One Dimensional Kinematics
Challenge Problem Solutions

Problem 1:

As a space shuttle burns up its fuel after take-off, it gets lighter and lighter and its acceleration larger and larger. Between the moment it takes off and the time at which it has consumed nearly all of its fuel, is the magnitude of the average velocity larger than, equal to, or smaller than half its final speed? Explain why.

Problem 1 Solution:

In Figure 1, the graph of velocity vs. time for both constant acceleration and non-constant acceleration are shown.

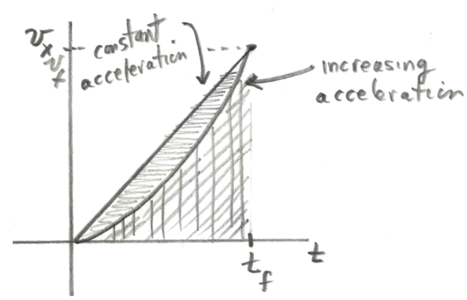


Figure 1: velocity graphs for constant and non-constant acceleration

When the acceleration is constant, the velocity is a linear function of time. Therefore the average velocity is given by (see Figure 1)

$$\overline{v_x} = \frac{1}{2}\left(v_x(t) + v_{x,0}\right) = \frac{1}{2}v_{x,f}\,. \tag{1.1}$$

Hence for an object starting at rest, the average velocity is one half of the final velocity. Suppose the space shuttle reaches the same final velocity but its acceleration is increasing. As see in Figure 1, the velocity is always less in the non-constant acceleration case than in the constant acceleration case, and so the average velocity for the non-constant acceleration case must be less than one half the final velocity.

Alternatively, we also defined average velocity as $\overline{v_x} \equiv \Delta x / t_f$. Since the displacement is the area under graph of velocity vs. time, we see from Figure 1 that for the case of non-constant acceleration the displacement is less than the constant acceleration case, hence the average velocity for the non-constant acceleration case is less than one half the final velocity (which is the average velocity for the constant acceleration case).

Problem 2

An airliner made an emergency landing at the Los Angeles airport with its nose wheel locked in a position perpendicular to its normal rolling position. The forces acting to stop the airliner arose from friction due to the nose wheel and the braking effort of the engine in reverse thrust mode. The sum of horizontal forces can be modeled as

$$F_{\text{horiz}}(t) = -F_0 + Bt \qquad (2.1)$$

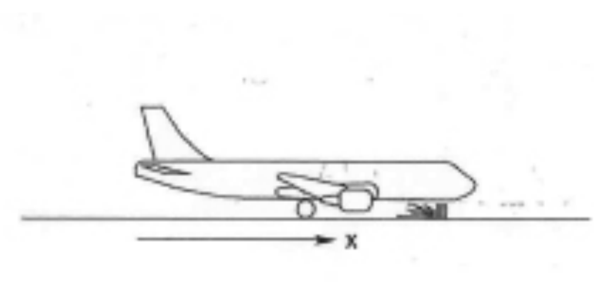
from touchdown at $t = 0$ until the plane comes to rest at $t = t_s$ where $F_0 > 0, B > 0$. Assume the mass of the plane is m. The point of this problem is to figure how far the plane traveled from touchdown until it came to a stop. In the questions below express your answers in terms of the known quantities F_0, B, t_s, m but not in terms of $v_{\text{horiz},0}$.

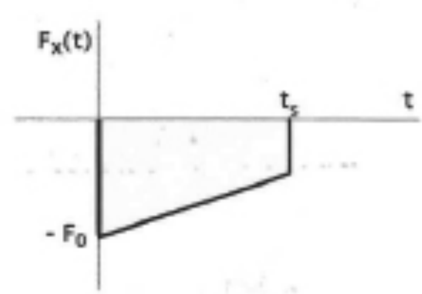
a. Find the horizontal component of the velocity when the airplane first touches the runway at $t = 0$.

(b) How far did the airplane travel on the ground before stopping?

