Special Relativity and Time

8.01t

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Einstein: Postulate of Relativity

Examples of this sort, together with the unsuccessful attempts to discover any motion of the earth relatively to the "light medium," suggest that the phenomena of electrodynamics as well as of mechanics possess no properties corresponding to the idea of absolute rest. They suggest rather that, as has already been shown to first order of small quantities, the same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good. We will raise this conjecture (the purport of which will hereafter be called the Principle of Relativity") to the status of a postulate, and also introduce another postulate, which is only apparently irreconcilable with the former, namely, that light is always propagated in empty space with a definite velocity c which is independent of the state of motion of the emitting body. ***[1]***

[1] A. Einstein, translated from "Zur Elektrodynamik bewegter Körper," Annalen der Physik, 17, 1905.

Postulates of Relativity

Postulate 1: All inertial frames are equivalent with respect to all the laws of physics.

Postulate 2: The speed of light in empty space always has the same value c.

Relatively Inertial Reference Frames

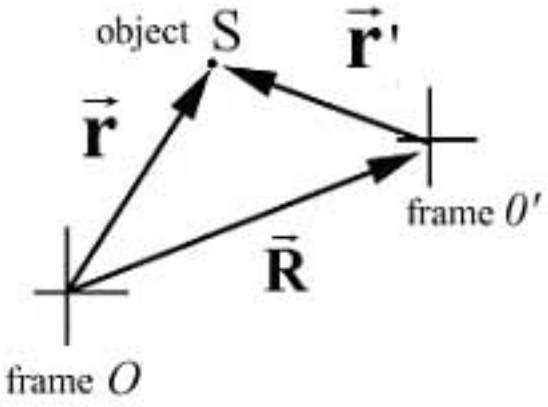
- Place an '*observer*' at the origin of coordinate system. The coordinate system with the observer acts as a '*reference frame'* .

- A *space-time event* : spatial and temporal coordinates. for describing the position, velocity, and acceleration of bodies.

- *Galilean Coordinate Transformations*.

$$\vec{\mathbf{r}}' = \vec{\mathbf{r}} - \vec{\mathbf{R}}$$

Relatively inertial reference frames



$$\vec{\mathbf{V}} = d\vec{\mathbf{R}}/dt$$

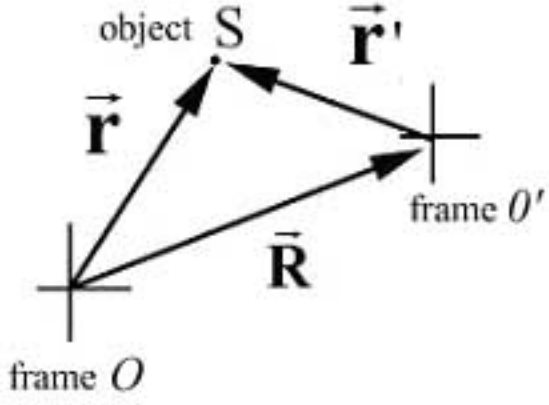
$$\vec{\mathbf{A}} = d\vec{\mathbf{V}}/dt = \vec{\mathbf{0}}$$

Law of Addition of Velocities: Newtonian Mechanics

Example: Relative Speed of Light in Newtonian Mechanics

$$\vec{\mathbf{v}}_2 = \vec{\mathbf{v}}_1 - \vec{\mathbf{V}}$$

Relative Velocity of Frames $\dfrac{d\vec{\mathbf{R}}}{dt} = \dfrac{1}{4}c\,\hat{\mathbf{i}}$



Light traveling with velocity $\dfrac{d\vec{\mathbf{r}}}{dt} = c\,\hat{\mathbf{i}}$

