Energy, Kinetic Energy, Work, Dot Product, and Power

8.01t

Oct 13, 2004

Energy Transformations

- Falling water releases stored ‘gravitational potential energy’ turning into a ‘kinetic energy’ of motion.

- Human beings transform the stored chemical energy of food into ‘catabolic energy

- Burning gasoline in car engines converts ‘chemical energy’ stored in the atomic bonds of the constituent atoms of gasoline into heat

Energy Transformations

- Stretching or compressing a spring stores ‘elastic potential energy’ that can be released as kinetic energy

- kinetic energy, gravitational energy, heat energy, elastic energy, electrical energy, chemical energy, electromagnetic energy, nuclear energy, or mass energy.

Energy Transformations

- Energy is always conserved

- converted from one form into another

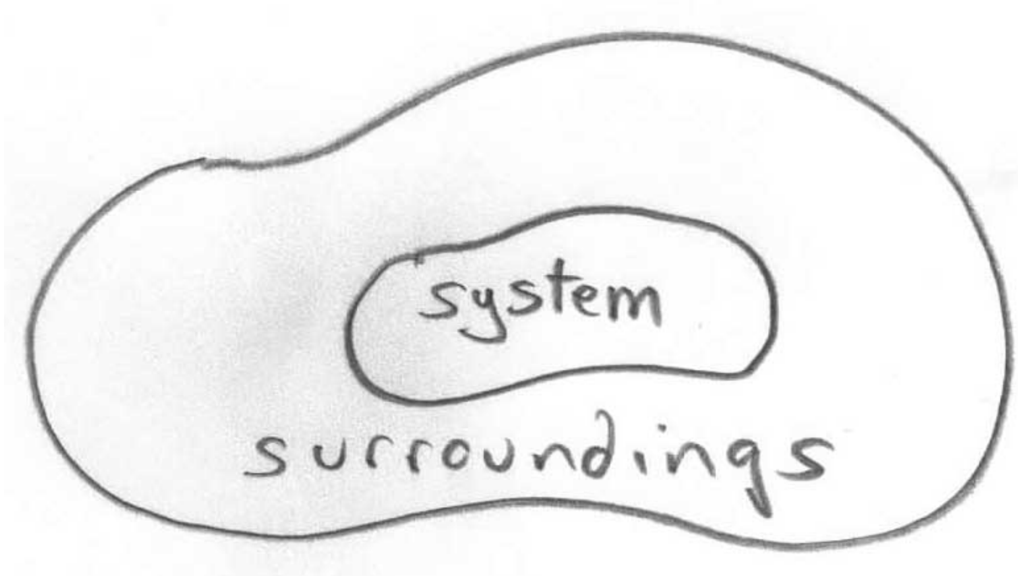
- ‘initial state’ that transforms into a ‘final state’.

$$\Delta E_i \equiv E_{final,i} - E_{initial,i}$$

- Conservation of energy

$$\Delta E_1 + \Delta E_2 + \ldots = \sum_{i=1}^{N} \Delta E_i = 0$$

System and Surroundings



$$\Delta E_{system} + \Delta E_{surroundings} = 0$$

Kinetic Energy

- positive scalar quantity

$$K = \frac{1}{2} m v^2$$

- SI unit is defined to be a Joule

$$1 J \equiv 1 kg \cdot m^2 \cdot s^{-2}$$

PRS Question 1

Compared to the amount of energy required to accelerate a car from rest to 10 miles per hour, the amount of energy required to accelerate the same car from 10 mph to 20 mph is

1) the same
2) twice as much
3) three times as much
4) four times as much

PRS Question 2

Consider two carts, of masses *m* and 2*m*, at rest on an air track. If you push first one cart for 3 s and then the other for the same length of time, exerting equal force on each, the kinetic energy of the light cart is

1) larger than
2) equal to
3) smaller than

the kinetic energy of the heavy car.

Work Done by Constant Force

Definition: Work done by a Constant Force

The work done by the constant force acting on the body is the product of the component of the force in the direction displacement with the displacement,

$$W_{applied} = F_x \Delta x$$

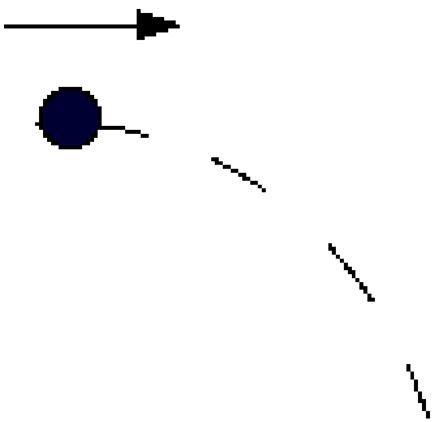
PRS Question 3

A ball is given an initial horizontal velocity and allowed to fall under the influence of gravity, as shown below. The work done by the force of gravity on the ball is:

1) positive

2) zero

3) negative



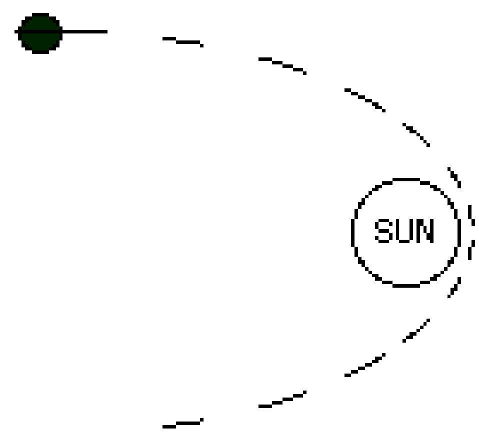
PRS Question 4

A comet is speeding along a hyperbolic orbit toward the Sun. While the comet is moving away from the Sun, the work done by the Sun on the comet is:

