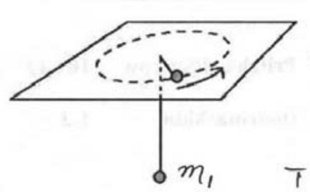
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

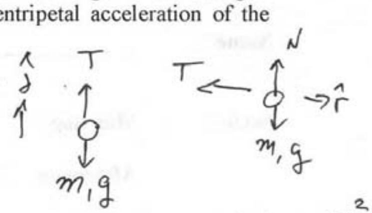
Physics 8.01T **Fall Term 2004**

Exam 2 Solutions

Problem 1: (20 points) Concept Questions: Circle the correct answer.

a) **(4 points)** A puck of mass M is moving in a circle at uniform speed on a frictionless table as shown above. It is held by a string which holds a suspended bob, also of mass M, at rest below the table. Half of the length of the string is above the tabletop and half below. What is the centripetal acceleration of the moving puck?





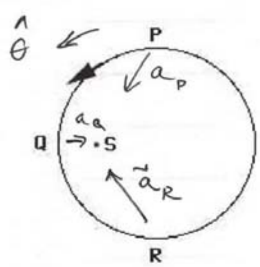
$T - m_1 g = 0$ $\quad -T = -\dfrac{m_1 v^2}{r}$

$\Rightarrow T = m_1 g$ $\quad m_1 g = m_1 \dfrac{v^2}{r}$

$\Rightarrow \dfrac{v^2}{r} = g$

1. less than g
2. g
3. greater than g
4. zero
5. insufficient information

b) **(4 points)** An object moves counter-clockwise along the circular path shown below. As it moves along the path its acceleration vector continuously points toward point *S*. The object



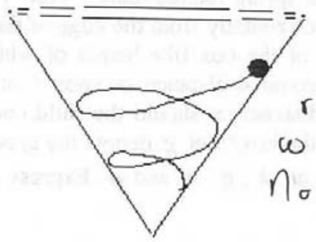
At point P: $\vec{a}_P$ has positive component in $\hat{\theta}$ direction indicating $a_{tan} > 0$, speeds up

At point S: $\vec{a}_S$ has only radially inward component.

At point R: $\vec{a}_R$ has negative component in $\hat{\theta}$ direction indicating $a_{tan} < 0$, slows down.

1. speeds up at *P*, *Q*, and *R*.
2. slows down at *P*, *Q*, and *R*.
3. speeds up at *P* and slows down at *R*.
4. slows down at *P* and speeds up at *R*.
5. speeds up at *Q*.
6. slows down at *Q*.
7. No object can execute such a motion.

c) **(4 points)** A ball is rolling without slipping in a spiral path down the inside of a hollow cone.



Since the ball rolls without slipping contact point is at rest, friction does no work.
Normal force ⊥ displacement so normal force does no work.

The work done by the inner surface of the cone on the ball is

1. positive
2. zero
3. negative
4. Impossible to determine

d) **(4 points)** Suppose you want to ride your mountain bike up a steep hill. Two paths lead from the base to the top, one twice as long as the other. You start and end at rest. Compared to the average force you would exert if you took the short path, the average force you exert along the longer path is

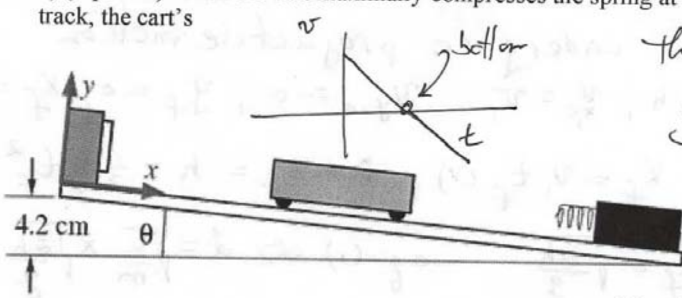
$W_{ext} = mgh = (F_{ave})_1 x_1 = (F_{ave})_2 x_2$

since $x_2 = 2x_1 \Rightarrow$

$(F_{ave})_2 = (F_{ave})_1 \dfrac{x_1}{x_2} = \dfrac{(F_{ave})_1}{2}$

1. one quarter as much.
2. one third as much.
3. one half as much.
4. the same.
5. undetermined-it depends on the time taken.