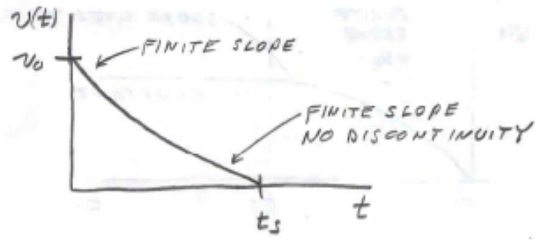
Problem 2 Solutions:

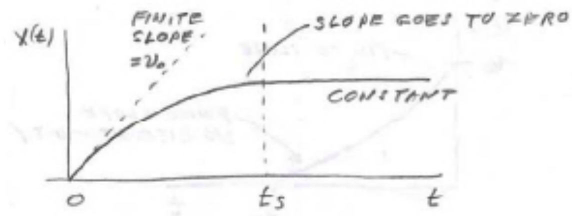
We shall model the landing of the airliner as a one-dimensional motion with a time-dependent force acting on the airliner. We shall assume the constant $B > 0$ and that for $t > t_s$ the force is zero. This means that there is a discontinuity in the force at $t = t_s$. By Newton's Second Law, the acceleration is time-dependent so we can find the find the equations that describe the position and the horizontal component of the velocity of the jetliner by integration. A motion diagram and graph of the force as a function of time are shown below.





The horizontal component of the velocity and the position graphs are also shown below.





We shall first find the acceleration and then integrate the acceleration to find an equation for how the horizontal component of the velocity varies with time. We shall use this equation to find the time that the plane comes to a stop since the horizontal component of the velocity is zero at that instant. Then we shall use the position equation to determine how far the airliner has traveled before it came to a stop.

(b) How far did the plane travel from touchdown until it came to a stop?

Answer:

Newton's Second Law in the horizontal direction is

$$-F_0 + Bt = ma_x. \tag{2.2}$$

Solving Eq. (2.2) for the horizontal component of the acceleration,

$$a_x = \frac{-F_0 + Bt}{m}. \tag{2.3}$$

We can integrate Eq. (2.3) with respect to time to find the change in the horizontal component of the velocity,

$$v_x(t) - v_{x,0} = \int_0^t a_x dt' = \int_0^t \left(\frac{-F_0 + Bt'}{m} \right) dt' = -\frac{F_0}{m}t + \frac{B}{2m}t^2. \tag{2.4}$$

When the plane comes to rest at $t = t_s$, the horizontal component of the velocity is zero, so we can solve Eq. (2.4) for the initial touchdown horizontal component of the velocity,

$$v_{x,0} = \frac{F_0}{m}t_s - \frac{B}{2m}t_s^{\,2}. \tag{2.5}$$

The change in the horizontal position as the airliner is landing is found by integrating the horizontal component of the velocity, so from Eq. (2.4)

$$x(t) - x_0 = \int_0^t v_x dt' = \int_0^t \left(v_{x,0} - \frac{F_0}{m}t' + \frac{B}{2m}t'^2 \right) dt' = v_{x,0}t - \frac{F_0}{2m}t^2 + \frac{B}{6m}t^3. \tag{2.6}$$

Let's choose our origin at the point where the airliner touched down on the runway, then set $t = t_s$, $x_0 = 0$, and substitute Eq. (2.5) in Eq. (2.6) to find the distance the airliner traveled before it came to a stop,

$$x(t_s) = \frac{F_0}{m}t_s^2 - \frac{B}{2m}t_s^3 - \frac{F_0}{2m}t_s^2 + \frac{B}{6m}t_s^3 = \frac{F_0}{2m}t_s^2 - \frac{B}{3m}t_s^3. \tag{2.7}$$

(c) Estimate $v_{\text{horiz},0}$, t_s, m, and how far the plane traveled from touchdown until it came to a stop in order to estimate the coefficients F_0 and B. What are the units of these coefficients?