1) positive

2) zero

3) negative

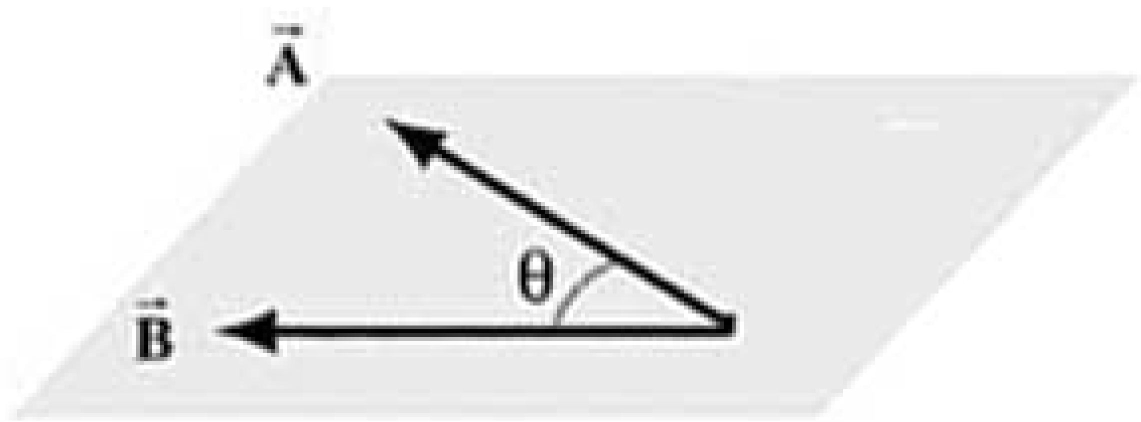


Dot Product

- The magnitude of the cross product

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = AB\cos\theta$$

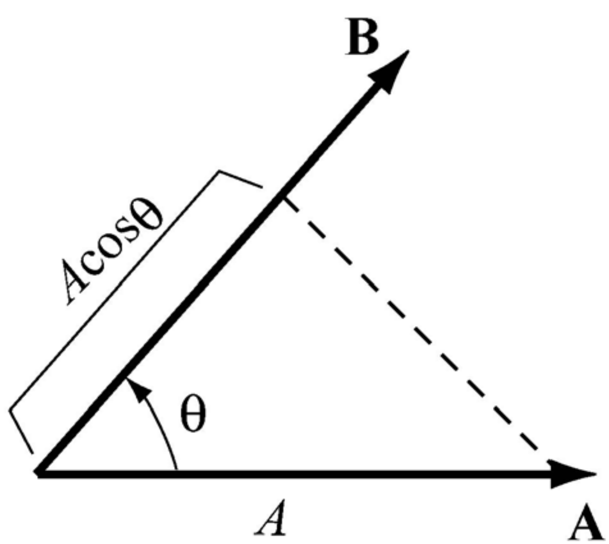
- The dot product can be positive, zero, or negative

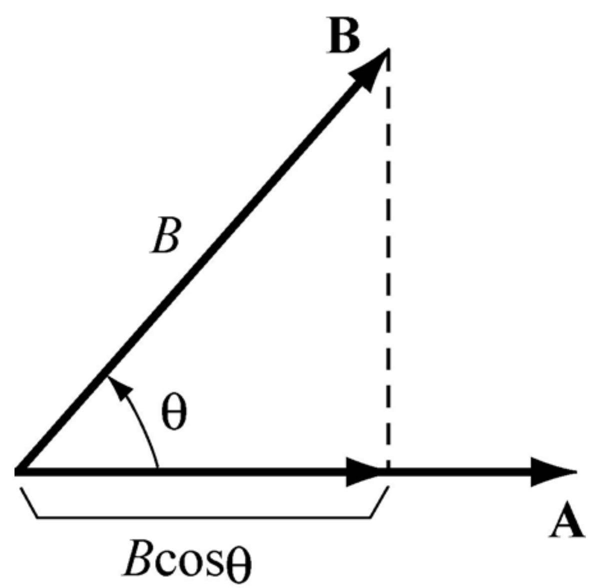


Projection and the Dot Product

Two types of projections

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = A(\cos\theta)B \qquad \vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = A(B\cos\theta)$$





Properties

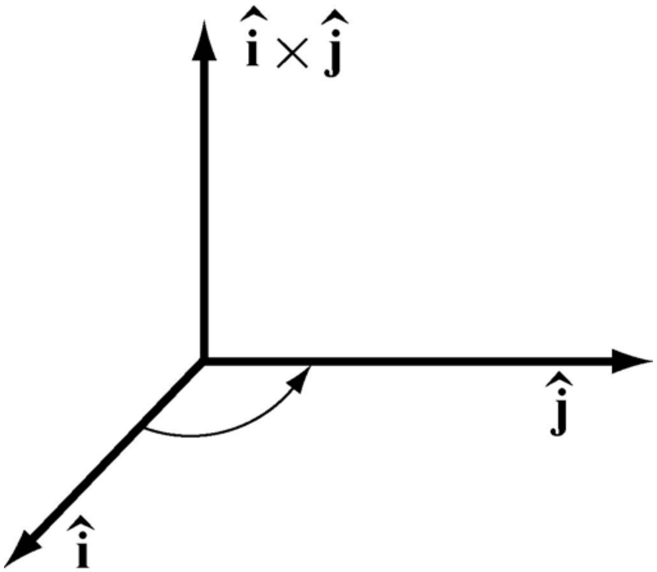
$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}}$$

$$c\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = c(\vec{\mathbf{A}} \cdot \vec{\mathbf{B}})$$

$$(\vec{\mathbf{A}} + \vec{\mathbf{B}}) \cdot \vec{\mathbf{C}} = \vec{\mathbf{A}} \cdot \vec{\mathbf{C}} + \vec{\mathbf{B}} \cdot \vec{\mathbf{C}}$$