e) **(4 points)** When the cart maximally compresses the spring at the bottom of the track, the cart's



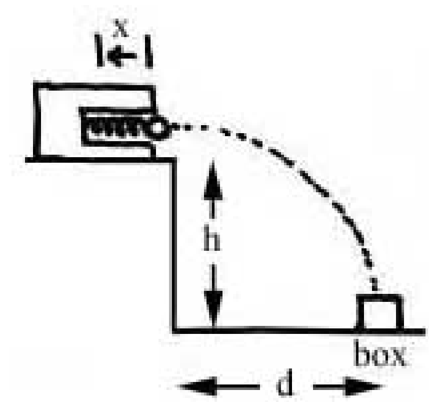
the spring force is maximum. The cart comes to rest but the spring force is accelerating the cart in the negative x-direction

1. velocity and acceleration are zero.
2. velocity is nonzero but its acceleration is zero.
3. acceleration is nonzero, but its velocity is zero.
4. velocity and acceleration are both nonzero.

1.

Part 2: Analytic Questions: Show all your work.

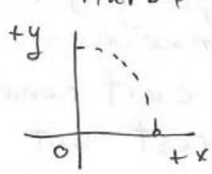
Problem 2: (20 points) A child is trying to shoot a marble of mass m in order to hit the center of a small box using a spring loaded marble gun. The marble gun is fixed on a table and shoots the marble horizontally from the edge of the table. The edge of the table is a height h above the top of the box (the height of which is negligibly small). The center of the box is some horizontal distance d away from the table. The spring has a spring constant k. By what distance x should the child compress the spring so that the marble lands in the center of the box? Let g denote the gravitational constant. The given quantities in this problem are m, k, g, h, and d. Express x in terms of whatever given quantities you may need.



Mechanical energy is conserved during launching of marble

$$E_0 = \frac{1}{2} k x^2 = \frac{1}{2} m v_1^2 = E_1 \Rightarrow v_1 = \sqrt{\frac{k}{m}}\, x$$

Marble then undergoes projectile motion



$y_0 = h$, $v_{x,0} = v_1$, $v_{y,0} = 0$, $y_f = 0$, $x_f = d$

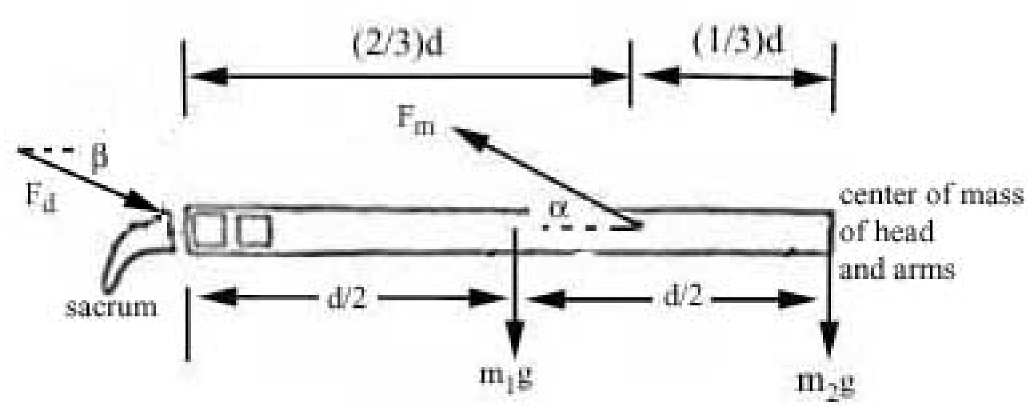
$x_f = v_1 t_f$ (1), $0 = y_f = h - \frac{1}{2} g t_f^2$ (2)

eq (2) $\Rightarrow t_f = \sqrt{\frac{2h}{g}}$ eq (1) $\Rightarrow d = \sqrt{\frac{k}{m}}\, x \sqrt{\frac{2h}{g}}$

$$\Rightarrow x = \sqrt{\frac{mg}{2kh}}\, d$$

Problem 3: (20 points) A Back Bending Exercise

When a person of mass m bends over or lifts an object, the main muscles that lift the back are the erector spinae (sacrospinal muscles). These muscles act approximately at a single point on the vertebral column and exert a force, $\vec{\mathbf{F}}_m$, at a point that is $2/3$ of the distance from base of the spine to the center of mass of the head and arms. The cord applies the force at an angle α relative to the axis of the vertebral column. The sacrum exerts a force, $\vec{\mathbf{F}}_d$, on the spine at an unknown angle β relative to the axis of the vertebral column.