Relative speed of light

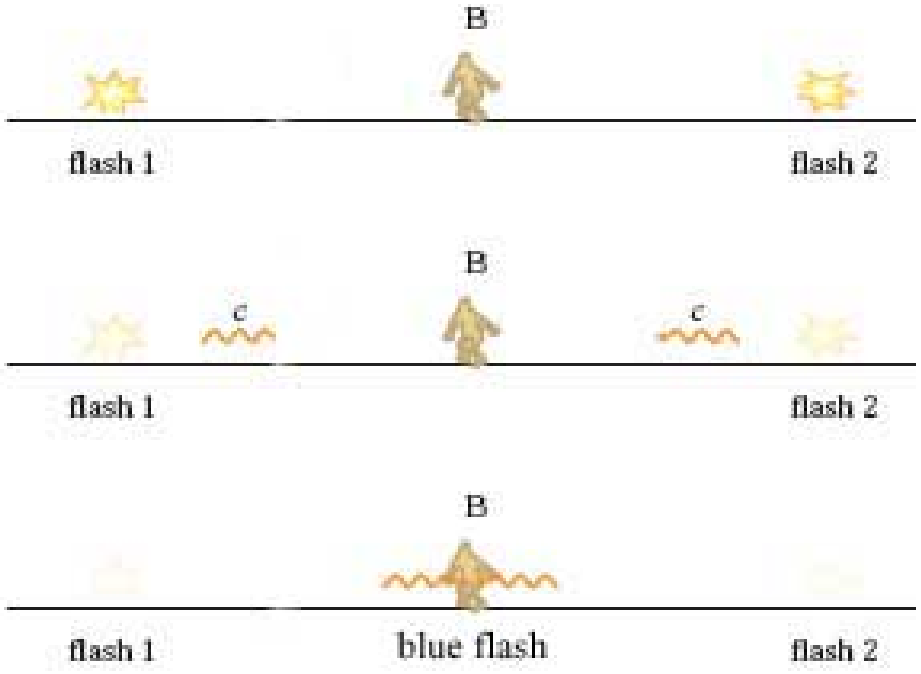
$$\frac{d\vec{\mathbf{r}}'}{dt'} = \frac{d\vec{\mathbf{r}}}{dt} - \frac{d\vec{\mathbf{R}}}{dt} = c\,\hat{\mathbf{i}} - \frac{1}{4}c\,\hat{\mathbf{i}} = \frac{3}{4}c\,\hat{\mathbf{i}}$$

Critique of Newtonian Mechanics:

- “absolute true and mathematical time, of itself and from its own nature, flows equably without relation to anything external”. ,
- $\Delta t_1 = \Delta t_2$, Newtonian Mechanics
- $\Delta t_1 \neq \Delta t_2$, Relativisitic Mechanics

Simultaneous events:

- ***Definition of Simultaneous events:***
- *If the detector at the point B midway between the points 1 and 2 emits a blue light indicating the two flashes reach it at the same instant then we define the two events at points 1 and 2 to have occurred simultaneously in time.*

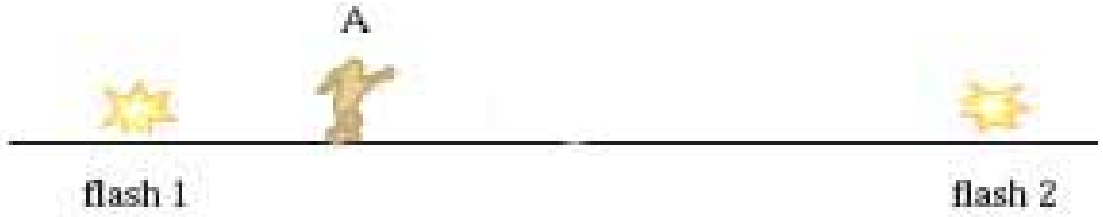


Time Ordering of Events:

- *If the detector emits a red light, then Flash 1 reaches it first, so Flash 1 occurred before Flash 2. If the detector emits a yellow light then Flash 2 reaches it first, so Flash 2 occurred before Flash 1.*

Concept Question:

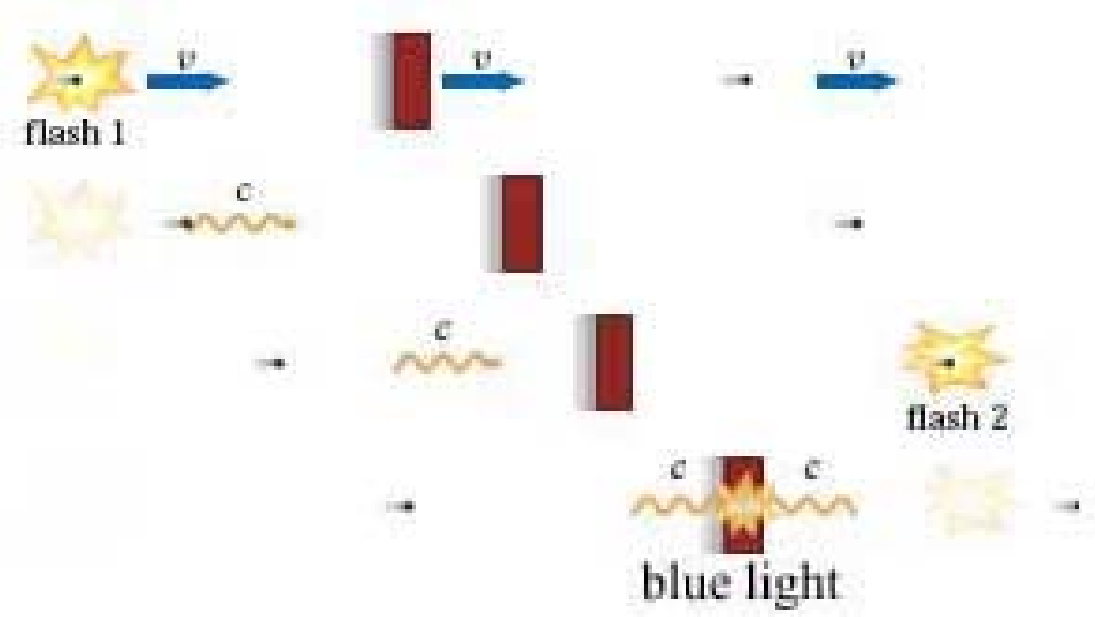
Suppose two flashes of light occur at the points 1 and 2. A flash detector is placed at A closer to event 1 as shown in Figure 1. The flash detector emits a blue light.



