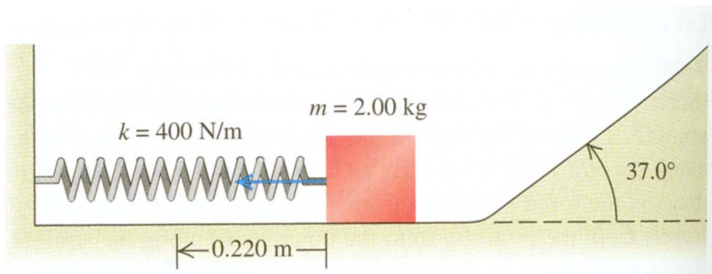
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01X Fall Term 2002

SOLUTIONS TO PROBLEM SET 8

1 Young & Friedman 7-38

A 2.00-kg block is pushed against a spring with negligible mass and force constant $k = 400\,\frac{\text{N}}{\text{m}}$, compressing it 0.220 m. When the block is released, it moves along a frictionless, horizontal surface and then up a frictionless incline with slope 37.0°.



(a) What is the speed of the block as it slides along the horizontal surface after having left the spring?

The forces involved here (spring force, gravity) are conservative, so the total mechanical energy remains unchanged throughout the motion.

$$\begin{aligned}
KE_{\text{horizontal sliding}} + PE_{\text{horizontal sliding}} &= KE_{\text{compressing spring}} + PE_{\text{compressing spring}} \\
\frac{1}{2}mv^2 + 0 &= 0 + \frac{1}{2}k\left(\Delta x_{\text{compressed}}\right)^2 \\
v &= \left(\Delta x_{\text{compressed}}\right)\sqrt{\frac{k}{m}} \\
&= (0.220)\sqrt{\frac{400}{2.00}} \\
&= 3.11\,\frac{\text{m}}{\text{s}}
\end{aligned}$$

(b) How far does the block travel up the incline before starting to slide back down?

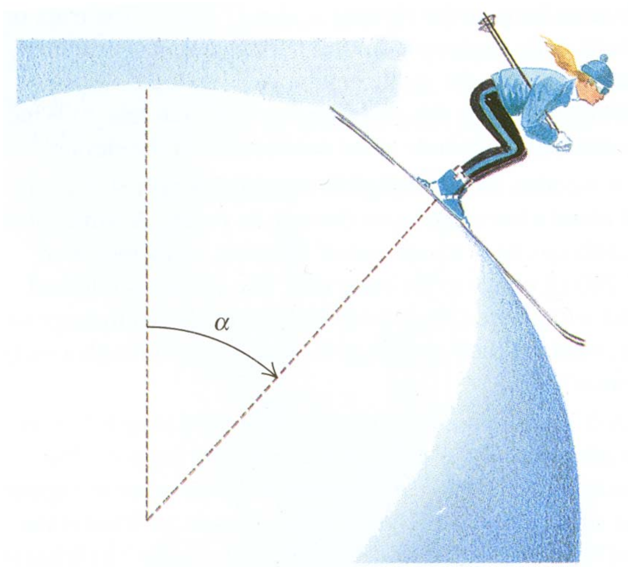
At the top of its trajectory, the block will be momentarily at rest. In part (a) we implicitly chose the zero of gravitational potential energy to be at the horizontal surface. When the block travels a distance ℓ up the ramp, it will be a height $\ell\sin\theta$ above that origin.

$$\begin{aligned}
KE_{\text{top}} + PE_{\text{top}} &= KE_{\text{compressing spring}} + PE_{\text{compressing spring}} \\
0 + mg\ell\sin\theta &= 0 + \frac{1}{2}k\left(\Delta x_{\text{compressed}}\right)^2
\end{aligned}$$

$$
\begin{aligned}
\ell &= \frac{k}{2mg\sin\theta}\left(\Delta x_{\text{compressed}}\right)^2 \\
&= \frac{400}{2\,(2.00)\,(9.81)\sin(37.0)}\,(0.220)^2 \\
&= 0.820\,\text{m}
\end{aligned}
$$

2 Young & Friedman 7-51

A skier starts at the top of a very large frictionless snowball, with a very small initial speed, and skis straight down the side. At what point does she lose contact with the snowball and fly off at a tangent? That is, at the instant she loses contact with the snowball, what angle α does a radial line from the center of the snowball to the skier make with the vertical?



Call the skier's mass m and the snowball's radius r. Choose the center of the snowball to be the zero of gravitational potential. We can look at the velocity v as a function of the angle α and find the specific α_{liftoff} at which the skier departs from the snowball.

