MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

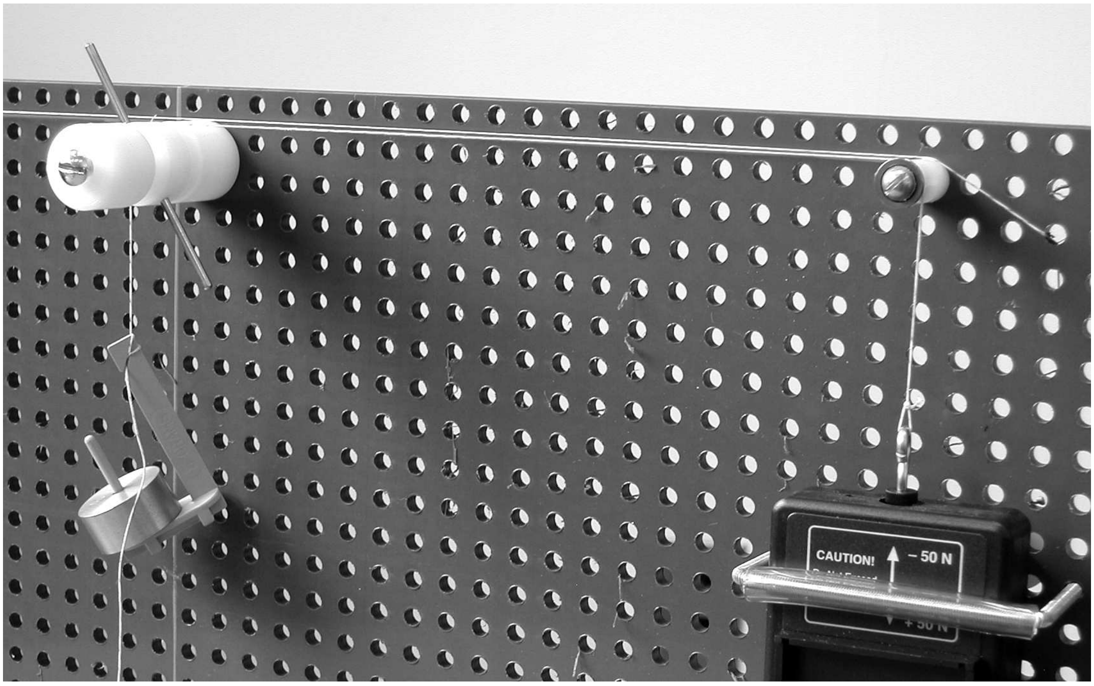
Physics 8.01T — Fall Term 2004

Experiment 05B: Friction

Purpose of the Experiment:
In this experiment you will study the friction between a string and a plastic cylinder. You should observe:

- The ideal case where there is a coefficient of static friction that is obviously greater than the coefficient of sliding friction may not apply in all situations.
- The friction force between the string and cylinder increases exponentially with the distance the string is wrapped around the cylinder.
- You will measure an effective coefficient of friction by fitting an exponential function to your data.

Setting Up the Apparatus:



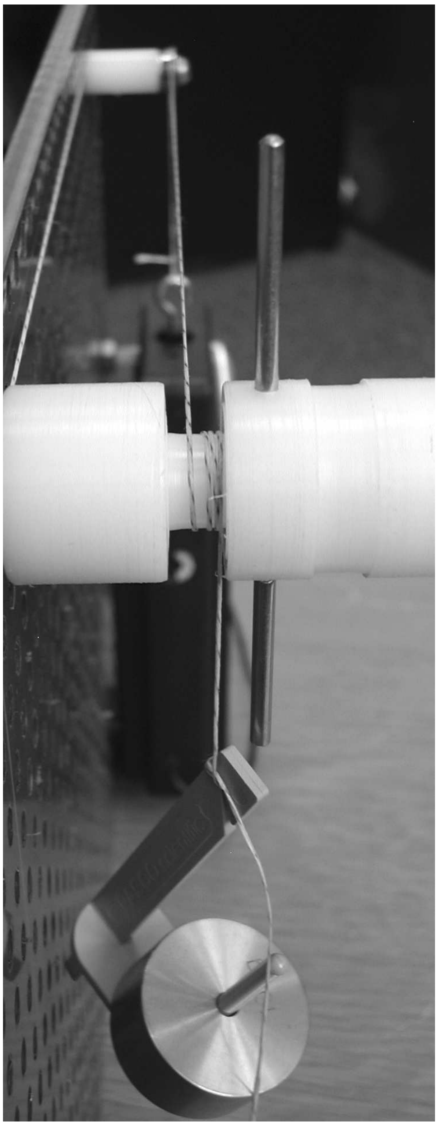
Use the same apparatus as for the static equilibrium experiment, but remove the sliding T-square and attach the plastic cylinder near the top center of the board. The cylinder has a handle so you can turn it and two regions for wrapping the string. One has a diameter of 0.5 in and the other of 1.0 in. You may use either one, but discuss with the other groups at your table to make sure that both diameters are being used at your table.

