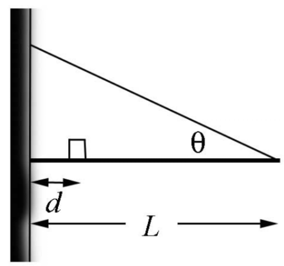
Static Equilibrium
Challenge Problems

Problem 1: Static Equilibrium: ***Steel Beam and Cable***

A uniform steel beam of mass $m_1 = 2.0\times10^2$ kg is held up by a steel cable that is connected to the beam a distance $L = 5.0$ m from the wall, at an angle $\theta = 30^\circ$ as shown in the sketch. The beam is bolted to the wall with an unknown force $\vec{\mathbf{F}}$ exerted by the wall on the beam. An object of mass $m_2 = 6.0\times10^1$ kg, resting on top of the beam, is placed a distance $d = 1.0$ m from the wall. Use $g = 9.8\ \text{m}\cdot\text{s}^{-2}$ for the gravitational acceleration.



a) Draw a free-body diagram for the beam.

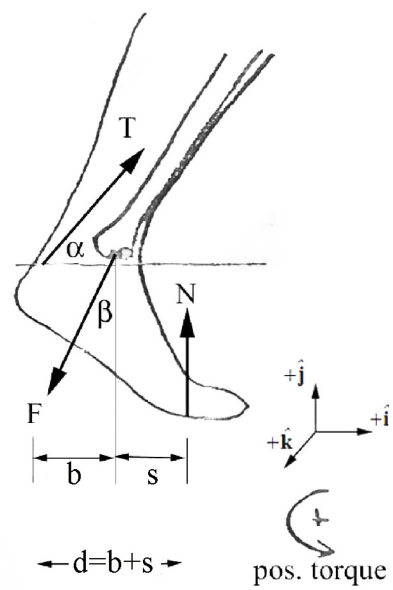
b) Find equations for static equilibrium for the beam (this will involve force equations and torque relations).

c) Find the tension in the cable.

d) Find the horizontal and vertical components of the force that the wall exerts on the beam.

Problem 2: Static Equilibrium *The Ankle*

A person of mass $m = 75\text{ kg}$ is crouching with his/her weight evenly distributed on both tiptoes. The forces on the skeletal part of the foot are shown in the diagram. In this position, the tibia acts on the foot with a force $\vec{\mathbf{F}}$ of magnitude $F = |\vec{\mathbf{F}}|$ and which makes an unknown angle β with the vertical. This force acts on the ankle a horizontal distance $s = 4.8\text{ cm}$ from the point where the foot contacts the floor. The Achilles tendon is under considerable tension $\vec{\mathbf{T}}$ and makes a given angle $\alpha = 37^0$ with the horizontal. The tendon acts on the ankle a horizontal distance $b = 6.0\text{ cm}$ from the point where the tibia acts on the foot. You may ignore the weight of the foot. Let $g = 9.8\text{ m}\cdot\text{s}^{-2}$ be the gravitational constant. In this problem you will express your answers symbolically. You may want to substitute in numbers if you have the time.

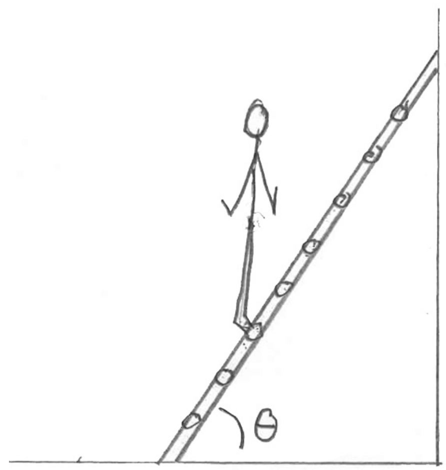


a) Find the magnitude of the tension in the Achilles tendon, $T \equiv |\vec{\mathbf{T}}|$.

b) Find the magnitude, $F \equiv |\vec{\mathbf{F}}|$, and the angle, β, of the tibia force on the ankle.

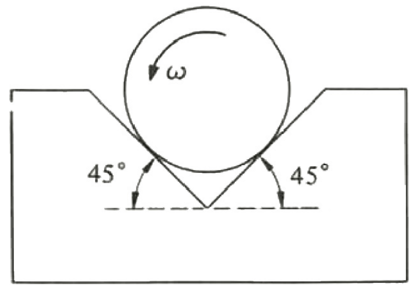
Problem 3:

A person of mass m_p is standing on a rung, one third of the way up a ladder of length d. The mass of the ladder is m_l, uniformly distributed. The ladder is initially inclined at an angle θ with respect to the horizontal. Assume that there is no friction between the ladder and the wall but that there is friction between the base of the ladder and the floor with a coefficient of static friction μ_s. In this problem you will try to find the minimum coefficient of friction between the ladder and the floor so that the person and ladder do not slip.