If we ignore snow-ski friction along with air resistance, then the one work-producing force in this problem, gravity, is conservative. Therefore the skier's total mechanical energy at any angle α is the same as her total mechanical energy at the top of the snowball.

$$
\begin{aligned}
KE(\alpha)+PE(\alpha) &= KE(\alpha=0)+PE(\alpha=0) \\
\frac{1}{2}m\left[v(\alpha)\right]^2+mgr\cos\alpha &= \frac{1}{2}m\left[v(\alpha=0)\right]^2+mgr \\
\frac{1}{2}m\left[v(\alpha)\right]^2+mgr\cos\alpha &\approx mgr
\end{aligned}
$$

One might be tempted to include a potential energy term arising from the normal force exerted on the skier by the snowball. Remember that the normal force is incapable of doing work because that force is always perpendicular to the direction of an object's motion. There cannot be a potential energy associated with it.

The last line of the above equations will be true if the "small" initial speed $v(\alpha = 0) \ll \sqrt{gr}$. In that case we can rearrange the above equation to find the requisite centripetal force for keeping the skier along the presumably circular path.

$$\frac{m[v(\alpha)]^2}{r} = 2mg(1 - \cos\alpha)$$

The centripetal force (due to gravity) will be $mg\cos\alpha$, so the skier will remain on the snowball as long as gravity can hold her to that path, *i.e.* as long as

$$mg\cos\alpha \geq 2mg(1 - \cos\alpha)$$

Any radial gravitational force beyond what is necessary for the circular motion will be balanced by the normal force—or else the skier will sink into the snowball.

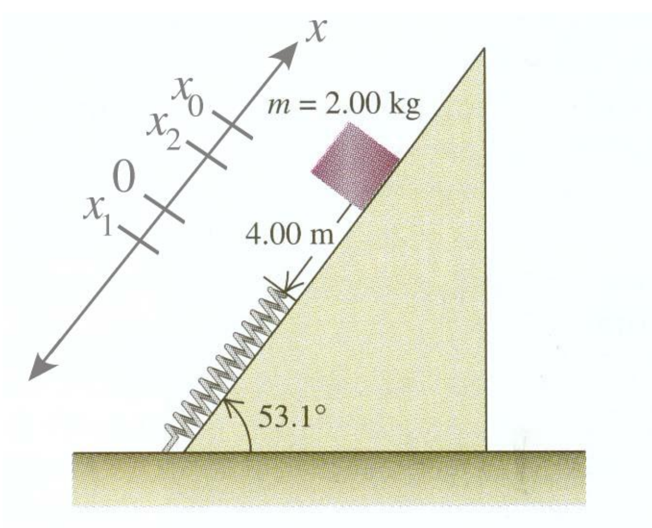
The expression for α_{liftoff} turns out to be very simple:

$$\begin{aligned} 3\cos\alpha &\geq 2 \\ \alpha_{\text{liftoff}} &= \arccos\frac{2}{3} = 48.2^\circ \end{aligned}$$

It has no dependence on r, m, or even g for that matter.

3 Young & Friedman 7-62

A 2.00 kg package is released on a 53.1° incline, 4.00 m from a long spring with force constant $120\,\frac{\text{N}}{\text{m}}$ that is attached at the bottom of the incline. The coefficients of friction between the package and the incline are $\mu_S = 0.40$ and $\mu_k = 0.20$. The mass of the spring is negligible.



(a) What is the speed of the package just before it reaches the spring?

Let's choose the $+x$-axis to be up the incline, with $x = 0$ at the end of the relaxed spring. Then the package starts at $x = x_0 \equiv 4.00\,\text{m}$. Energy conservation demands that

$$KE(x=0) + PE(x=0) - KE(x=x_0) - PE(x=x_0) = W_{\text{nonconservative}}$$

$$
\begin{aligned}
\frac{1}{2}m\left[v\left(x=0\right)\right]^2+0-0-mgx_0\sin\theta &= \\
v\left(x=0\right) &= \sqrt{2\left(\frac{W_{\mathrm{nc}}}{m}+gx_0\sin\theta\right)}
\end{aligned}
$$

where $\theta \equiv 53.1°$.

Before the release static friction may exert a force on the package, but it does no work because whenever static friction is present, the object it acts on is motionless. In contrast, kinetic friction performs nonconservative work on the package during sliding.

$$
\begin{aligned}
W_{\mathrm{nc}} &= \int_{x_0}^{0} f_{\mathrm{k}}\,\mathrm{d}x \\
&= \int_{x_0}^{0} \mu_{\mathrm{k}} mg\cos\theta\,\mathrm{d}x \\
&= -\mu_{\mathrm{k}} mg\cos\theta x_0
\end{aligned}
$$

So

$$
\begin{aligned}
v\left(x=0\right) &= \sqrt{2\left(\frac{-\mu_{\mathrm{k}} mg\cos\theta x_0}{m}+gx_0\sin\theta\right)} \\
&= \sqrt{2gx_0\left(\sin\theta-\mu_{\mathrm{k}}\cos\theta\right)} \\
&= 7.30\,\frac{\mathrm{m}}{\mathrm{s}}
\end{aligned}
$$

(b) What is the maximum compression of the spring?