Unit Vectors and the Dot Product

- Unit vectors



$$\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = |\hat{\mathbf{i}}||\hat{\mathbf{i}}|\cos(0) = 1$$

$$\hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = 1$$

$$\hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = |\hat{\mathbf{i}}||\hat{\mathbf{j}}|\cos(\pi/2) = 0$$

$$\hat{\mathbf{i}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{k}} = 0$$

Vector Components of Dot Product

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$

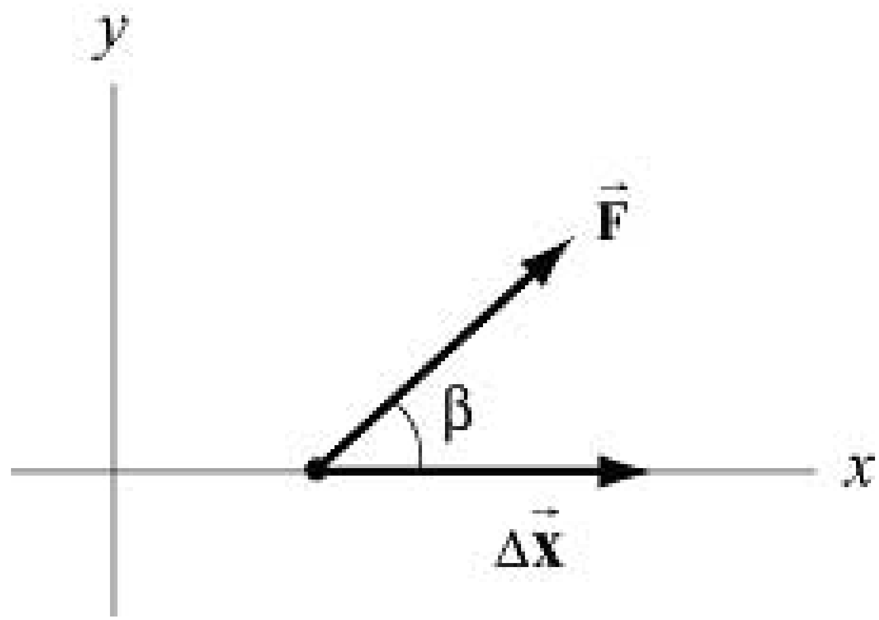
$$\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$$

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = A_x B_x + A_y B_y + A_z B_z$$

Work and the Dot Product

$$\vec{\mathbf{F}} = F_x\,\hat{\mathbf{i}} + F_y\,\hat{\mathbf{j}}$$

$$\Delta\vec{\mathbf{x}} = \Delta x\,\hat{\mathbf{i}}$$



$$W = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{x}} = (F_x\,\hat{\mathbf{i}} + F_y\,\hat{\mathbf{j}}) \cdot (\Delta x\,\hat{\mathbf{i}}) = F_x\,\Delta x$$

PRS Question 5

Consider two blocks stacked on a table. Someone pulls the bottom block to the right with a rope in such a way that both bocks accelerate to the right but no slipping occurs at the interface between the top and bottom blocks. Friction at the interface between the two blocks does

1. Positive work on the top block.
2. No work on the top block.
3. Negative work on the top block.

PRS Question 6

When a person walks, the force of friction between the floor and the person's feet accelerates the person forward. The floor does

1. Positive work on the person.
2. Negative work on the person.
3. No work on the person.

Work-Kinetic Energy Theorem

$$W_{f,i} = F_x \Delta x = m a_x \Delta x = m a_x (v_{x,i} t + \frac{1}{2} a_x t^2)$$

$$= m \frac{(v_{x,f} - v_{x,i})}{t} (v_{x,i} t + \frac{1}{2} (\frac{(v_{x,f} - v_{x,i})}{t}) t^2$$

$$= \frac{1}{2} m v_{x,f}{}^2 - \frac{1}{2} m v_{x,i}{}^2$$

$$= \Delta K$$

$$K_f = K_i + W_{f,i}$$

Work-Kinetic Energy Theorem

- one dimension

$$W = \int_{x_0}^{x_f} F_x dx = \int_{x_0}^{x_f} m \frac{dv_x}{dt} dx = \int_{x_0}^{x_f} m \frac{dx}{dt} dv_x = \int_{v_{x,0}}^{v_{x,f}} m v_x dv_x$$

$$W = \int_{v_{x,0}}^{v_{x,f}} m v_x dv_x = \int_{v_{x,0}}^{v_{x,f}} d\left(\frac{1}{2} m {v_x}^2\right) = \frac{1}{2} m {v_{x,f}}^2 - \frac{1}{2} m {v_{x,0}}^2 = \Delta K$$