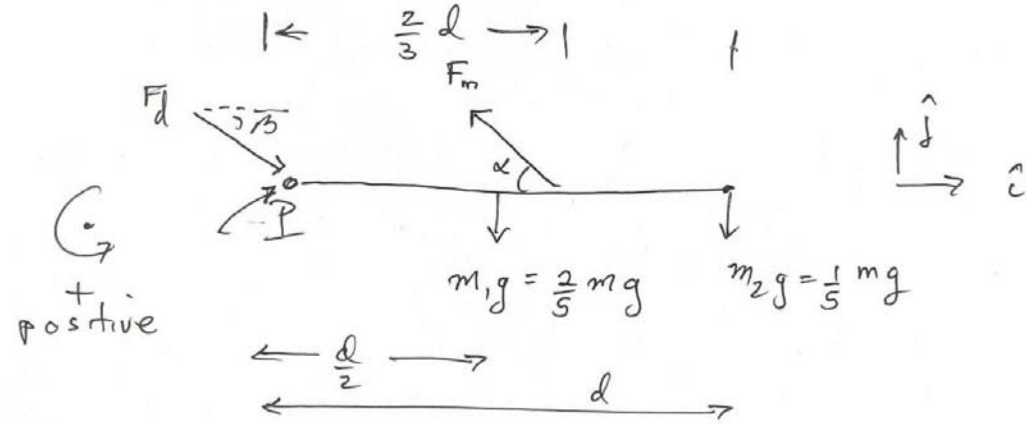
Assume that mass of the head and arms is $m_2 = (1/5)m$ where m is the mass of the person. The center of mass of the trunk is $1/2$ of the distance from the base of the spine to the center of mass of the head and arms. Assume that mass of the trunk is $m_1 = (2/5)m$, where m is the mass of the person. Let g denote the gravitational constant.

In this problem assume the given quantities are m, α, and g. Express your answers below in terms of these quantities.

a) What is the magnitude of the force $\vec{\mathbf{F}}_m$ that the sacrospinal muscles exert on the spine?

b) What is the angle β that the force of the sacrum on the spine, $\vec{\mathbf{F}}_d$, makes relative to the axis of the vertebral column?

c) What is the magnitude of the force $\vec{\mathbf{F}}_d$?



$$\vec{F} = m\vec{a}$$

$\hat{i}: -F_m \cos\alpha + F_d \cos\beta = 0$ (1)

$\hat{j}: F_m \sin\alpha - F_d \sin\beta - m_1 g - m_2 g = 0$ (2)

$$\vec{\tau}_P = 0$$

choose P to be the point where the sacrum applies the force on the spine. Choose ⊕ = .

$$\vec{\tau}_P = 0$$

$\frac{2}{3} d\, F_m \sin\alpha - \frac{d}{2} m_1 g - d\, m_2 g = 0$ (3)

Use $m_1 g = \frac{2}{8} mg$, $m_2 g = \frac{1}{5} mg$

(1) $-F_m \cos\alpha + F_d \cos\beta = 0$

(2) $F_m \sin\alpha - F_d \sin\beta - \frac{2}{8} mg - \frac{1}{5} mg = 0$

(3) $\frac{2}{3} d\, F_m \sin\alpha - \frac{d}{2}\left(\frac{2}{5} mg\right) - d\, \frac{1}{5}\, mg = 0$

Eq (3) becomes

$$\frac{2}{3} d \ F_m \sin\alpha = \frac{2}{5} d m g$$

$$\Rightarrow \quad F_m \sin\alpha = \frac{3}{5} m g \quad \Rightarrow F_m = \frac{3}{5} \frac{mg}{\sin\alpha}$$

Eq (2) becomes

$$\frac{3}{5} mg - F_d \sin\beta - \frac{3}{5} mg = 0$$

$$\Rightarrow \quad F_d \sin\beta = 0 \quad \Rightarrow \beta = 0 \quad \text{since } F_d \neq 0$$

