reached the summit.

Now I realize that the view is not spectacular for all of you yet, because often at the summit there is some fog.

But the fog will clear.

And I can assure you that from here on on, it's climbing downhill.

I think this moment is worth celebrating, and therefore I bought 600 daffodils for this occasion.

[laughter].

And I would like you to come at the, at the end of the lecture and pick up one of these daffodils and take it back to your dormitory.

I don't know whether I have enough for all these high school students and all their parents, but why not, give it a shot, and take one.

And when you look at it tonight at home and tomorrow, remember that you only once in your life go through this experience, that for the first time you see all four Maxwell's equations complete, hold it, and that you're capable of appreciating them, at least in principle.

This will never happen again.

You will never be the same.

[laughter].

To put it in simple terms, as far as 8.02 is concerned, you are no longer unspoiled virginal material [laughter].

You've lost your virginity.

[laughter].

Congratulations! [laughter].