Work done by Non-Constant Force

- Infinitesimal work is a scalar

$$\Delta W_i = (F_x)_i \Delta x_i$$

- add up these scalar quantities to get the total work

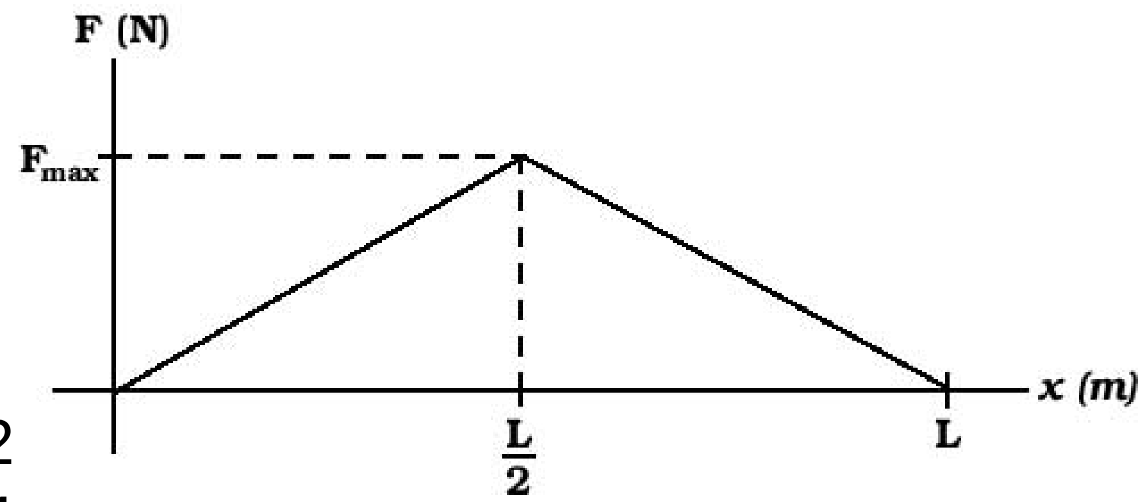
$$W_N = \sum_{i=1}^{i=N} \Delta W_i = \sum_{i=1}^{i=N} (F_x)_i \Delta x_i$$

- limit as $N \to \infty$ and $\left|\Delta x_i\right| \to 0$

$$W = \lim_{\substack{N \to \infty \\ \Delta x_i \to 0}} \sum_{i=1}^{i=N} (F_x)_i \Delta x_i = \int_{x=x_0}^{x=x_f} F_x dx$$

PRS Question 7

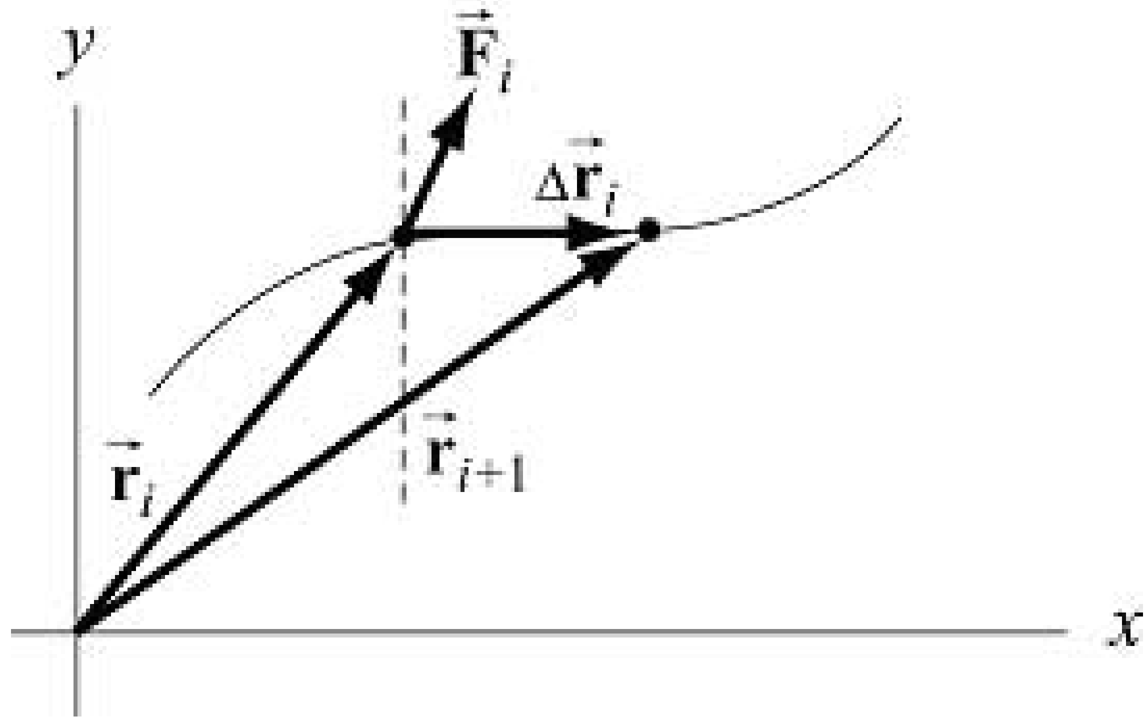
A particle starts from rest at x = 0 and moves to x = L under the action of a variable force F(x), which is shown in the figure. What is the particle's kinetic energy at x=L/2 and at x=L?



1. Fmax L/2, FmaxL
2. Fmax L/4, 0
3. Fmax L, 0
4. Fmax L/4, Fmax L/2
5. Fmax L/2, Fmax L/4

Work Done Along an Arbitrary Path

$$\Delta W_i = \vec{\mathbf{F}}_i \cdot \Delta\vec{\mathbf{r}}_i$$



$$W = \lim_{\substack{N\to\infty \\ |\Delta\vec{\mathbf{r}}_i|\to 0}} \sum_{i=1}^{i=N} \vec{\mathbf{F}}_i \cdot \Delta\vec{\mathbf{r}}_i = \int_{r_0}^{r_f} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$$

Line Integrals

- force vector $\vec{\mathbf{F}} = F_x\,\hat{\mathbf{i}} + F_y\,\hat{\mathbf{j}} + F_z\hat{\mathbf{k}}$

- line element $d\vec{\mathbf{r}} = dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}} + dz\hat{\mathbf{k}}$