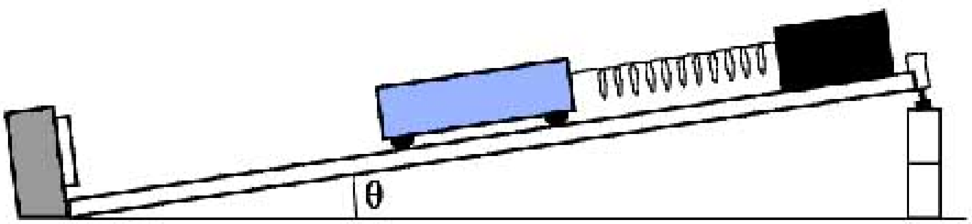
Eq (1) becomes

$$F_m \cos\alpha = F_d \cos\beta = F_d \quad \text{since } \beta = 0$$

$$F_d = \frac{3}{5} mg \frac{\cos\alpha}{\sin\alpha} = \frac{3}{5} mg \cot\alpha$$

Problem 4: (20 points)

Consider an ideal spring that has an unstretched length l_0. Assume the spring has a constant k. Suppose the spring is attached to an cart of mass m that lies on a frictionless plane that is inclined by an angle θ from the horizontal. Let g denote the gravitational constant. The given quantities in this problem are l_0, m, k, θ, and g.

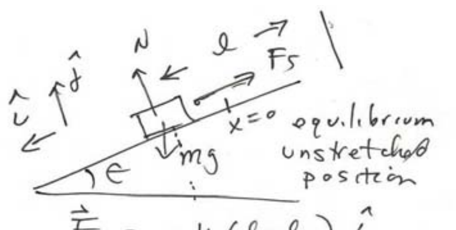


a) The spring stretches slightly to a new length $l > l_0$ to hold the cart in equilibrium. Find the length l in terms of the given quantities.

b) Now move the cart up along the ramp so that the spring is <u>compressed</u> a distance x from the unstretched length l_0. Then the cart is released from rest. What is the velocity of the cart when the spring has first returned to its unstretched length l_0?

c) What is the <u>period</u> of oscillation of the cart?

a)



$$\vec{F}_s = -k(l - l_0)\,\hat{i}$$

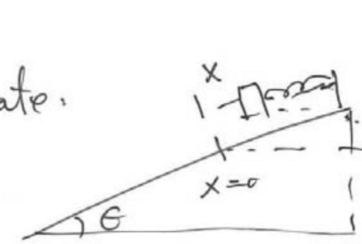
$$\vec{F} = m\vec{a}$$

$\hat{i}$: $mg\sin\theta - F_s = 0$ $\Rightarrow$ $mg\sin\theta - k(l - l_0) = 0$

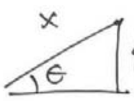
$$\Rightarrow l = \frac{mg\sin\theta}{k} + l_0$$

b) since surface is frictionless, mechanical energy is conserved.

Initial state:



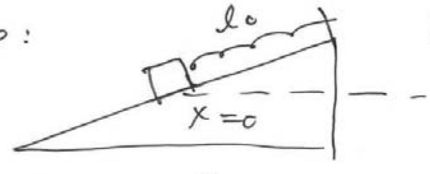
$U = 0$ at unstretched position



$\{x \sin\theta$ height above zero potential energy

$$E_0 = mgx\sin\theta + \frac{1}{2}kx^2$$

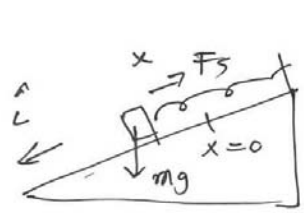
final state:



$$E_f = \frac{1}{2}m v_f^2$$

$$E_0 = E_f \Rightarrow \frac{1}{2}kx^2 + mgx\sin\theta = \frac{1}{2}m v_f^2$$

$$\Rightarrow \quad v_f = \left(2gx\sin\theta + \frac{k}{m}x^2\right)^{1/2}$$



$\vec{F}_s = m\vec{a}$	
$mg\sin\theta - kx =$	$m\dfrac{d^2x}{dt^2}$

$$\Rightarrow \quad \frac{d^2x}{dt^2} + \frac{k}{m}x = g\sin\theta$$

Solution: $\quad x = A\cos\left(\frac{2\pi}{T}t\right) + B\sin\left(\frac{2\pi}{T}t\right) + C$