The diameter you are using should be placed closest to the mounting board; note that the two diameters use different mounting holes. Choose the mounting hole so that a string passing over the pulley to the force sensor and around the plastic cylinder will be horizontal.

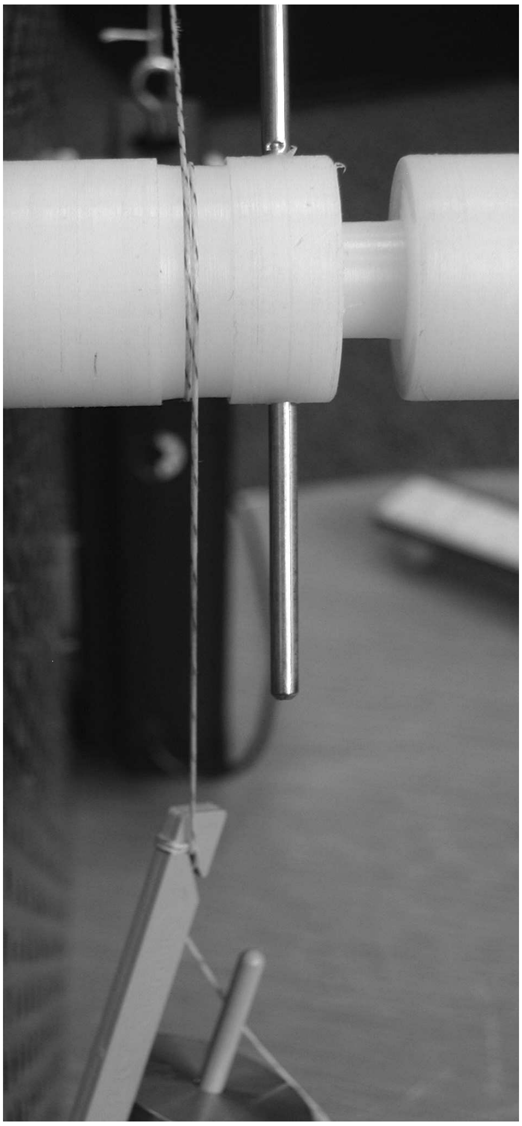
Here are photos showing more details of how the cylinder may be used.

One end of the string passes over or wraps around the cylinder and is attached to a brass weight by a plastic holder. The other passes over a bearing as a low-friction pulley to transmit the tension force to the force sensor. **It is very important for the string to pass over this pulley for all your measurements so that the force will be measured correctly.** Try to avoid wrapping the string over itself.

Choose a brass weight of mass 100 gm; then the weight of the brass plus holder (105 gm mass) will be 1.03 N.