- total work

$$W = \int_{\vec{\mathbf{r}}=\vec{\mathbf{r}}_0}^{\vec{\mathbf{r}}=\vec{\mathbf{r}}_f} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \int_{\vec{\mathbf{r}}=\vec{\mathbf{r}}_0}^{\vec{\mathbf{r}}=\vec{\mathbf{r}}_f} F_x dx + \int_{\vec{\mathbf{r}}=\vec{\mathbf{r}}_0}^{\vec{\mathbf{r}}=\vec{\mathbf{r}}_f} F_y dy + \int_{\vec{\mathbf{r}}=\vec{\mathbf{r}}_0}^{\vec{\mathbf{r}}=\vec{\mathbf{r}}_f} F_z dz$$

Power

- *The average power of the applied force is defined to be the rate of doing work*

$$P_{ave} = \frac{\Delta W}{\Delta t} = \frac{F_{applied,x}\,\Delta x}{\Delta t} = F_{applied,x} v_{x,ave}$$

- SI units of power are called Watts

$$[1W] \equiv [1J \cdot s^{-1}] \equiv [1kg \cdot m^2 \cdot s^{-3}]$$

Instantaneous Power

- limit of the average power

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = F_{applied,x} \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = F_{applied,x} \, v_x$$

Class Problem 1

A person pushes a cup of mass m for a time t along a horizontal table with a force of magnitude F at an angle θ with respect to the horizontal for a distance d . The coefficient of friction between the table and the cup is μ_k. The cup was initially at rest.

Class Problem 1 (con’t)

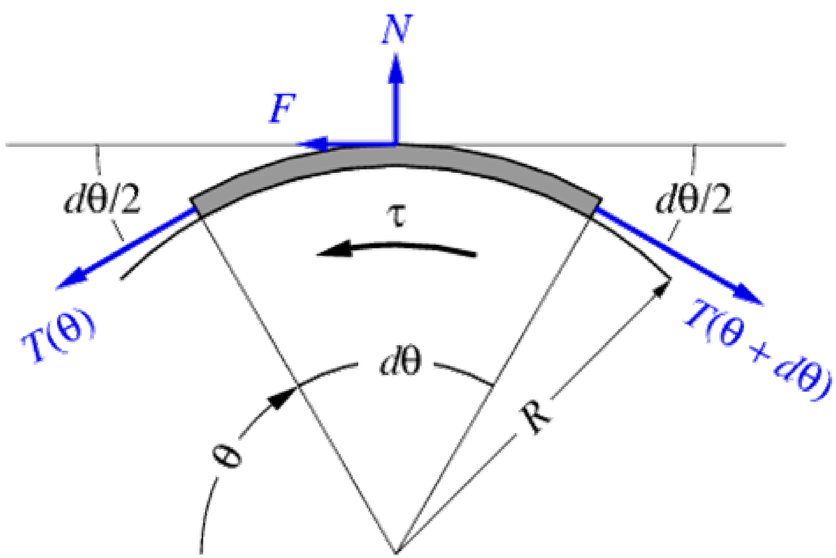
a) Calculate the work done by the pushing force.
b) Calculate the work done by the friction force.
c) What is the average power of the pushing force?
d) What is the average power of the kinetic friction force?

MIT

8.01T Physics I

Experiment 5B: Friction

Goal

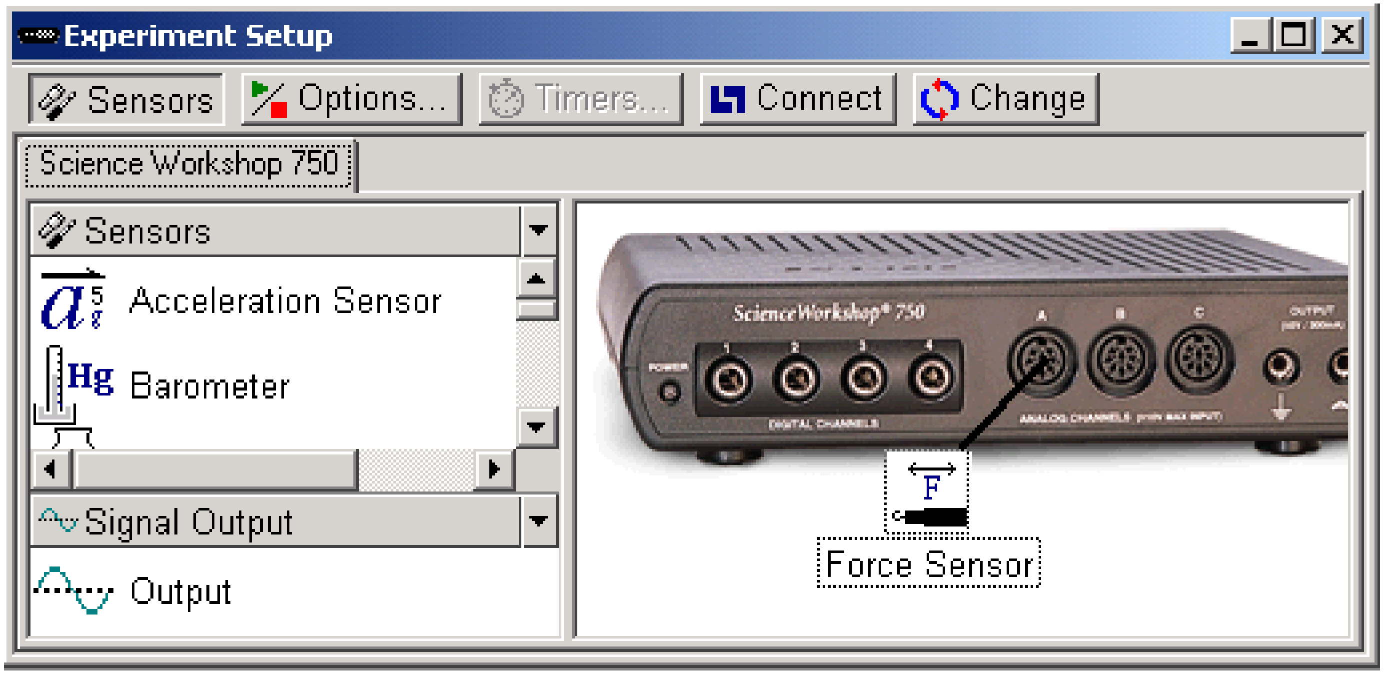


To investigate the friction of a string wrapped around a cylinder, observe that it increases exponentially, and to measure the friction coefficient μ.