$$\frac{dx}{dt} = -\frac{2\pi}{T}A\sin\left(\frac{2\pi}{T}t\right) + \frac{2\pi}{T}B\cos\left(\frac{2\pi}{T}t\right)$$

$$\frac{d^2x}{dt^2} = -\left(\frac{2\pi}{T}\right)^2\left(A\cos\left(\frac{2\pi}{T}t\right) + B\sin\left(\frac{2\pi}{T}t\right)\right)$$

$$= -\left(\frac{2\pi}{T}\right)^2(x - C)$$

$$\Rightarrow \quad -\left(\frac{2\pi}{T}\right)^2(x - C) + \frac{k}{m}x = g\sin\theta$$

$$\Rightarrow \left(-\left(\frac{2\pi}{T}\right)^2 + \frac{k}{m}\right)x \quad + \left(\frac{2\pi}{T}\right)^2 c - g\sin\theta = 0$$

$$\Rightarrow \left(\frac{2\pi}{T}\right)^2 = \frac{k}{m} \quad \text{and} \quad c = (g\sin\theta)\left(\frac{T}{2\pi}\right)^2$$

$$\Rightarrow \frac{2\pi}{T} = \sqrt{\frac{k}{m}} \quad \text{and} \quad c = mg\frac{\sin\theta}{k}$$

so period $T = 2\pi\sqrt{\frac{m}{k}}$

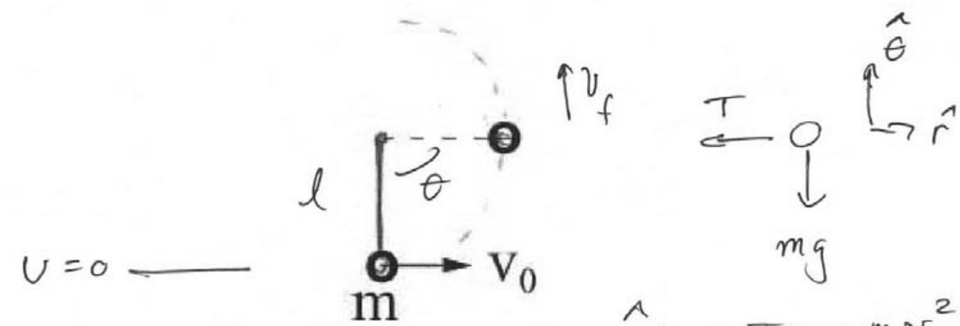
note: An acceptable solution would also simply state that

$$\frac{2\pi}{T} = \sqrt{\frac{k}{m}} \quad \text{or} \quad T = 2\pi\sqrt{\frac{m}{k}}$$

and the period is unaffected by the constant gravitational force acting on the mass-spring throughout its motion.

Problem 5: (20 points)

A ball of negligible radius and mass m hangs from a string of length l. It is hit in such a way that it then travels in a vertical circle (i.e., the tension in the string is always greater than zero). The initial speed of the ball after being struck is v_0. You may ignore air resistance. Let g denote the gravitational constant.



$\hat{r}: \quad -T = -\frac{m v_f^2}{l}$

a) Find the speed of the ball when the string is horizontal.

b) Find the tension in the string when the string is horizontal. $\Rightarrow \quad T = \frac{m v_f^2}{l}$

Answer: Since $\vec{T} \perp d\vec{r}$, $W_T = \vec{T} \cdot d\vec{r} = 0$.

So tension does no work, hence mechanical energy is constant.

initial state: bottom of circle, choose $U = 0$ at bottom. $E_0 = \frac{1}{2} m v_0^2$

final state: half-way around, $\theta = \pi$

$$E_f = \frac{1}{2} m v_f^2 + mgl$$

$$E_0 = E_f \Rightarrow \frac{1}{2} m v_0^2 = \frac{1}{2} m v_f^2 + mgl$$

$$\Rightarrow \frac{m v_f^2}{l} = \frac{m v_0^2}{l} - 2mg$$

$$T = \frac{m v_f^2}{l} = \frac{m v_0^2}{l} - 2mg$$

$$v_f = (v_0^2 - 2gl)^{1/2}$$

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