1) Flash 1 occurred before Flash 2
2) Flash 1 occurred at the same time as Flash 2
3) Flash 1 occurred after Flash 2

The Relativity of Simultaneity

- Now suppose the flash lamps, and the observer with the flash detector (midway between the flashes), are placed on a train moving to the right with speed v.



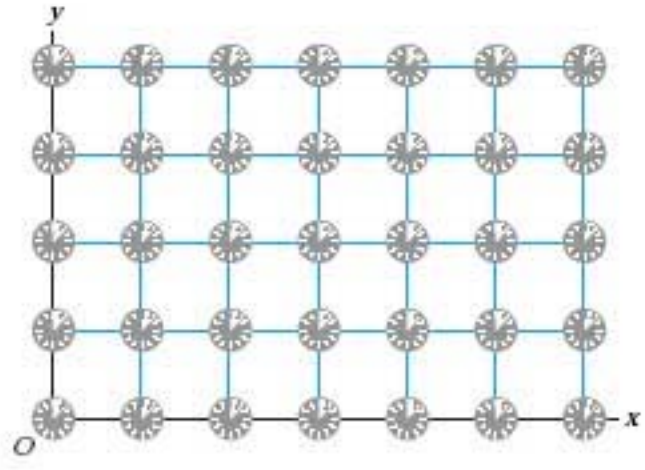
Concept Question:

Suppose the observer on the ground determines that Flash 1 and Flash 2 on the moving train occurred simultaneously. The detector midway between the flash lamps on the train emits

1. A red light indicating that Flash 1 occurred before Flash 2
2. A yellow light indicating that Flash 2 occurred before Flash 1
3. A blue light indicating Flash 1 occurred at the same time as Flash 2.

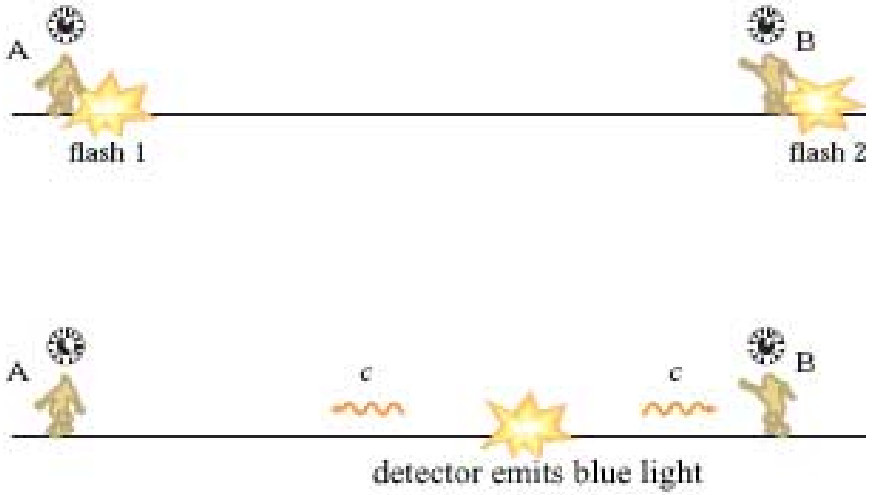
Synchronization of clocks:

- two identical clocks that are at rest in a reference frame at different spatial points will run at identical rates.
- a lamp at the origin emits a pulse of light at time .
- Every point in space has a clock that will begin running when the light pulse reaches it.
- Each clock has been pre-set to the time t = d/c where d is the distance from the clock to the origin and is the speed of light.
- When the light pulse reaches the clock, it begins to run at the same identical rate as all the other clocks.