(A theoretical expression is derived in the appendix to the write up for the experiment.)

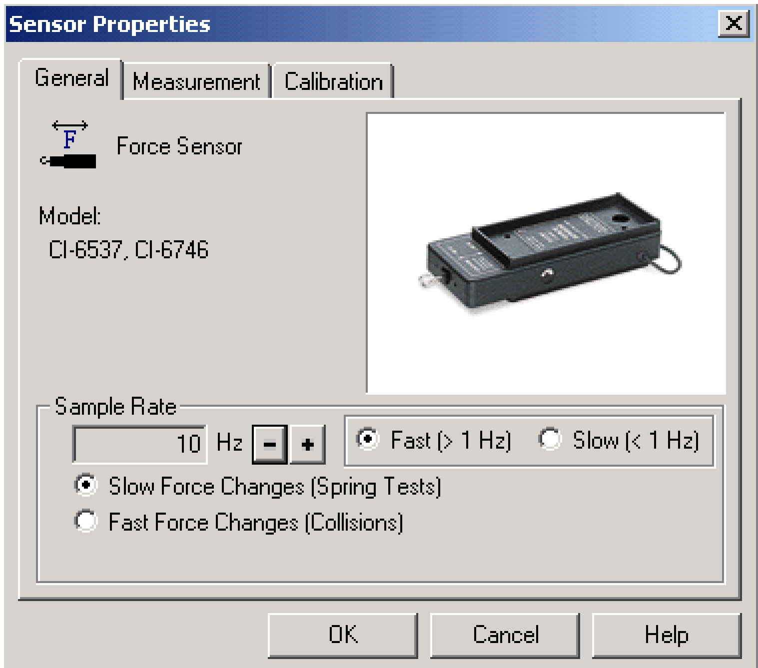
Starting *DataStudio*:

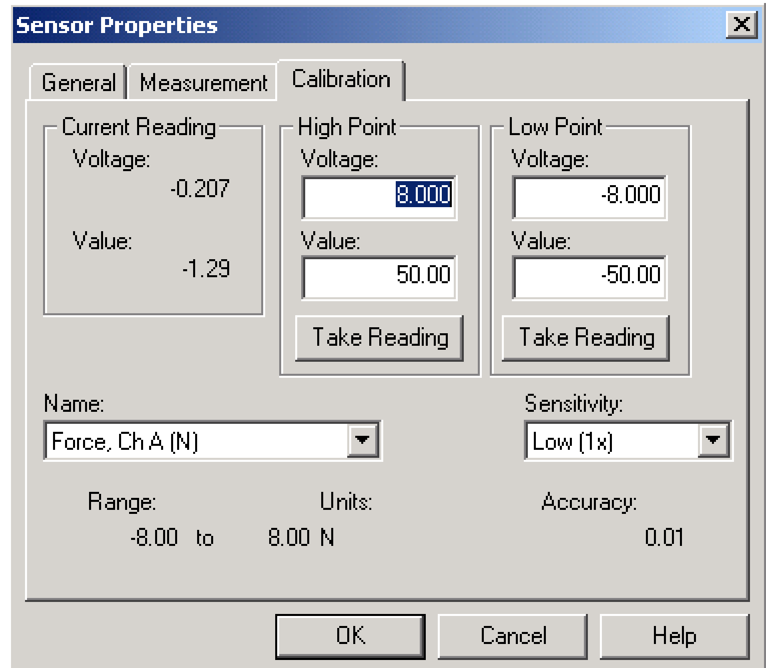
Create a new experiment. Plug a force sensor into the drag it to the input in the Setup window.



Double-click the Force Sensor icon.

Force Sensor:

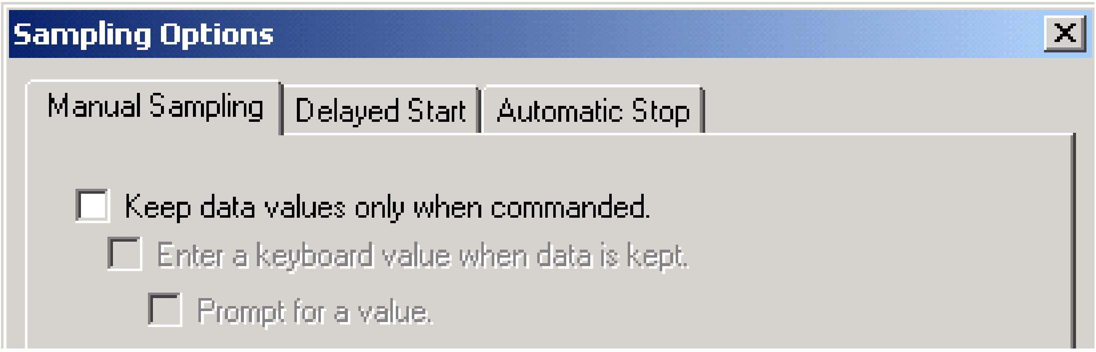




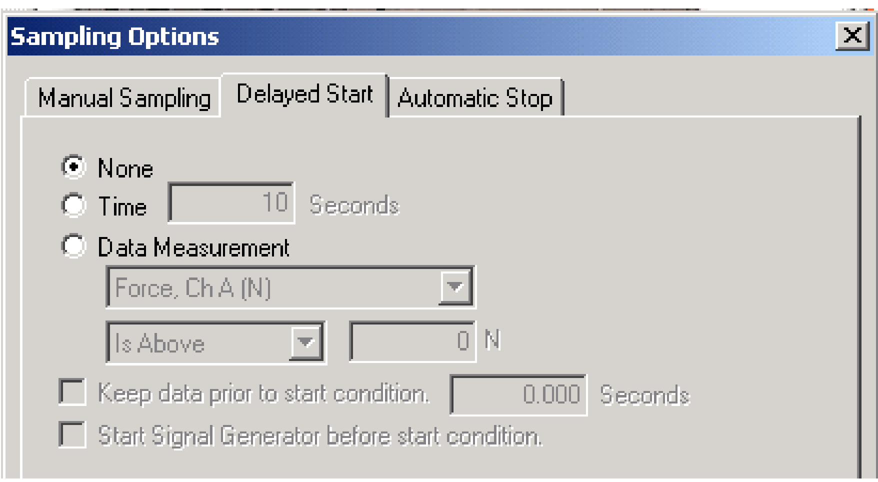
Set it for 10 samples/s and low sensitivity.

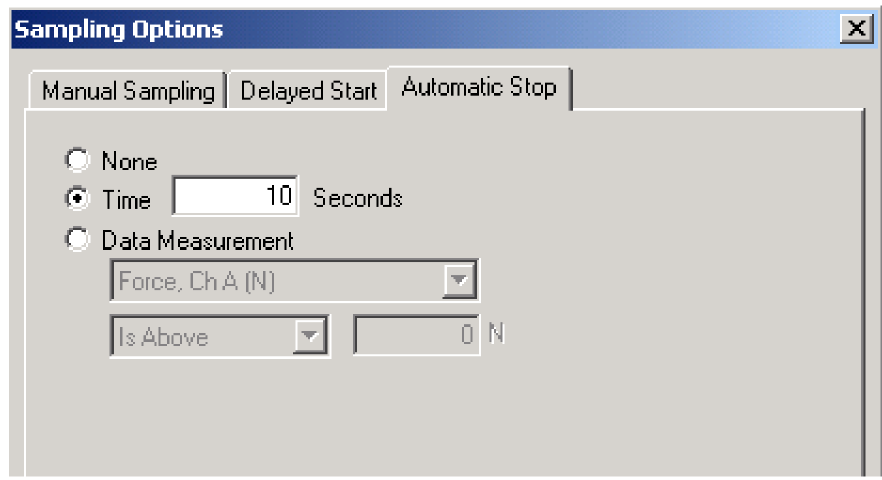
Click 

Sampling Options:



No boxes checked.





No delayed start.

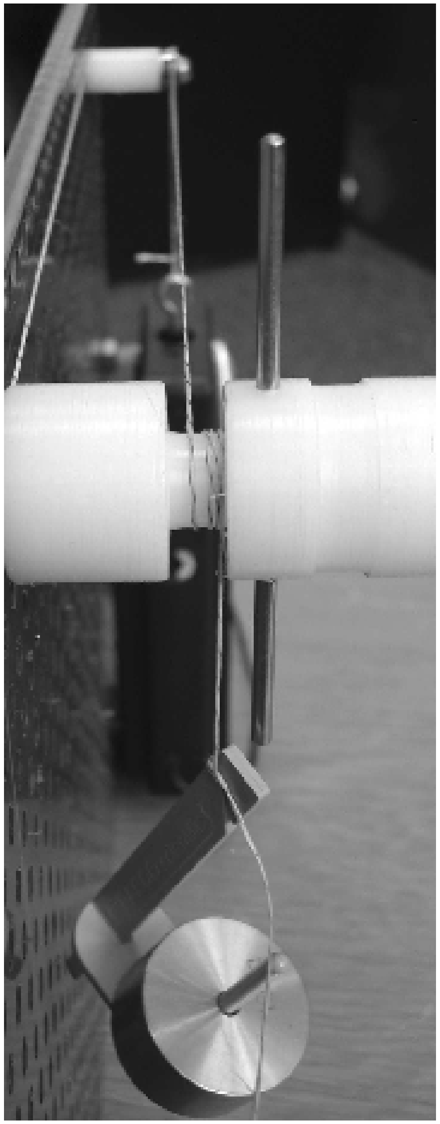
Stop after 10 s.

Set Up :

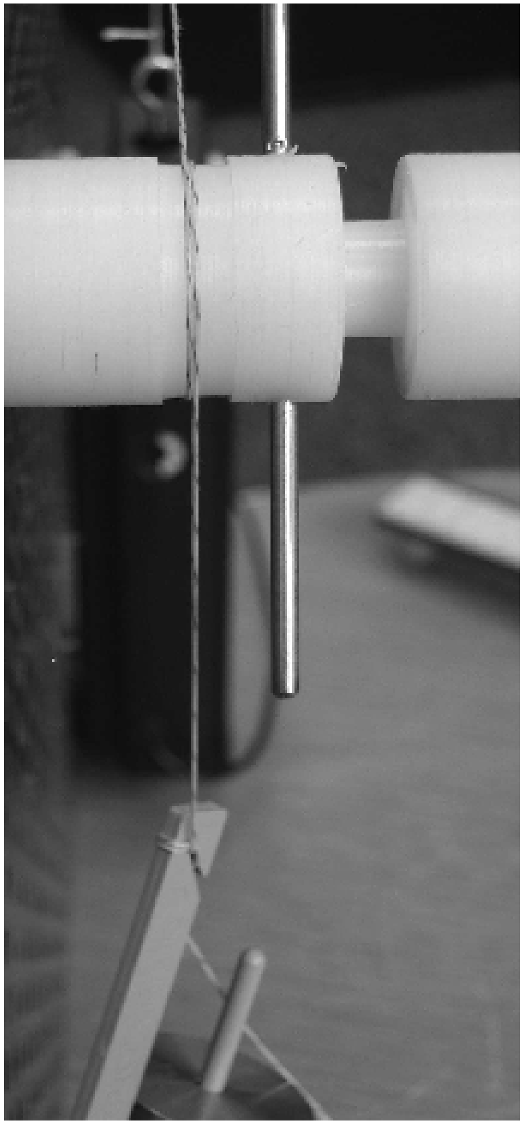
Choose large or small cylinder.