Answer:

A typical airliner landing speed (fuel tanks nearly empty) is about $v_{x,0} \simeq 100\,\text{m}\cdot\text{s}^{-1}$. (This is a little fast, landing speeds are around $v_{x,0} \simeq 70\,\text{m}\cdot\text{s}^{-1}$.) Let's model an airliner as a cylinder with a diameter $d \simeq 10\,\text{m}$, about $l \simeq 30\,\text{m}$ long. The volume is then

$$V \simeq \frac{l\pi d^2}{4} = \frac{(30\,\text{m})(\pi)(10\,\text{m})^2}{4} \simeq 2\times10^3\,\text{m}^3. \tag{2.8}$$

Let's assume that if an airliner crashes in the water it should float until it fills with water so we will approximate its average density as 1/100 the density of water. Then an estimate for the mass of the airliner is

$$m \simeq \frac{\rho_{\text{water}}}{100}V = \frac{1}{100}\left(10^3\text{ kg}\cdot\text{m}^{-3}\right)\left(2\times10^3\,\text{m}^3\right) \simeq 2\times10^4\text{ kg}. \tag{2.9}$$

(If I look up the mass of a Boeing 737 I find a value equal to 2.7×10^4 kg with a maximum take-off weight 5×10^4 kg weight so we are in the right range.) At take-off the fuel is about 80% of the weight so let's assume that at landing the fuel is about 10% of the weight and so we can estimate the mass of the airliner and fuel to be $m \simeq 3\times10^4$ kg. Let's assume it takes about $t_s \simeq 30\,\text{s}$ to stop after the airline travels an estimated $x(t_s) \simeq 1.0\times10^3\,\text{m}$ (this landing distance is likely to be an underestimate, but we only want an order of magnitude). We can use these estimates and Eqs. (2.5) and (2.7) to estimate the two constants F_0, B.

We begin by solving Eq. (2.5) for F_0:

$$F_0 = \frac{mv_{x,0}}{t_s} + \frac{B}{2}t_s. \tag{2.10}$$

Then substitute Eq. (2.10) into Eq. (2.7) to obtain

$$x(t_s) = \frac{1}{2m}\left(\frac{mv_{x,0}}{t_s} + \frac{B}{2}t_s\right)t_s^{\,2} - \frac{B}{3m}t_s^{\,3}. \tag{2.11}$$

After a small amount of algebra we can solve Eq. (2.11) for B,

$$x(t_s) = \frac{v_{x,0}t_s}{2} + B\left(\frac{1}{4m} - \frac{1}{3m}\right)t_s^{\,3}. \tag{2.12}$$

$$\begin{aligned} B &= \frac{x(t_s) - \frac{v_{x,0}t_s}{2}}{\left(\frac{1}{4m} - \frac{1}{3m}\right)t_s^{\,3}} = \frac{\left(1\times10^3\ \mathrm{m}\right) - \frac{\left(100\,\mathrm{m\cdot s^{-1}}\right)\left(30\,\mathrm{s}\right)}{2}}{\left(\frac{1}{4\left(3\times10^4\ \mathrm{kg}\right)} - \frac{1}{3\left(3\times10^4\ \mathrm{kg}\right)}\right)\left(30\,\mathrm{s}\right)^3} \\ B &\simeq 7\times10^3\,\mathrm{kg\cdot m\cdot s^{-3}} \end{aligned} \tag{2.13}$$

which to the nearest order of magnitude is

$$B \approx 1\times10^4\,\mathrm{kg\cdot m\cdot s^{-3}}. \tag{2.14}$$

Finally Eq. (2.10) becomes

$$\begin{aligned} F_0 &\approx \frac{\left(4\times10^4\ \mathrm{kg}\right)\left(100\,\mathrm{m\cdot s^{-1}}\right)}{\left(30\,\mathrm{s}\right)} + \frac{\left(1\times10^4\,\mathrm{kg\cdot m\cdot s^{-3}}\right)}{2}\left(30\,\mathrm{s}\right) \\ F_0 &\approx 3\times10^5\,\mathrm{kg\cdot m\cdot s^{-2}} \end{aligned} \tag{2.15}$$

which to the nearest order of magnitude is

$$F_0 \approx 1\times10^5\,\mathrm{kg\cdot m\cdot s^{-2}}. \tag{2.16}$$

These estimates are very rough and so they are only order of magnitude estimates.