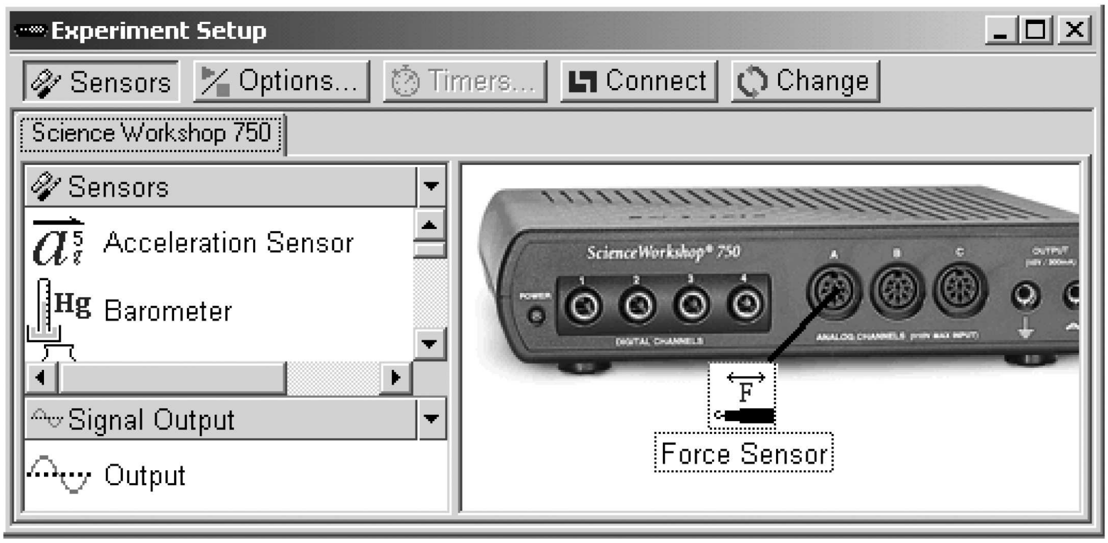
Small Cylinder



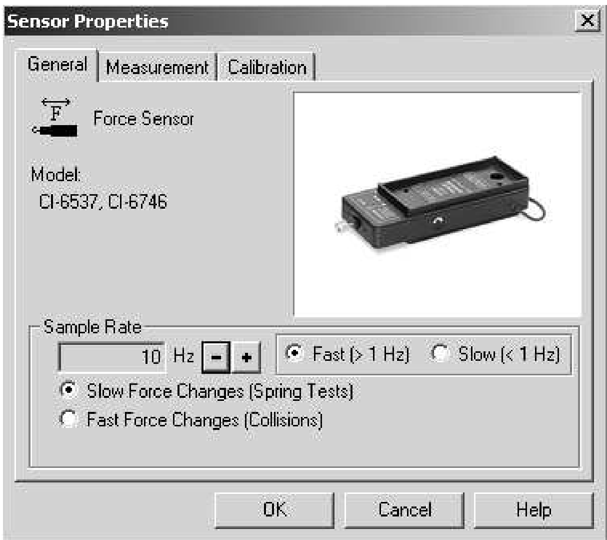
Large Cylinder

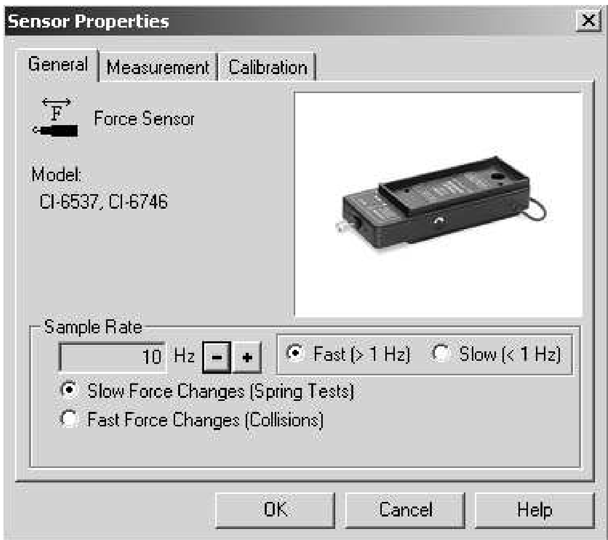
Setting Up DataStudio:

To begin, plug the force sensor into the 750 interface and start up *DataStudio*.