Concept Question:

Flash 1 and Flash 2 reach the detector at a point closer to observer B than observer A. The two observers use their local synchronized clocks to determine the time of the events. When they compare times later on,



1. Flash 1 occurred before Flash 2
2. Flash 2 occurred after Flash 1
3. Flash 1 and Flash 2 occurred at the same time

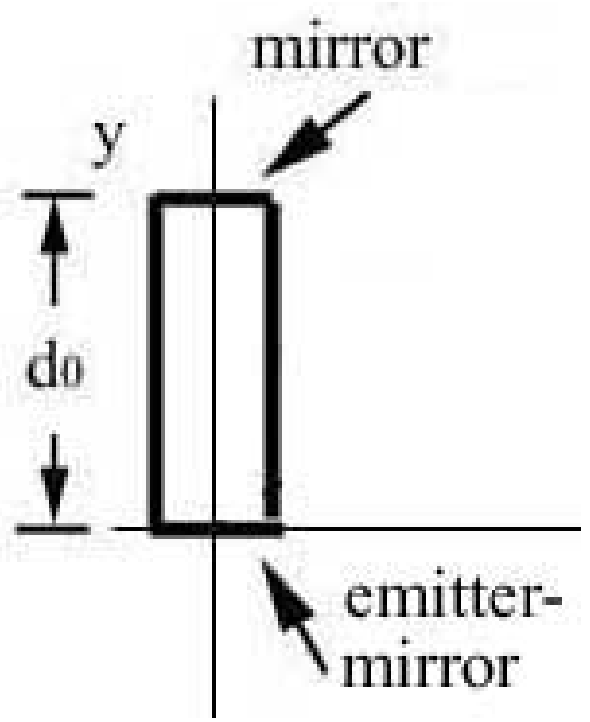
Causality:

- Events that are simultaneous in one reference frame are not simultaneous in a reference frame moving with a constant relative speed.

- Any two causally related events in one reference frame must preserve the same causal relation in any other reference frame. The time ordering cannot be reversed! If event 1 is causally related to event 2 and event 1 occurred first in one reference frame then it occurs first in all other relatively inertial reference frames.

Time Dilation: Light Clock at Rest

- The clock consists of pulse emitter, and two mirrors. One mirror is located a distance from the emitter and the other mirror is located at the emitter. The pulse is emitted and then bounces back and forth between the mirrors. Every time the pulse completes one path, the clock makes a tick.

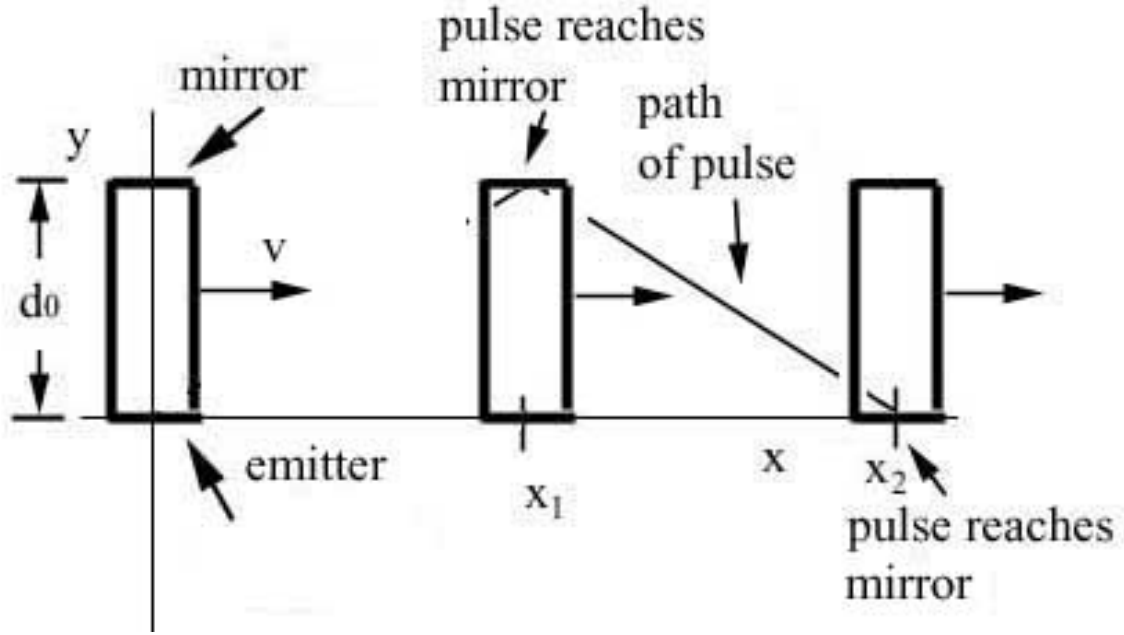


- Time interval for tick

$\Delta t = 2d_0/c$

Time dilation: Light Clock Moving

- Time Interval:



$$\Delta t' = 2d'/c = \frac{\sqrt{(v\Delta t')^2 + (d_0)^2}}{c}$$

$$\Delta t' = 2d_0 / c\sqrt{(1 - v^2/c^2)}$$

$$\Delta t' = \gamma 2d_0 / c = \gamma \Delta t$$

$$\gamma = 1/\sqrt{(1 - v^2/c^2)}$$