Experiment AM Angular Momentum

Introduction

If an object, e.g., a heavy ball, comes straight at you and you catch it, you may stagger back while exerting the impulse -- some force for some time -- needed to bring its linear momentum down to zero. If you were on a frictionless surface you wouldn't be able to exert any forces to stop the ball, and you and it would move together, conserving linear momentum. Were you to catch the same ball as it passes beside you, with your arms outstretched to one side, you would have to exert an *angular* impulse, applying a *torque* for some interval of time, to bring its *angular* momentum down to zero. Suppose while catching the ball you were sitting on a stool firmly fixed to the ground but the seat of which could turn without friction. If you hung on to the ball, the ball and you would spin together indefinitely. If the stool were not anchored but free to slide and you caught the ball, then you, the ball and the stool would move in a combination of translational and rotational motion. Both the linear momentum and the angular momentum would be conserved during this collision since there would are no external forces or torques to provide impulses to change the total of each kind of momentum.

Just as conservation of linear momentum is a hidden part of many everyday happenings at all size scales, so is the conservation of angular momentum. The collisions of particles, the emission and absorption of radiation, the leaps and gyrations of cats, dancers, athletes, the motions of astronomical objects, all involve angular impulses and exchanges of angular momentum. In any isolated system angular momentum is conserved (as is linear momentum), despite mechanical energy losses ranging from zero (elastic collisions) to the maximum consistent with the conservation of linear momentum.

Part of the richness of phenomena and complexity of description in rotational dynamics comes from the fact that the same torque can be obtained with different combinations of lever arm $\vec{r}$, force $\vec{F}$ and angle between them θ. The same angular momentum can arise with different combinations of radius r, mass m, and velocity $\vec{v} = \vec{\omega} \times \vec{r}$ or moment of inertia I and angular velocity $\vec{\omega}$. Some or all of these can change in any given situation.

Experiments

You'll investigate two kinds of rotational phenomenon using a motor and a hub as a fixed axis of rotation.

In the first experiment you will measure the total frictional torque responsible for the slowing down of a rotating washer. You will place a washer on the hub. When the motor is turned on, the angular velocity of the shaft is increases. When the motor is turned off, the total frictional torque decreases the angular velocity until the assembly comes to a stop. During the deceleration,

the motor will act as a generator providing a voltage that is a measure of the instantaneous angular velocity.

In the second experiment, you will measure the change in angular momentum due to an inelastic rotational collision in which a stationary washer is dropped on a spinning washer. During the collision there is a rotational frictional torque between the washers, slowing one washer down and speeding the other washer up until the washers are moving at the same angular velocity. The total angular momentum is nearly conserved during this collision. You will measure how closely angular momentum is conserved.

Theory

When a **torque**, $\vec{\tau}_S$, is applied to a body about a point S, the body will acquire an *angular acceleration*, $\vec{\alpha}$. If the body is constrained to rotate about a *fixed axis of rotation* then the component of angular acceleration will be proportional to the component of torque about the axis,

$$\tau_S^{total} \propto \alpha,$$

where S denotes the point where the fixed axis passes through the center of the orbit. The constant of proportionality is called the *moment of inertia about the axis passing through S, I_S,* and it is a measure of how the mass is distributed about the axis of rotation,

$$\tau_S^{total} = I_S \alpha$$

The moment of inertia of a set of N masses about an axis of rotation is given by

$$I_S = \sum_{i=1}^{i=N} \Delta m_i (r_{\perp,i})^2$$

where $r_{\perp,i}$ is the distance the ith mass m_i lies from the axis of rotation. For a continuous body the sum over all the masses becomes an integral over the body

$$I_S = \int_{body} dm (r_\perp)^2$$

where $r_\perp$ is the radius of the circular orbit of the mass element dm.

Let the angle θ parameterize some point on the rigid body in a plane perpendicular to the axis of rotation. The *angular velocity*, $\vec{\omega}$, about the axis of rotation is the rate of change of the angle θ, and the component is given by

$$\omega = \frac{d\theta}{dt}.$$

The component of angular acceleration is defined to be the rate of change in time of the component of angular velocity,

$$\alpha = \frac{d\omega}{dt}.$$

Consequently the applied torque will either increase or decrease the angular velocity.