Keeping $x=0$ at the end of the uncompressed spring, call the position of the package at maximum spring compression $x=x_1$ ($x_1$will be negative). This part is asking for $|x_1|$. Again, energy conservation comes to the rescue:

$$
\begin{aligned}
KE\left(x=x_1\right)+PE\left(x=x_1\right)-KE\left(x=x_0\right)-PE\left(x=x_0\right) &= W_{\mathrm{nc}} \\
0+\left(mgx_1\sin\theta+\frac{1}{2}kx_1^2\right)-0-mgx_0\sin\theta &= \mu_{\mathrm{k}} mg\cos\theta\left(x_1-x_0\right) \\
\frac{k}{2mg\left(\sin\theta-\mu_{\mathrm{k}}\cos\theta\right)}x_1^2+x_1-x_0 &= 0
\end{aligned}
$$

At this point it becomes convenient to introduce the shorthand notation $A \equiv \frac{k}{2mg(\sin\theta-\mu_{\mathrm{k}}\cos\theta)} = 4.50$ for the parameters in this problem.

$$
\begin{aligned}
Ax_1^2+x_1-x_0 &= 0 \\
x_1 &= \frac{1}{2A}\left(-1\pm\sqrt{1+4Ax_0}\right) \\
&= -1.06\,\mathrm{m}
\end{aligned}
$$

We discard the positive answer for x_1, 0.838 m, on the grounds that we are looking for a compression of the spring. The spring is compressed 1.06 m.

(c) The package rebounds back up the incline. How close does it get to its initial position?

Define $x = x_2$ as the farthest point reached on the rebound.

$$\begin{aligned} KE\,(x=x_2)+PE\,(x=x_2)-KE\,(x=x_0)-PE\,(x=x_0) &= W_{\text{nc}} \\ 0+mgx_2\sin\theta-0-mgx_0\sin\theta &= \int_{x_0}^{x_1}\mu_k mg\cos\theta\,dx+\int_{x_1}^{x_2}(-\mu_k mg\cos\theta)\,dx \end{aligned}$$

Note that on the second (upward) leg of the trip, the kinetic friction force points down the incline, as opposed to up during the first leg. Hence the minus sign in the second integral above.

$$\begin{aligned} mg\sin\theta\,(x_2-x_0) &= \mu_k mg\cos\theta\,(x_1-x_0)-\mu_k mg\cos\theta\,(x_2-x_1) \\ &= \mu_k mg\cos\theta\,(2x_1-x_2-x_0) \\ (\sin\theta+\mu_k\cos\theta)\,x_2 &= x_0\,(\sin\theta-\mu_k\cos\theta)+2x_1\mu_k\cos\theta \\ x_2 &= \frac{x_0\,(\tan\theta-\mu_k)+2x_1\mu_k}{\tan\theta+\mu_k} \\ &= 2.68\,\text{m} \end{aligned}$$

The package rebounds to $x_0 - x_2 = 4.00 - 2.68 = 1.32\,\text{m}$ beneath its starting point.

Note that we could instead have started with the equation

$$KE\,(x=x_2)+PE\,(x=x_2)-KE\,(x=x_1)-PE\,(x=x_1)=\int_{x_1}^{x_2}(-\mu_k mg\cos\theta)\,dx$$

and reached the same conclusion.

4 Young & Friedman 15-30

While running, a 70-kg student generates thermal energy at a rate of 1200 W. To maintain a constant body temperature of 37°C, this energy must be removed by perspiration or other mechanisms. If these mechanisms failed and the heat could not flow out of the student's body, for what amount of time could a student run before irreversible body damage occurs? (Protein structures in the body are irreversibly damaged if body temperature rises to 44°C or above. The specific heat capacity of a typical human body is 3480 J/kg•K, slightly less than that of water. The difference is due to the presence of protein, fat, and minerals, which have lower specific heat capacities.)

With a constant power, P, the total thermal energy generated over the time t before damage occurs is Pt.

$$\begin{aligned} Pt=Q &= m_{\text{body}}c_{\text{body}}\Delta T \\ t &= \frac{m_{\text{body}}c_{\text{body}}\Delta T}{P} \\ &= \frac{(70)\,(3480)\,(44-37)}{1200} \\ &= 1400\,\text{sec} \end{aligned}$$

or about 24 minutes.