Drag the force sensor icon to the interface in the Experiment Setup window. Double-click the force sensor icon to open a window to set the Sensor Properties. Under the "General" tab, choose a sample rate of 10 Hz, and "Slow Force Changes (Spring Tests)". Under the "Calibration" tab, choose Sensitivity: "Low (1x)". (The force sensor records force as a negative number when you pull on it.)

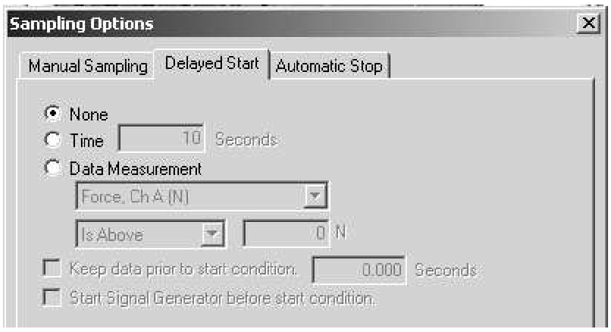


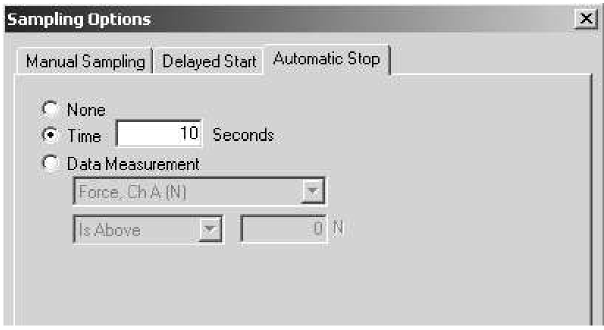


Next, click the Options 

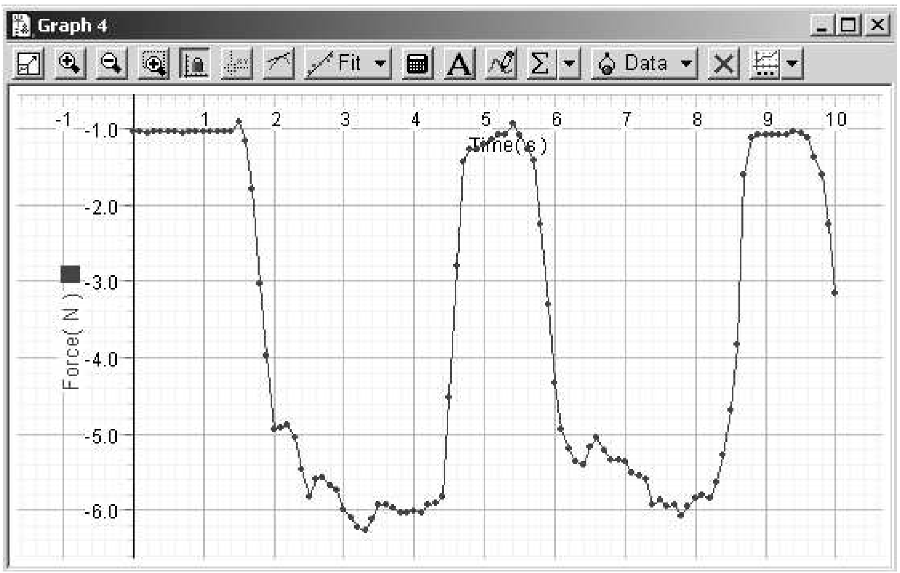
button.

On the Manual Sampling tab of the Sampling Options window that opens, make sure that none of the boxes are checked. Set the delayed start to none, and automatic stop to 10 s.





Measuring the Friction:
Wind the string $2 + \frac{1}{4}$ turns around the cylinder and hang the weight from it, as in the photos on the second page. Hold the weight so there is no tension in the string and tare the force sensor. **Make sure the string passes over the pulley**, the string does not overlap itself as it wraps around the cylinder, and let the weight hang. Grasp the cylinder and twist it counter-clockwise. As you increase your twisting force you should reach a point where the string starts to slip. Practice this a few times so you get the feel of it and are able to smoothly increase the torque to the point where slipping starts and are able to keep it slipping for a second or two. When you are ready, open a graph by dragging the Force entry from the Data window onto the Graph entry in the Displays window. Have a partner click the Start button. For 10 s, the force will be plotted on the graph. This should be a long enough time to ramp the torque to the slipping point, maintain slow rotation for a second or so, then back off and repeat the process once or twice. You should get a graph something like this one.

