One Dimensional Kinematics Non-Uniform Acceleration
Concept Questions

Question 1 Non-Uniform Acceleration: ***Terminal Velocity of Raindrop***

A raindrop of initial mass m_0 starts falling from rest under the influence of gravity. and approaches a constant terminal speed v_t. If we assume the air resistance is proportional to the square of the speed, the resulting acceleration is given by the equation

$$\frac{dv}{dt} = g - k\,v^2$$

where $g = 9.8\ \mathrm{m \cdot s^{-2}}$ and k is a constant. What is the terminal speed?

1. Impossible to tell without integrating the expression for acceleration to find the speed.

2. $v_t = g / k$

3. $v_t = \sqrt{g / k}$

4. $v_t = 0$

Answer 3. When the raindrop approaches terminal speed, the rate of change of the speed approaches zero, so $g - k\,v^2 \to 0$, $v \to v_t = \sqrt{g / k}$.

Question 2 A particle, starting at rest at $t = 0$, experiences a non-constant acceleration $a_x(t)$. It's change of position can be found by

1) Differentiating $a_x(t)$ twice.
2) Integrating $a_x(t)$ twice.
3) $\frac{1}{2}a_x(t)t^2$.
4) None of the above.
5) Two of the above.

Answer 2. The change in x-component of the velocity for an interval $[0,t]$ is the integral of the x-component of acceleration with respect to time,

$$v_x(t) - v_x(0) = \int_{t'=0}^{t'=t} a_x(t')dt' .$$

The displacement for an interval $[0,t]$ is the integral of the x-component of acceleration with respect to time,

$$x(t) - x(0) = \int_{t'=0}^{t'=t} v_x(t')dt' .$$

When $a_x(t)$ is independent of time, i.e. constant, then

$$\begin{aligned} &v_x(t) - v_x(0) = a_x t \\ &x(t) = x(0) + v_x(0)t + a_x t \frac{1}{2} a_x t^2 \end{aligned},$$

however in this question, $a_x(t)$ is non-constant so the correct answer is that you need to integrate twice.

Question 3 An airliner made an emergency landing at the Los Angeles airport with its nose wheel locked in a position perpendicular to its normal rolling position. The component of the acceleration in the horizontal direction of motion is given by

$$a_{\text{horiz}}(t) = -B_0 + B_1 t$$

from touchdown at $t = 0$ until the plane comes to rest at $t = t_{\text{s}}$ where $B_0 > 0, B_1 > 0$. What was the horizontal speed of the airplane at time $t = 0$ when it first touched down

1) $v_{\text{horiz}}(t = 0) = -B_0$
2) $v_{\text{horiz}}(t = 0) = -B_0 t_s + B_1 \dfrac{t_s^{\,2}}{2}$
3) $v_{\text{horiz}}(t = 0) = B_0 t_s - B_1 \dfrac{t_s^{\,2}}{2}$
4) $v_{\text{horiz}}(t = 0) = (-B_0 + B_1 t_s) t_s$
5) $v_{\text{horiz}}(t = 0) = -(-B_0 + B_1 t_s) t_s$

Answer 3. The change in the horizontal component of the velocity of the plane is found by integrating the horizontal component of the acceleration

$$v_{\text{horiz}}(t_s) - v_{\text{horiz}}(0) = \int_{t'=0}^{t'=t_s} a_x(t')dt' = \int_{t'=0}^{t'=t_s} (-B_0 + B_1 t')dt' = -B_0 t_s + \frac{B_1}{2} t_s^{\,2} \ .$$

The plane came to a stop at $t = t_{\text{s}}$ so $v_{\text{horiz}}(t_s) = 0$. Therefore $-v_{\text{horiz}}(0) = -B_0 t_s + \dfrac{B_1}{2} t_s^{\,2}$ and so $v_{\text{horiz}}(0) = B_0 t_s - \dfrac{B_1}{2} t_s^{\,2}$.