

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
Department of Physics

Physics 8.01T

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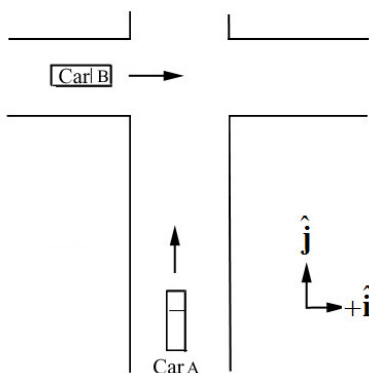
**In-Class Problems 7-10: Newton's Laws of Motion Solutions**

We would like each group to apply the problem solving strategy with the four stages (see below) to answer the following two problems.

- I. Understand – get a conceptual grasp of the problem
- II. Devise a Plan - set up a procedure to obtain the desired solution
- III. Carry out your plan – solve the problem!
- IV. Look Back – check your solution and method of solution

**In-Class Problem 7: Law of Addition of Velocities (Morning Session)**

a) Suppose two cars, Car A, and Car B, are traveled along roads that are perpendicular to each other. An observer is at rest with respect to the ground. A second observer is in Car A. According to the observer on the ground, Car A is moving with a velocity  $\vec{v}_A = v_A \hat{j}$ , and Car B is moving with a magnitude of velocity  $\vec{v}_B = v_B \hat{i}$ .



What is the velocity of Car B according to the observer in Car A? Express your answer as a vector, and in terms of the direction and magnitude of the velocity.

in frame of ground:  $\vec{v}_B = v_B \hat{i}$   
 $\vec{v}_A = v_A \hat{j}$

define new reference frame, car A, moving with velocity with respect to ground frame

$$\vec{V} = \vec{v}_A = v_A \hat{j}$$

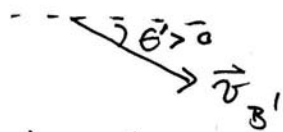
then  $\vec{r} = \vec{r}' + \vec{R}$  and  $\vec{v} = \vec{v}' + \vec{V}$

or  $\vec{v}' = \vec{v} - \vec{V}$  velocity of car B

with respect to car A

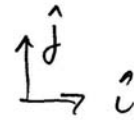
$$\vec{v}'_B = v_B \hat{i} - v_A \hat{j}, \quad |\vec{v}'_B| = (v_B^2 + v_A^2)^{1/2}$$

car A frame:



$$\tan \theta' = +v_B / v_A$$

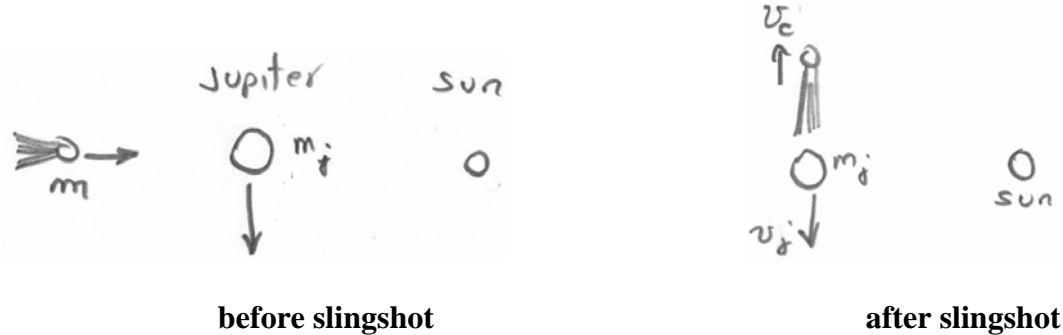
$$\theta' = \tan^{-1}(v_B / v_A)$$



## In-Class Problem 8: Gravitational slingshot

A comet of mass  $m$  is initially in the Oort Cloud which is a distance far enough away from the sun that you may approximate it as infinity. Due to random motions in the cloud, the comet picks up a negligibly small initial velocity but it now moves out of the cloud and heads directly towards the sun. Denote the mass of the sun by  $m_s$ . The planet Jupiter is directly in the way of the comet. Let  $m_j$  denote the mass of Jupiter. Assume Jupiter is in a nearly circular orbit around the sun with radius  $r_{s,j}$  and has tangential velocity  $v_j$ , as measured in a reference frame at rest with the sun. When the comet reaches the orbit of Jupiter, the comet has a velocity,  $v_c = \sqrt{2}v_j$  directed towards the sun. Jupiter's gravity changes the direction of the comet's path by  $90^\circ$  until the comet is moving opposite the direction of Jupiter's velocity. As seen by an observer moving with the planet Jupiter, the comet changes its direction but not the magnitude of its velocity  $v_c'$ .