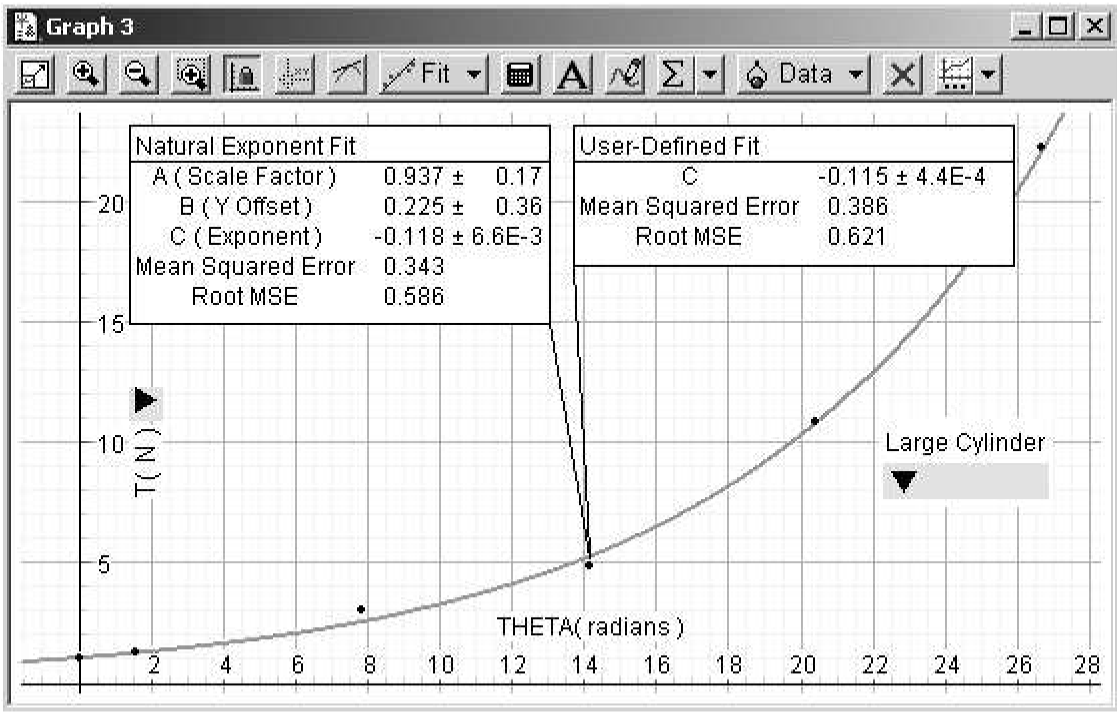


As you can see from the graph, and may have noticed while watching the string as you rotated the cylinder, the ideal behavior of a clean threshold (determined by the coefficient of static friction μ_s) at which slipping occurs which is then followed by a lower frictional force (determined by the coefficient of sliding friction μ_k) is not really observed. With a waxed string and a plastic cylinder that is not carefully prepared, that is perhaps to be expected. My observation of the string was that it did not slide smoothly over the cylinder, that it was rather jerky as the string stretched, slipped, and relaxed. This suggests that the string-plastic interface may not be well described by the simple idea of a μ_s and a μ_k.

So, assume that some effective coefficient μ characterizes the interaction between the string and the plastic and determines the maximum friction force you observe. The Smart Tool or the Statistics Tool Σ can help to find the maximum tension T (the minimum F, as $F < 0$). In this way, you may proceed to see how the friction changes with the amount of string wrapped around the cylinder. The geometry of the experiment restricts the wrapping to be $n + \frac{1}{4}$ turns, where n is an integer. The wrappings you should try are in the table below and, to save you some calculation, I have put in the corresponding wrapping angle θ.