The *angular momentum*, $\vec{\mathbf{L}}_S$, of the mass about the axis of rotation passing through S is proportional to angular velocity, $\vec{\omega}$, with the moment of inertia I_S as the constant of proportionality,

$$\vec{\mathbf{L}}_S = I_S \vec{\omega}.$$

Differentiating the above equation shows that the rate of change in time of angular momentum is equal to the applied torque

$$\frac{d\vec{L}_S}{dt} = I_S \frac{d\vec{\omega}}{dt} = \vec{\tau}_S.$$

When the total applied torque on a rigid body about some point S is zero then the angular momentum, $\vec{\mathbf{L}}_S$, is conserved. If there is a constant applied torque, $\vec{\tau}_S$, over an interval of time $\Delta t = t_f - t_0$, then the change in angular momentum, $\Delta \vec{\mathbf{L}}_S = \vec{\mathbf{L}}_{S,f} - \vec{\mathbf{L}}_{S,0}$, which is known as the *angular impulse*, and is given by

$$\Delta \vec{\mathbf{L}}_S = \vec{\mathbf{L}}_{S,f} - \vec{\mathbf{L}}_{S,0} = \int_{t_0}^{t_f} \vec{\tau}_S dt.$$

The *rotational work*, W, which an applied torque, $\vec{\tau}_S$, does on a body in rotating that body about a fixed axis through an angle $\Delta\theta = \theta_f - \theta_0$ is given by

$$W = \int_{\theta_0}^{\theta_f} \tau_S d\theta.$$

The rate of change in time of the work done by the torque is the *instantaneous rotational power* and is given by

$$P = \frac{dW}{dt} = \tau_S \omega.$$

Apparatus

You have a permanent-magnet dc motor of the kind used in portable tape players which serves three purposes: (*i*) it provides a pivot, bearing or axis about which things can rotate with relatively low, but significant, friction, (*ii*) it is, of course, a motor which, when powered by your LVPS, can exert torque, rotate objects and give them angular momentum, and (*iii*) it can act as a DC generator that produces a dc voltage proportional to the angular velocity of its motor if it is made to rotate mechanically. In the latter mode, it can be used as an angular velocity sensor. The voltage generated by the motor is linearly proportional to the angular velocity of the motor, $\omega = \beta V$, where β is the constant of proportionality. You'll need a calibration of your generator to determine β, and we'll describe how to do this later. We'll use the motor in all these ways in this experiment.

You have a plastic bushing used to attach casters to tubular furniture, a wooden dowel in which we've drilled a hole, and a piece of rubber insulation. These can be assembled to make up a hub on which washers can be placed so that they can be rotated and so that their angular velocity can be measured.

The double pole, double throw slide switch (DPDT) will be used to connect the motor to the low voltage power supply (LVPS) when you want to accelerate a rotating object, and to the digital multimeter (DMM) when you want to measure angular velocity as the object coasts down.

To make readings at uniform intervals of, let's say, one second, you'll either have to call out seconds and record what your partner reads, or use the fact that the DMM flashes at a rate of two flashes a second.

Assembling the Apparatus
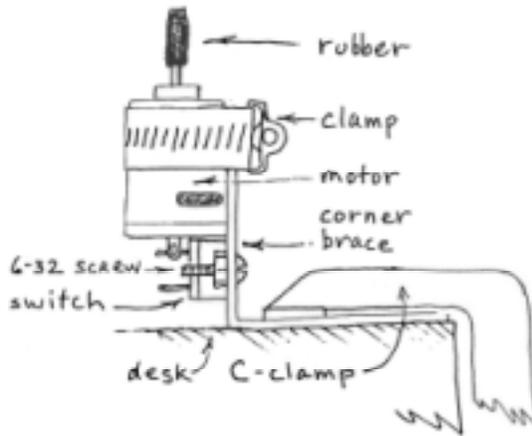
Figure 1: Overall assembly diagram   **Figure 2: Hub diagrams**
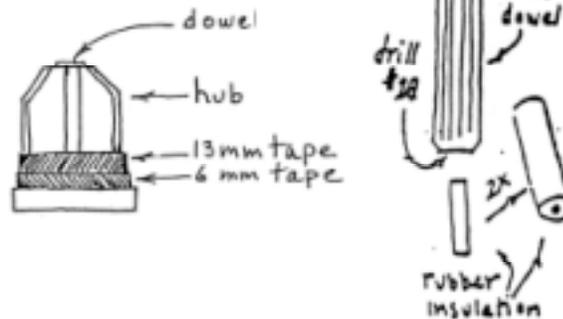
The assembly of the apparatus involves the following steps:

•   construct the axis of rotation using the dowel, hub, tape, and rubber insulation as connector to shaft of motor (steps 1-4 below);

•   connect the double pole double throw switch (DPDT) and low pass filter to motor (steps 5-7 below);

•   connect the hub to shaft of motor clamped to table (steps 8-9 below);

•   connect the LVPS to DPDT and motor (step 10 below).

1) Look at the white plastic object that will serve as a hub to carry the washers. It has a flat base and a conical top. Press the wooden dowel into the hub. The end of the dowel without a hole should be at the top of the hub and the end with the hole at the base.

