I would like to do one more case, which is perhaps a little bit more difficult, and that is to calculate the moment of inertia of a rotating disk, like a turntable. This is a disk, and the disk is like a cylinder: it has a thickness a, and it's rotating about this axis of rotation, perpendicular to the paper. It's rotating in this direction, or it could oscillating back and forth. It has a total mass m, it has a radius R, and thickness a. The mass is uniformly distributed throughout this cylinder.

In order to now evaluate the moment of inertia about this axis of rotation, I divide here a very thin cylinder. This cylinder has a thickness d little r, and it has a radius little r. This is little r, and the thickness is dr. So which fraction of mass of the total cylinder is in this cylinder-- very thin cylinder that's like a ring-- which has a height a?

What is the volume of this ring? The volume is 2 pi r times dr-- that's the surface of this ring-- multiplied by a, and that's the volume. What is the total volume of this cylinder? That would be pi r squared times a.

The mass that is in this ring-- this is the fraction-- and so the total mass in this ring, dm, is the fraction of the total mass multiplied by the mass of the disk. As you see, a cancels, and pi cancels. Now when I want to calculate the moment of inertia about this axis of rotation right through the center, perpendicular to the paper, I have to do an integral over the r, from 0 to capital R-- an integral from 0 to capital R. I get 2m divided by R squared. I r d r, but I also have an r squared, because remember with moments of inertia, you have to multiply by the distance from the mass element to the axis of rotation, which is r squared.

I'm going to get here r to the third dr, and this integral becomes 2m divided by R squared 1/4 r to the fourth. Evaluate it from 0 to capital R, and that becomes 1/2 m r squared. This, too, is a very well known result. You can find it in any book when you look at the tables that the moment of inertia of rotation of a cylinder about its axis of symmetry if the mass is uniformly distributed throughout the cylinder is 1/2 mr squared. Don't remember it, but it can be derived; it's not all that difficult to derive it.

For a given density, if I take the disk of a given material-- let's say it's aluminum-- we now ask the question, what happens when we make the disk thicker, and what happens when we make the disk larger? The moment of inertia of that cylinder as we just derived it was 1/2 mr times r squared. Suppose

I double a-- I double the thickness of the cylinder-- but there's no change in R. What now happens with the moment of inertia?

The first thing you may say is that it's independent over a, so nothing changes, but that's not true. The mass depends on a: if you make the disk twice as thick, the mass will double. If the mass doubles, the moment of inertia will double, so I will double.

Let's now take a situation whereby we double the radius R, but there's no change in a, so the disk thickness remains the same. What now happens with the moment of inertia? You have to be careful, now-- there are two terms. There is a mass term m, and there is R squared. If you double the radius, the mass will go up by a factor of four, because the surface air of the cylinder-- the flat parts-- go with R squared.

The mass will go up by a factor 4, but R squared will also go up by a factor of 4, so the moment of inertia will go up by a factor of 16. That's not so intuitive, is it? If you double the thickness of the disk, the moment of inertia will only double. If you double the radius of the cylinder, the moment of inertia will become 16 times larger.