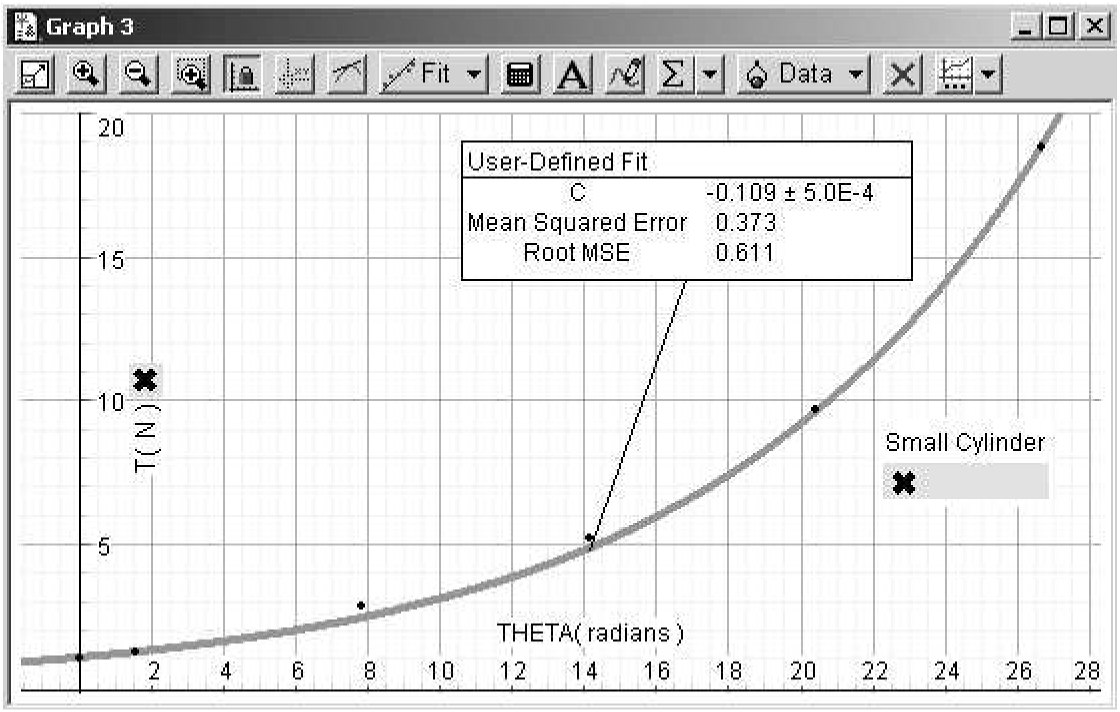
Wrap (turns)	0	$\frac{1}{4}$	$1 + \frac{1}{4}$	$2 + \frac{1}{4}$	$3 + \frac{1}{4}$	$4 + \frac{1}{4}$
θ (radians)	0	1.57	7.85	14.14	20.42	26.70
T (N)	1.03					

Of course, T would be 1.03 N if you could measure it with $\theta = 0$, so I put that in the table. In your experiment you should complete this table and then type the results into a table in *DataStudio*. Put θ as the independent (X or left column) variable, and T as the dependent variable. Drag the resulting table from the Data window onto the Graph icon in the Displays window to make a graph.



The graph has my data for the large (1 in. dia.) cylinder. From the theory in the appendix, I expect $T = 1.03\,e^{\mu\theta}$ N. First I tried the Natural Exponent Fit built into *DataStudio*. This function includes a background term, which should be zero according to the model (and to common sense!) if the force sensor was properly tared. It fit the data, but with large uncertainties in the parameters. Thus I turned to a User-Defined Fit with the definition of the fit function as `1.03*exp(-C*x)`. This fit the data and gave the value $\mu = 0.12$. The Natural Exponent Fit is not plotted on the graph, but it would be difficult to distinguish from the User-Defined Fit that is plotted.

I also did the experiment for the small cylinder (0.5 in. dia.) and here are my results for it.