**Reference frame at rest with respect to sun:**



- a) As seen in a reference frame moving with the planet Jupiter, what is the magnitude of comet's velocity  $v_c'$  before the collision? Express your answer in terms of  $v_j$ .
- b) As seen in a reference frame at rest with the sun, what is the magnitude of the comet's final velocity after it's flyby of Jupiter? Did it speed up, stay the same speed, or slow down? Express your answer in terms of the initial velocity of the comet  $v_c = \sqrt{2} v_j$

Before Encounter in Sun frame

$$\vec{v}_{c,b} = v_{c,b} \hat{c} = \sqrt{2} v_j \hat{c}$$

$$\vec{v} = v_j \hat{j}$$

in the frame of Jupiter

$$\vec{v}'_{c,b} = \vec{v}_{c,b} - \vec{v} = \sqrt{2} v_j \hat{c} - v_j \hat{j}$$

$$|\vec{v}'_{c,b}| = \sqrt{3} v_j$$

after encounter in frame of Jupiter:

$$|\vec{v}'_{c,f}| = |\vec{v}'_{c,b}| = \sqrt{3} v_j$$

$$v'_j = 0$$

after collision in frame of sun

$$\vec{v}_{c,f}$$

$$\vec{v}_0$$

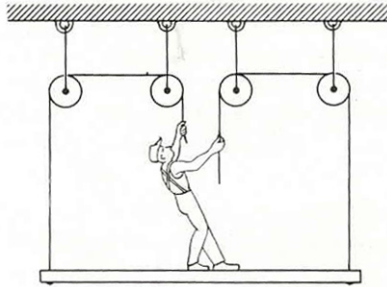
$$\vec{v}_{c,f} = \vec{v}'_{c,f} + \vec{v}$$

$$= -\sqrt{3} v_j \hat{j} + v_j \hat{j}$$

$$\vec{v}_{c,f} = (1 - \sqrt{3}) v_j \hat{j} \quad \text{with respect to sun}$$

### In-Class Problem 9: Painter on a Platform

A painter of mass  $m_1$  stands on a platform of mass  $m_2$  and pulls himself up by two ropes which hang over massless pulleys. He pulls each rope with a force  $F$  and accelerates upward with a uniform acceleration  $a$ . Find the acceleration  $a$ .



Separate Free Body Diagrams on Plank and Painter

$$a_1 = a_2 = a$$

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$\vec{F}_2 = m_2 \vec{a}$

$\vec{F}_1 = m_1 \vec{a}_1$

$\uparrow: 2F - N - m_2 g = m_2 a$

$2F - m_1 g + N = m_1 a$

$$2F - N - m_2 g = m_2 a$$

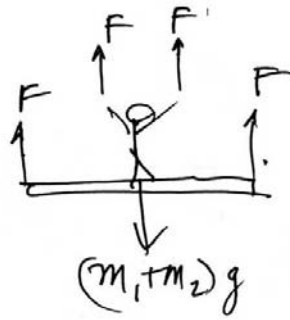
$$2F - m_1 g + N = m_1 a$$

$$4F - (m_1 + m_2)g = (m_1 + m_2)a$$

$$a = \frac{4F - (m_1 + m_2)g}{m_1 + m_2}$$

$$a = \frac{4F}{m_1 + m_2} - g$$

Free Body Diagram on System consisting of Plank and Painter



$$4F - (m_1 + m_2)g = (m_1 + m_2)a$$

Lose all information about the internal normal force between painter and plank.

### In-Class Problem 10: Glass on a tablecloth (Afternoon Session)

Consider a glass that is resting on top of a tablecloth. The glass has mass  $m_1$  and the tablecloth has mass  $m_2$ . Suppose the coefficient of static friction between the tablecloth and glass is given by  $\mu_s$ , and the table is highly polished so it is nearly frictionless. What is the maximum force that the tablecloth can be pulled horizontally so that the glass and table cloth move together without slipping?

$$\vec{F}_2 = m_2 \vec{a}_2 \qquad \vec{F}_1 = m_1 \vec{a}_1$$

$$\hat{i}: F - f_s = m_2 a_2 \qquad f_s = m_1 a_1$$

$$\hat{j}: N_0 - N - m_2 g = 0 \qquad N - m_1 g = 0$$

constraint  $\cdot a_1 = a_2 \equiv a$

maximum static friction  $f_s = \mu_s N$

$$N = m_1 g$$

$$F - \mu_s N = m_2 a$$

$$\mu_s N = m_1 a$$

$$F - \mu_s m_1 g = m_2 a$$

$$\mu_s m_1 g = m_1 a$$

$$\Rightarrow a = \mu_s g$$

$$F = \mu_s m_1 g + m_2 a$$

$$= \mu_s m_1 g + m_2 \mu_s g$$

$$F = \mu_s (m_1 + m_2) g$$