Problem 3

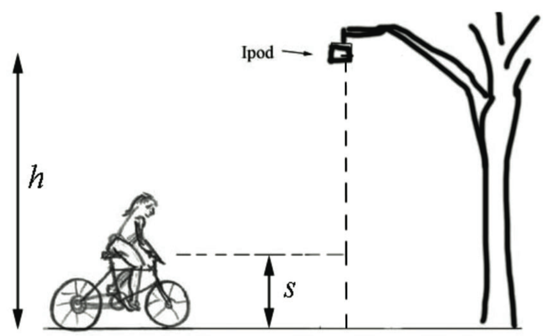
A bicycle rider is traveling at a constant speed along a straight road and then gradually applies the brakes during a time interval $0 < t < t_f$ until the bicycle comes to a stop. The combined mass of the cyclist and bicycle is m. Assume that the magnitude of the braking acceleration increases linearly in time according to

$$|\vec{\mathbf{a}}| = bt / m, \qquad\qquad 0 < t < t_f$$

where $b > 0$ is a constant. At the instant the person applies the brakes, a horizontal distance from the rider, the wind blows and snaps an iPod off the branch of the tree. With a speed v_{po} directed downwards. The ipod was initially a height h above the ground. The cyclist catches the iPod at the instant the cyclist has come to a stop. You may assume that the cyclist catches it at a height s above the ground. How far did the bicycle travel while it was braking? The gravitational constant is g.

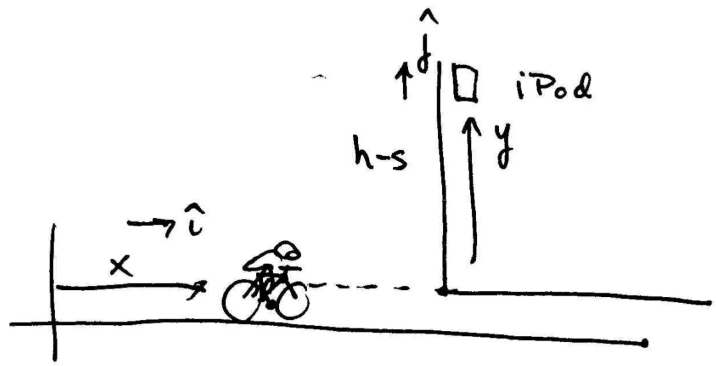
a) At what time t_1 did the cyclist catch the ipod? Express your answer in terms of the quantities, b, m, h, s, v_{po}, d, and g as needed.

b) What was the initial speed of the cyclist? You may leave your answer in terms of t_1 from part a) and any other quantities as needed.



Problem 3 Solutions:

There are two objects moving, the bicyclist and the iPod. We shall choose a coordinate system for the bicyclist and the iPod as shown in the figure below.



We can integrate the acceleration to find the x-component of the velocity of the bicyclist. We then use the equations for free fall to determine the time it takes the iPod to fall a distance $h-s$. The x-component of the velocity of the bicyclist is zero at this instant so we can solve for the initial x-component of the velocity of the bicyclist. The x-component of the acceleration is

$$a_x = -\frac{b}{m}t \tag{3.1}$$

We can now integrate the x-component of the acceleration of the bicyclist to get the x-component of the velocity of the bicyclist

$$v_{bx}(t) - v_{bx0} = \int_0^t a_x dt = \int_0^t \left(-\frac{b}{m}t\right) dt = -\frac{b}{2m}t^2 . \tag{3.2}$$