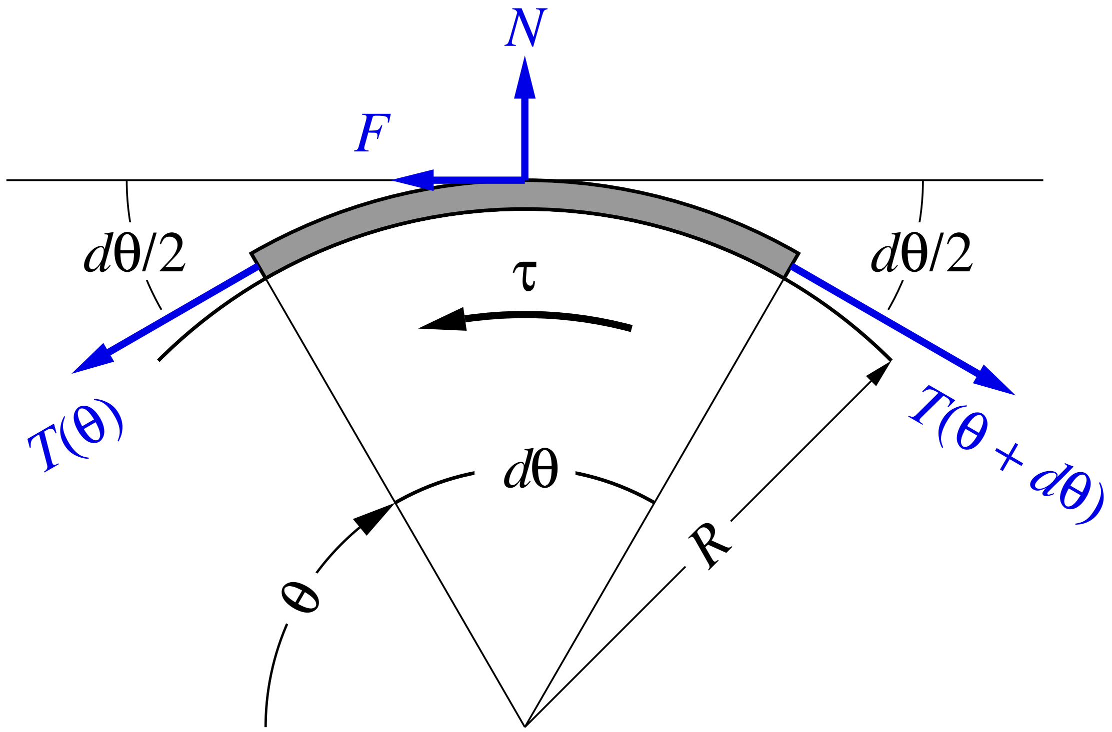
The fit gave $\mu = 0.11$, in good agreement with the result for the large cylinder. (Again, the Natural Exponent Fit worked, but the parameter values were unreasonable, a case of too many fit parameters for somewhat noisy data being a bad thing.)

My conclusion is that friction between the string and plastic can be be described by an effective coefficient μ with a value about 0.11, that the friction of a string wrapped around a cylinder (or a rope around a tree) increases exponentially with the wrapping angle, and that this result does not depend on the diameter of the cylinder.

Note: I used PASCO's "physics string", which appears to be waxed. You may have different string (e.g., black nylon fishing line) and get somewhat different results than I did.

Appendix: Derivation of Friction Formula

This is a derivation of the equation that describes the friction of a string wrapped around a cylinder. You will not be responsible to repeat this derivation on an exam, but reading it will give you some insight into how calculus can be used to solve physics problems.



The figure above shows a free body diagram for a piece of the string, which is wrapped around a cylinder of radius R. The piece of string we consider subtends an angle $d\theta$, and we will be interested in the situation when $d\theta$ is infinitesimally small. (In the figure $d\theta$ is about 60°, but that is so we can see what is going on.) Some external force is trying to rotate the cylinder in the direction (ccw) shown by the arrow labeled τ. Then the friction force F acting on the piece of string will be to the left, as shown. The other forces acting on the small piece of string are a normal force N from the cylinder and the tension forces exerted by the rest of the string. The problem is easiest to solve if we choose the angle around the cylinder, θ, as a coordinate. In the free body diagram, consider the force components that are normal to and along the piece of string at its center.