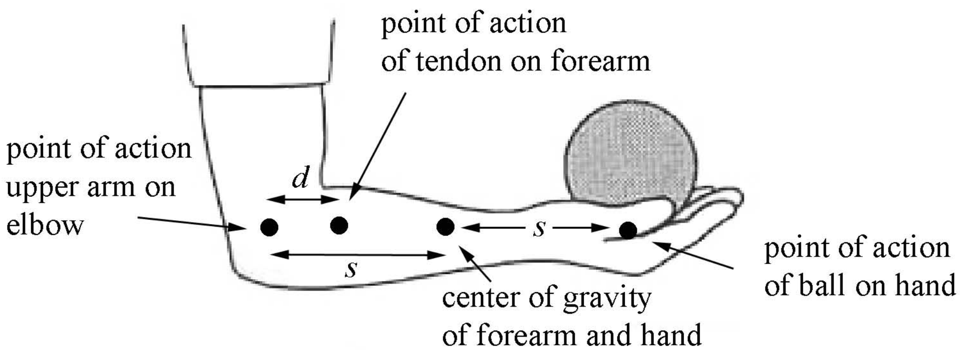
Problem 4: V-Groove Frictional Torque and Fixed Axis Rotation

A cylinder of mass m and radius R is rotated in a V-groove with constant angular velocity ω_0. The coefficient of friction between the cylinder and the surface is μ. What external torque must be applied to the cylinder to keep it rolling?



Problem 5: Static Equilibrium *Arm*

You are holding a ball of mass m_2 in your hand. In this problem you will solve for the upward force $\vec{\mathbf{T}}$ that the tendon of your biceps muscle exerts to keep the forearm horizontal and the downward force $\vec{\mathbf{F}}$ that the upper arm exerts on the forearm at the elbow joint. Assume the outstretched arm has a mass of m_1, the center of mass of the outstretched arm is a distance s from the elbow, the tendon attaches to the bone a distance d from the elbow, and the ball is a distance $2s$ from the elbow. (Taking $\vec{\mathbf{T}}$ to be upward and $\vec{\mathbf{F}}$ to be downward, with no horizontal components, indicates that this is a simplified model.) A schematic representation, but not showing the forces, of this situation is shown below:

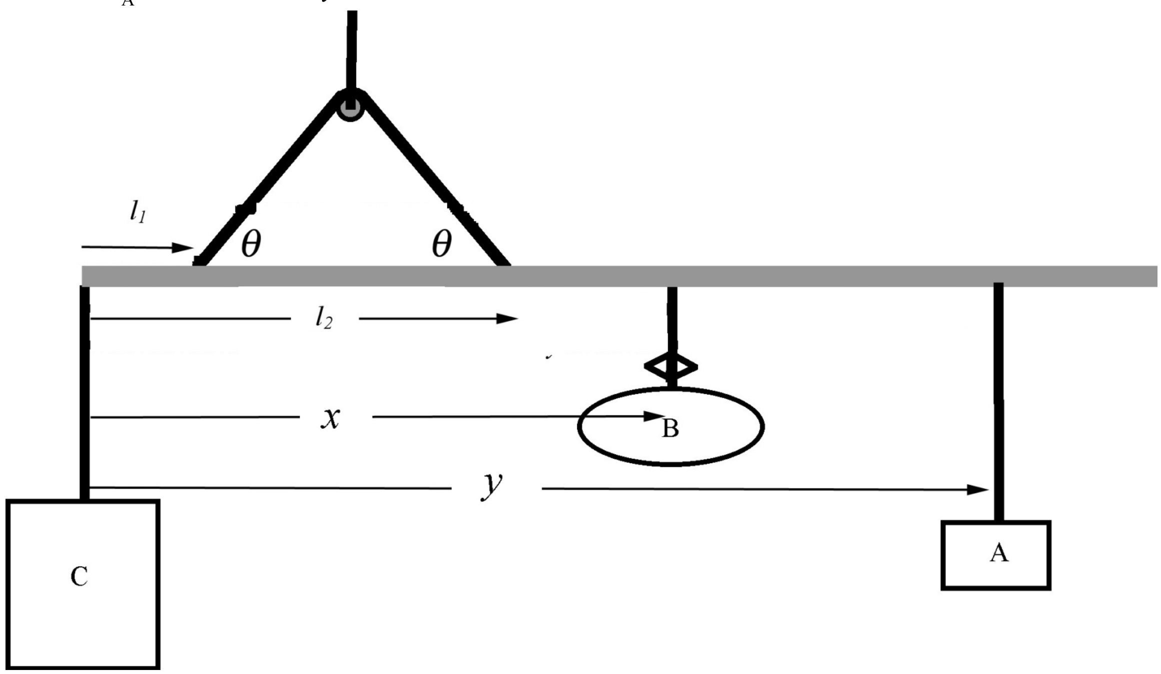


a) What is the magnitude of the tension $T \equiv \left|\vec{\mathbf{T}}\right|$ in the tendon?

b) What is the magnitude of the force $\vec{\mathbf{F}}$ that the upper arm exerts on the forearm at the elbow joint?

Problem 6: Crane

A crane is configured as below, with the beam (which we can define as massless, for simplicity) suspended at two points l_1 and l_2 by each end of a cable passing over a frictionless pulley. The two ends of the cable each make an angle θ with the beam. A counterbalance object C with mass m_C is fixed at one end of the beam. A balance object B of mass m_B is attached to the beam and can move horizontally in order to maintain static equilibrium. The crane lifts an object A with mass m_A at a distance y from the counterbalance.



a) Draw a free body force diagram for the beam.
b) What is the tension in the cable that runs over the pulley, as a function of the masses of the hanging objects and the angle θ between the cable and the beam? Show all your work. Answers without work will not receive any credit.
c) At what horizontal position should one put the balance object B such that the crane doesn't tilt? Show all your work. Answers without work will not receive any credit.