2) Cut off a length of black electrical tape about 300 mm long. Stick one end to your desk so that you can use scissors to cut the tape lengthwise into 2 strips, one narrow, about 6 mm wide, the other wider, 13 mm wide. Carefully wind the wider piece around the lower part of the hub, taking care not to cover the rim at the base. Now wind the narrow piece around the lowest part of the hub, again taking care not to cover the rim at the base. See if one of the washers can be pressed onto the taped hub. The washer should fit neither loosely nor so tightly that it's a struggle to press it on. Add or remove tape as appropriate to get the right fit.

3) Strip off a 12 mm piece of rubber insulation from the length of red 5 kV test lead in your kit. Look at the piece of insulation and remove any thread or strands of wire. What you want is

essentially a small piece of rubber tubing. Press it onto the motor shaft so that 9 mm are on the shaft and the rest isn't.

4) Clamp the motor to the corner brace with the stainless steel hose clamp. The clamp should be flush with the end of the corner brace and the top of the shaft end of the motor should be a few mm above that. Tighten the clamp firmly.

The double pole double throw switch (DPDT) will connect the motor alternately to the LVPS or to the DMM. The hookup diagram below shows the circuit.
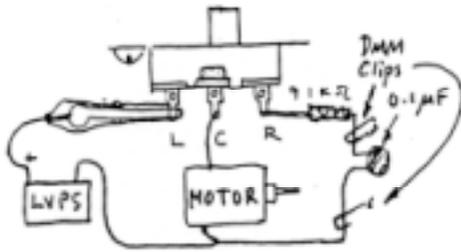
Figure 3: Hookup diagram for DPDT   **Figure 4: DPDT Switch Connections**
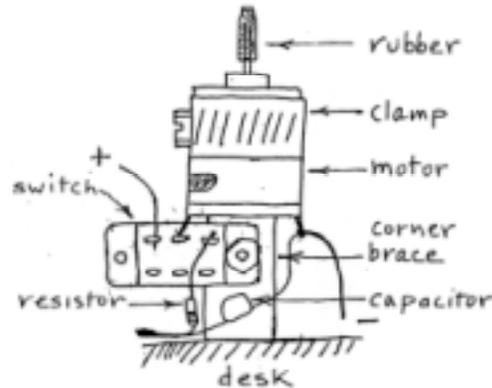
5) Attach the DPDT switch to the corner brace below the motor using the 6-32 screw and nut.

6) One of the motor terminals may be so near the center terminal of the DPDT switch that you can simply solder them together. If not use a short length of bare wire to connect them.

7) Solder the 15 kW resistor to the end terminal of the DPDT switch that's right under the motor. Solder the $0.1\mu F$ capacitor from the free end of the resistor to other motor terminal. This low pass filter reduces commutator ripple that would otherwise cause wrong readings on the DMM. You will clip your digital multimeter (DMM) leads across the $0.1\mu F$ capacitor.

8) Press the hub and washer assembly onto the motor shaft with its rubber tube. It should be pressed on far enough so that no rubber is visible but still with some clearance (1 to 2 mm) so that nothing rubs on the brass bearing at the top of the motor.

9) Clamp the corner brace firmly to your desk or table with the C clamp.

10) Use clip leads to connect the negative output of the LVPS to the motor terminal that has one end of the $0.1\mu F$ capacitor soldered to it. Connect the positive output of the LVPS to the end terminal of the DPDT switch that has nothing soldered to it.

Running the Experiment

The DPDT switch allows you to turn the motor on and off. When it's off the motor is connected to your voltmeter and acts as a generator whose output is proportional to its angular velocity.
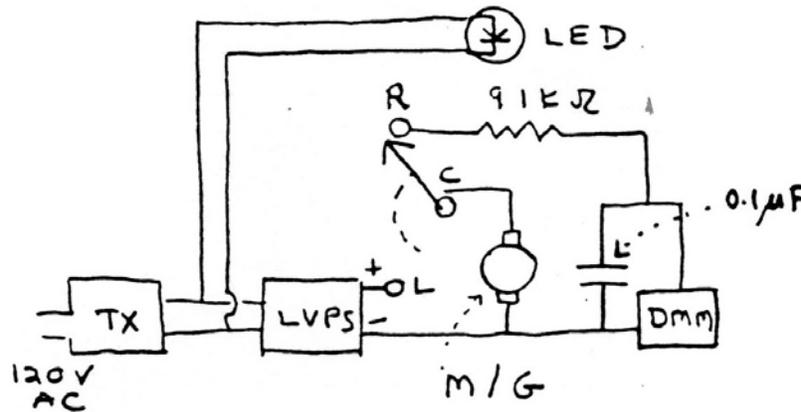
Figure 5: Circuit Diagram for Expt AM

Place one washer on the hub. **(Make sure you note the mass; it is printed in grams on the washer.)** Let the motor come up to speed, as you can tell by listening, operate the switch and observe the readings on your DMM. Since the voltage readings change twice a second on the DMM (sampling rate) they are hard to read. The first thing to do is to blank off the end digit on the right with a piece of black tape so you have 2 or 3 digits to read instead of 3 or 4.