Both kinds at each table.

100 gm brass weight + 5 gm holder = 1.03 N.



Small Cylinder



Large Cylinder

Measuring technique:

Tare the force sensor.

Ensure string passes over pulley before all measurements.

Do not wrap the string over itself.

Wrap string 2 ¼ turns around cylinder.

Increase ccw torque on cylinder smoothly to the slipping point, then turn slowly for 1 or 2 seconds.

Practice this, so you get 1 or 2 peaks of the force in 10 s, then record the force with *DataStudio*.

Measuring the Friction:

Choose the largest <u>magnitude</u> of Force as the friction.
Use the Smart Tool or the Statistics (Σ) Tool.

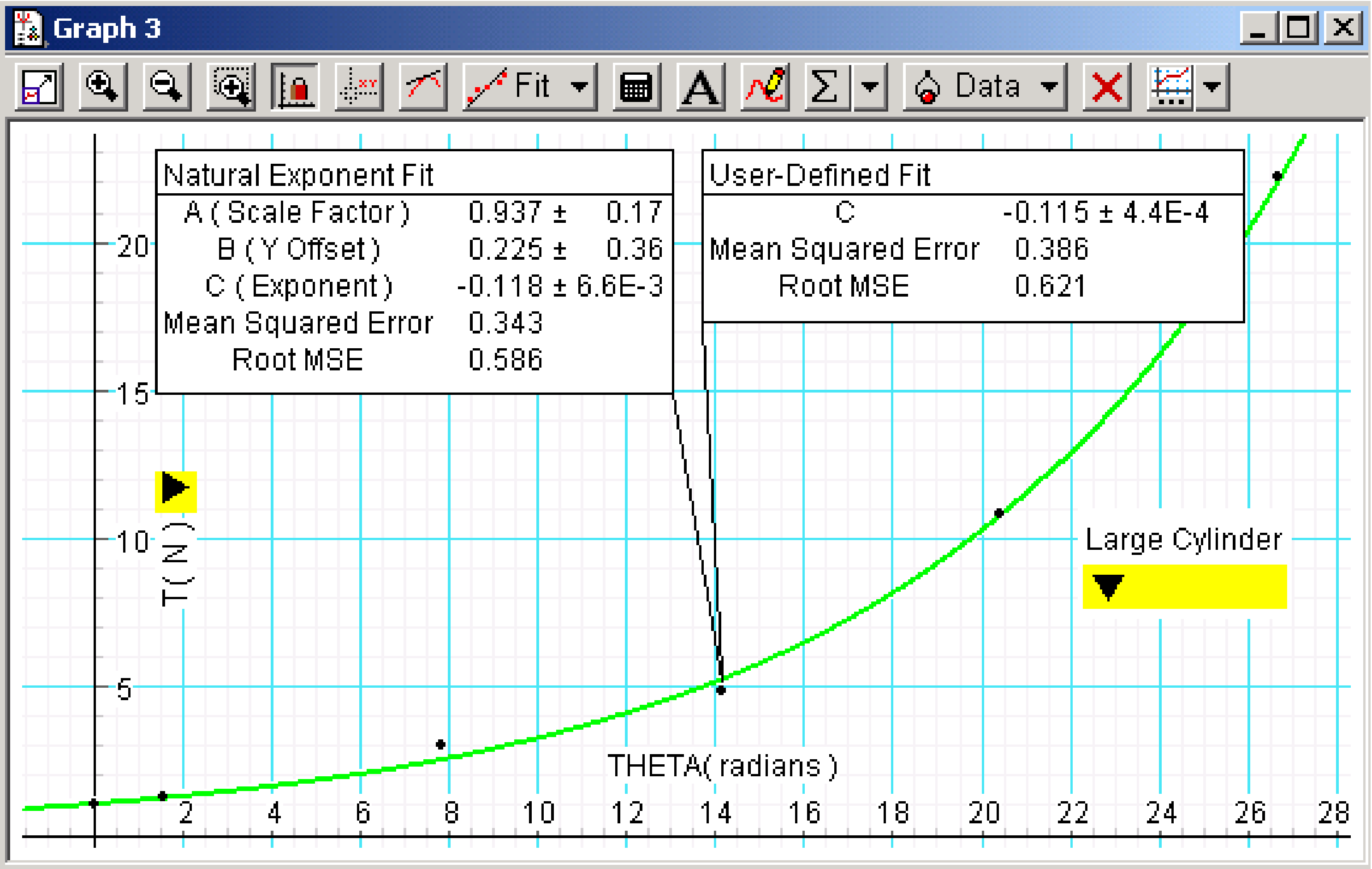
Recording the Friction:

Measure the friction for the string wrapped ¼, 1¼, 2¼, 3¼ and 4¼ turns. Express these in radians and type them as the independent variable, along with the friction force as the dependent variable, into a table in DataStudio.

Include the force of 1.03 N when the wrap angle is 0 as an entry in your table.

Plot the table on a graph and carry out a User-Defined Fit with the function `1.03*exp(-C*x)`.

Fit Result (large cylinder):



Fit Result (small cylinder):