MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 TEAL **Fall Term 2004**

Exam 3: Equation Summary

Momentum:

$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$, $\vec{\mathbf{F}}_{ave}\Delta t = \Delta\vec{\mathbf{p}}$, $\vec{\mathbf{F}}_{ext}^{total} = \dfrac{d\vec{\mathbf{p}}^{total}}{dt}$

Impulse: $\vec{\mathbf{I}} \equiv \int_{t=0}^{t=t_f} \vec{\mathbf{F}}(t)dt = \Delta\vec{\mathbf{p}}$

Torque: $\vec{\boldsymbol{\tau}}_S = \vec{\mathbf{r}}_{S,P} \times \vec{\mathbf{F}}_P$ $\left|\vec{\boldsymbol{\tau}}_S\right| = \left|\vec{\mathbf{r}}_{S,P}\right|\left|\vec{\mathbf{F}}_P\right| sin\,\theta = r_\perp F = rF_\perp$

Static Equilibrium:
$\vec{\mathbf{F}}_{total} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + ... = \vec{\mathbf{0}}$; $\vec{\boldsymbol{\tau}}_S^{total} = \vec{\boldsymbol{\tau}}_{S,1} + \vec{\boldsymbol{\tau}}_{S,2} + ... = \vec{\mathbf{0}}$.

Rotational dynamics: $\vec{\boldsymbol{\tau}}_S^{total} = \dfrac{d\vec{\mathbf{L}}_S}{dt}$

Angular Velocity: $\vec{\boldsymbol{\omega}} = \left(d\theta/dt\right)\hat{\mathbf{k}}$

Angular Acceleration: $\vec{\boldsymbol{\alpha}} = \left(d^2\theta/dt^2\right)\hat{\mathbf{k}}$

Fixed Axis Rotation: $\vec{\boldsymbol{\tau}}_S = I_S\vec{\boldsymbol{\alpha}}$

$\tau_S^{total} = I_S\alpha = I_S\dfrac{d\omega}{dt}$

Moment of Inertia: $I_S = \int_{body} dm(r_\perp)^2$

Angular Momentum: $\vec{\mathbf{L}}_S = \vec{\mathbf{r}}_{S,m} \times m\vec{\mathbf{v}}$,

$\left|\vec{\mathbf{L}}_S\right| = \left|\vec{\mathbf{r}}_{S,m}\right|\left|m\vec{\mathbf{v}}\right| sin\,\theta = r_\perp p = rp_\perp$

Angular Impulse:
$\vec{\mathbf{J}}_S = \int_{t_0}^{t_f} \vec{\boldsymbol{\tau}}_S dt = \Delta\vec{\mathbf{L}}_S = \vec{\mathbf{L}}_{S,f} - \vec{\mathbf{L}}_{S,0}$

Rotation and Translation:
$\vec{\mathbf{L}}_S^{\ total} = \vec{\mathbf{L}}_S^{orbital} + \vec{\mathbf{L}}_{cm}^{\ spin}$,

$\vec{\mathbf{L}}_S^{orbital} = \vec{\mathbf{r}}_{S,cm} \times \vec{\mathbf{p}}^{total}$,

$\vec{\mathbf{L}}_{cm}^{\ spin} = I_{cm}\vec{\boldsymbol{\omega}}_{spin}$

$\vec{\boldsymbol{\tau}}_S^{orbit} = \dfrac{d\vec{\mathbf{L}}_S^{\ orbit}}{dt}$, $\vec{\boldsymbol{\tau}}_{cm}^{spin} = \dfrac{d\vec{\mathbf{L}}_{cm}^{\ spin}}{dt}$

Rotational Energy: $K_{cm} = \frac{1}{2} I_{cm} \omega_{cm}{}^2$

Rotational Power: $P_{rot} \equiv \frac{dW_{rot}}{dt} = \vec{\boldsymbol{\tau}}_S \cdot \vec{\boldsymbol{\omega}} = \tau_S \omega = \tau_S \frac{d\theta}{dt}$

One Dimensional Kinematics: $\vec{\mathbf{v}} = d\vec{\mathbf{r}} / dt$, $\vec{\mathbf{a}} = d\vec{\mathbf{v}} / dt$

$$v_x(t) - v_{x,0} = \int_{t'=0}^{t'=t} a_x(t')dt' \qquad x(t) - x_0 = \int_{t'=0}^{t'=t} v_x(t')dt'$$

Constant Acceleration:

$$x(t) = x_0 + v_{x,0}(t - t_0) + \frac{1}{2} a_x (t - t_0)^2 \qquad v_x(t) = v_{x,0} + a_x (t - t_0)$$

$$y(t) = y_0 + v_{y,0}(t - t_0) + \frac{1}{2} a_y (t - t_0)^2 \qquad v_y(t) = v_{y,0} + a_y (t - t_0)$$

where $x_0, v_{x,0}, y_0, v_{y,0}$ are the initial position and velocities components at $t = t_0$