Equating the normal components one finds

$$N = T(\theta)\sin(d\theta/2) + T(\theta + d\theta)\sin(d\theta/2)\,,$$

and setting the components along the string to be equal gives

$$F + T(\theta)\cos(d\theta/2) = T(\theta + d\theta)\cos(d\theta/2)\,.$$

Here is where the power of calculus helps us out. If $d\theta$ is very small, then $\sin(d\theta/2) = d\theta/2$ and $\cos(d\theta/2) = 1$. We may also ignore all terms having $(d\theta)^2$ and higher powers. And finally, we may write

$$T(\theta + d\theta) = T(\theta) + \frac{dT}{d\theta}\, d\theta$$

All of these approximations become exact in the limit $d\theta \to 0$. The two equations on the previous page then become enormously simpler.

$$N = T(\theta) d\theta\,,$$

and

$$F + T(\theta) = T(\theta) + \frac{dT}{d\theta}\, d\theta\,,$$

or

$$F = \frac{dT}{d\theta}\, d\theta\,.$$

When we use the relationship $F = \mu N$, where μ is the coefficient of friction, we get a single equation

$$\mu T(\theta) d\theta = \frac{dT}{d\theta}\, d\theta\,.$$

Divide both sides by $d\theta$ to get

$$\mu T = \frac{dT}{d\theta} \quad \text{or} \quad \frac{dT}{T} = \mu\, d\theta\,.$$

This equation is the result of the free body analysis of an infinitesimal piece of string. To see how a larger piece behaves, we must use the rules of calculus to put all of the tiny pieces together. (In 18.01, that is called integrating the differential equation.) To find the solution, we need to recognize that dT/T is the derivative of $\ln T$; then we simply integrate the derivatives on both sides of the equation

$$\frac{dT}{T} = d(\ln T) = \mu\, d\theta\,.$$

Suppose the string starts to wrap around the cylinder at $\theta = 0$ and we know the tension $T(\theta = 0) = T_0$ at that point. We would like to find the tension at some value $\theta > 0$. What we have to do is integrate the differential equation from the starting point $\theta = 0$ (where we know the tension) to the point θ (where we want to calculate the tension). That is:

$$\int_0^\theta d(\ln T) = \mu \int_0^\theta d\theta\,.$$

Doing the integrals gives

$$\ln[T(\theta)] - \ln[T_0] = \ln\left[\frac{T(\theta)}{T_0}\right] = \mu\theta$$

or, after taking e to the power of each side,

$$T(\theta) = T_0\, e^{\mu\theta}\,.$$

This expression relates the tension in the string when it first contacts the cylinder, T_0, to the tension at any point in the string up to the angle at which it leaves the cylinder. It is interesting that the answer does not depend on the radius R of the cylinder. Can you explain in words why that is? (Naively, one might think that a larger diameter cylinder would have more string in contact with it, hence greater friction.)

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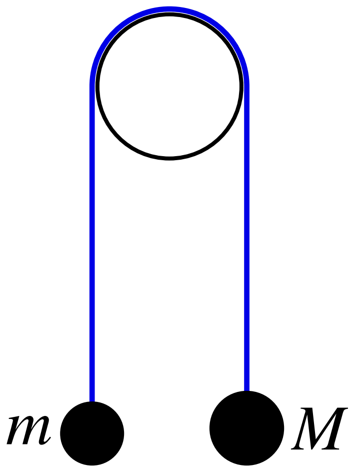
Part of Problem Set 06

Section and Group: ______________________

Your Name: ______________________

Two unequal masses ($M > m$) are suspended by a string over a rod. Suppose the behavior is ideal: the coefficient of static friction between the string and the rod is μ_s, and the coefficient of sliding friction is $\mu_k < \mu_s$.



1. What is the largest ratio M/m before the string starts to slide?
2. What is the magnitude of the acceleration of the masses after sliding has begun?