Make three runs in which you allow the motor to speed up and then coast down, taking voltmeter readings at regular intervals.

One person should read the meter and calls out numbers while the other writes them down (very rapidly) on a sheet of paper. This will not be easy; so practice and work out some procedure for taking reliable measurements at constant intervals of time.

Calibration of the Motor/Generator

Connect the LED from Experiment CF to the 12V AC output from your wall transformer with leads sufficiently long to allow you to position the LED over your motor. (Note the transformer output will also be connected to the LVPS.)

Put a single washer on the hub. Place a small narrow piece of electrical tape completely across the top of the hub, as shown in figure 6. Since there are 60 flashes of light per second, the

tape should appear stationary when you reach a rotational speed of 30 Hz. (Wobble may make it sort of blurry.)
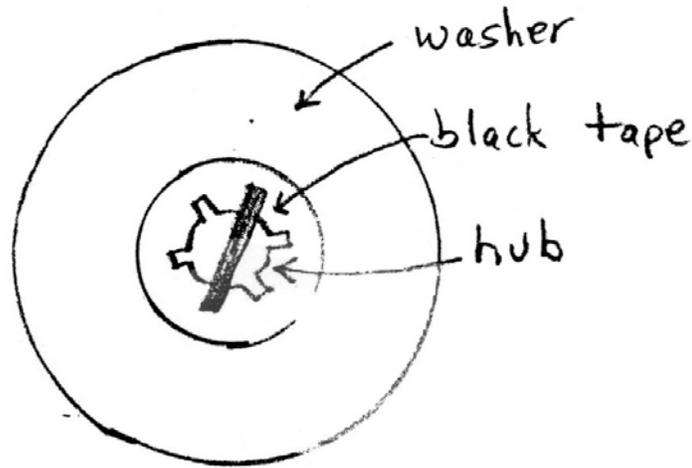
Figure 6: Setting up the strobe pattern

Accelerate the washer to as high a speed as you can achieve without the washer flying off the hub. Then disconnect the motor with the switch and read your voltmeter, as the motor coasts down. As the hub approaches 30 Hz, the LED will reflect off the tape at the top of the hub and the stroboscopic pattern will appear to slow down and spin the other way. (This will appear blurry but it is noticeable.) Just as the strobe pattern stops, you have reached 30 Hz. Record the value of the voltage. Repeat this three times and average your voltages. The angular velocity w of the motor is linearly proportional to the voltage V generated by the motor, $\omega = \beta V$, where β is the constant of proportionality. Recall that the angular frequency $\omega = 2\pi f$, where f is the frequency. Do not confuse the two quantities or their units: ω [rad/sec] and f [Hz]. Calculate your value for β. From this data you can now calibrate your motor generator output so that you can determine angular velocity for any measured voltage with an assumption of linearity down to zero volts and zero Hz.

Experiment One

With one washer on the hub, set the LVPS to 3.0 volts. Let the motor come up to speed. Turn the motor off and measure the voltages generated by the motor as it slows down (see above for ways to record the voltages).

Plot the voltage readings versus time for the three runs, arranging to have the first reading always at the zero of time. Because you are not starting each run at the same rotational speed the initial readings may differ, but one would expect the deceleration (represented by the slope) to be the same for each run.

Experiment Two

Make three runs in which after about 6 s you drop a washer on the spinning hub, producing an inelastic rotational collision. Take data as before. Make sure you have at least three or four data points before and after the collision.

Plot voltage readings versus time (in arbitrary units) for the three runs on three sheets of linear graph paper. The collision takes about 0.5 s; that's the time during which the dropped washer is being accelerated and the one beneath decelerated by the friction torque between them. The meter reading process introduces additional uncertainty.

Parts for Experiment AM

From previous experiments:
C-clamp,
2" corner brace,
10-32 screws,
2 LVPS,
DC permanent magnet motor,
stainless steel hose clamp,
LED

New Items:
1   5/16" hardwood dowel with #18 drilled hole
1   1" plastic chair caster bushing
2"   5-kV test lead wire
2   1" US Standard Washer (2-1/2" OD, approx. 5/32" thick)
1   paper clip #1
1   slide type DPDT switch
1   6-32 X 1/2 RH screw
1   6-32 nut
1   resistor 91 kΩ 1/2 W
1   capacitor, 0.1 μF