Newton's Second Law: Force, Mass, Acceleration

$$\vec{\mathbf{F}} \equiv m\vec{\mathbf{a}} \qquad \vec{\mathbf{F}}^{total} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 \qquad F_x{}^{total} = ma_x \qquad F_y{}^{total} = ma_y \qquad F_z{}^{total} = ma_z$$

Newton's Third Law: $\vec{\mathbf{F}}_{1,2} = -\vec{\mathbf{F}}_{2,1}$

Force Laws:

Universal Law of Gravity: $\vec{\mathbf{F}}_{1,2} = -G \frac{m_1 m_2}{r_{1,2}{}^2} \hat{\mathbf{r}}_{1,2}$, attractive

Gravity near surface of earth: $\vec{\mathbf{F}}_{grav} = m_{grav} \vec{\mathbf{g}}$, towards earth

Contact force: $\vec{\mathbf{F}}_{contact} = \vec{\mathbf{N}} + \vec{\mathbf{f}}$, depends on applied forces

Static Friction: $0 \le f_s \le f_{s,\max} = \mu_s N$ direction depends on applied forces

Kinetic Friction: $f_k = \mu_k N$ opposes motion

Hooke's Law: $F = k\left|\Delta x\right|$, restoring

Kinematics Circular Motion: arc length: $s = R\theta$; angular velocity: $\omega = d\theta / dt$ tangential velocity: $v = R\omega$; angular acceleration: $\alpha = d\omega / dt = d^2\theta / dt^2$; tangential acceleration $a_\theta = R\alpha$.

Period: $T = \dfrac{2\pi R}{v} = \dfrac{2\pi}{\omega}$; **frequency:** $f = \dfrac{1}{T} = \dfrac{\omega}{2\pi}$,

Radial Acceleration: $|a_r| = R\,\omega^2$; $|a_r| = \dfrac{v^2}{R}$; $|a_r| = 4\pi^2 R\, f^2$; $|a_r| = \dfrac{4\pi^2 R}{T^2}$

Center of Mass: $\vec{\mathbf{R}}_{cm} = \sum_{i=1}^{i=N} m_i \vec{\mathbf{r}}_i / m^{total} \to \int_{body} dm\vec{\mathbf{r}} / m^{total}$;

Velocity of Center of Mass: $\vec{\mathbf{V}}_{cm} = \sum_{i=1}^{i=N} m_i \vec{\mathbf{v}}_i / m^{total} \to \int_{body} dm\vec{\mathbf{v}} / m^{total}$

Torque: $\vec{\boldsymbol{\tau}}_S = \vec{\mathbf{r}}_{S,P} \times \vec{\mathbf{F}}_P$ $|\vec{\boldsymbol{\tau}}_S| = |\vec{\mathbf{r}}_{S,P}||\vec{\mathbf{F}}_P| \sin\theta = r_\perp F = rF_\perp$

Static Equilibrium: $\vec{\mathbf{F}}_{total} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \ldots = \vec{\mathbf{0}}$; $\vec{\boldsymbol{\tau}}_S^{total} = \vec{\boldsymbol{\tau}}_{S,1} + \vec{\boldsymbol{\tau}}_{S,2} + \ldots = \vec{\mathbf{0}}$.

Kinetic Energy: $K = \dfrac{1}{2} m v^2$; $\Delta K = \dfrac{1}{2} m {v_f}^2 - \dfrac{1}{2} m {v_0}^2$

Work: $W = \int_{r_0}^{r_f} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$; **Work- Kinetic Energy:** $W^{total} = \Delta K$

Power: $P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}} = dK/dt$

Potential Energy: $\Delta U = -W_{conservative} = -\int_A^B \vec{\mathbf{F}}_c \cdot d\vec{\mathbf{r}}$

Potential Energy Functions with Zero Points:

Constant Gravity: $U(y) = mgy$ with $U(y_0 = 0) = 0$.

Inverse Square Gravity: $U_{gravity}(\mathrm{r}) = -\dfrac{G m_1 m_2}{r}$ with $U_{gravity}(\mathrm{r}_0 = \infty) = 0$.

Hooke's Law: $U_{spring}(x) = \dfrac{1}{2} k x^2$ with $U_{spring}(x = 0) = 0$.

Work- Mechanical Energy: $W_{nc} = \Delta K + \Delta U^{total} = \Delta E_{mech} = \left(K_f + U_f^{\;total}\right) - \left(K_0 + U_0^{\;total}\right)$