So the x-component of the velocity of the cyclist as a function of time is given by

$$v_{bx}(t) = v_{bx0} - \frac{b}{2m}t^2 \tag{3.3}$$

The y-component of the position of the iPod is given by

$$y(t) = (h-s) - \frac{g}{2}t^2 \tag{3.4}$$

The bicyclist catches the iPod at the instant when

$$y(t = t_1) = 0 = (h-s) - \frac{g}{2}t_1^{\ 2} \tag{3.5}$$

We can solve eq. (3.5) for the time the iPod falls:

$$t_1 = \sqrt{\frac{2(h-s)}{g}} \tag{3.6}$$

The bicyclist comes to a full stop at the instant t_1 so we can substitute Eq. (3.6) into Eq. (3.3) and solve for the initial x-component of the velocity of the bicyclist

$$v_{bx0} = \frac{b}{2m}t_1^2 = \frac{b(h-s)}{mg} \tag{3.7}$$

We can integrate Eq. (3.3) to find that the x-component of the position the bicyclist when it just came to a stop

$$x_b(t_1) - x_{b0} = \int_0^{t_1} v_{bx}\,dt = \int_0^{t_1}\left(v_{bx0} - \frac{b}{2m}t^2\right)dt = v_{bx0}t_1 - \frac{b}{6m}t_1^3. \tag{3.8}$$

We chose $x_0 = 0$ and so substituting $x_{b0} = 0$ and Eq. (3.6) into Eq. (3.8) yields

$$x_b(t_1) = v_{bx0}t_1 - \frac{b}{6m}t_1^3 \tag{3.9}$$

Substituting the time t_1 and the initial x-component of the velocity of the bicyclist v_{bx0}, gives the distance the cyclist traveled while braking

$$x_b(t_1) = \frac{b(h-s)}{mg}\sqrt{\frac{2(h-s)}{g}} - \frac{b}{6m}\left(\frac{2(h-s)}{g}\right)^{3/2} = \frac{b}{3m}\left(\frac{2(h-s)}{g}\right)^{3/2} \tag{3.10}$$

Problem 4: Model Rocket

Roxana launches a home-built model rocket straight up into the air. At $t = 0$ the rocket is at rest at $y = 0$ with $v_{y,0} = 0$. The acceleration of the rocket is given by

$$a_y = \begin{cases} -g + \alpha g - \beta t^2; & 0 < t < t_b \\ -g; & t > t_b \end{cases} \tag{4.1}$$

where $t_b = (\alpha g / \beta)^{1/2}$ is the time that the fuel burns out, g is the acceleration of gravity, and $\alpha > 3/2$ is a positive dimensionless number greater than 3/2. After the fuel burns out, the rocket is still moving upwards.

The goal of this problem is to figure out how high the rocket went up.

You may find the following integration formula useful

$$\int_{t_1}^{t_2} t^n dt = \frac{1}{n+1}\left(t_2^{\,n+1} - t_1^{\,n+1}\right); \; n \neq -1 \tag{4.2}$$

a) Make a qualitative graphical representation of the velocity and position of the rocket before answering the questions below.

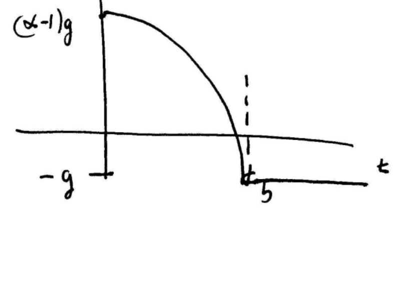
b) Explain how you will model the motion of the rocket i.e. how will you determine the equations for the position and velocity of the rocket.

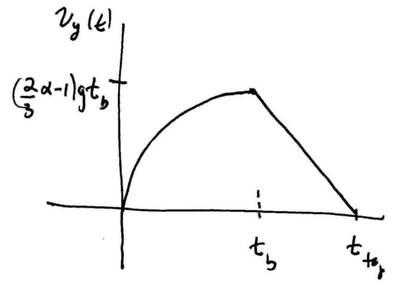
c) Explain your strategy that you will use to determine how high the rocket went up.

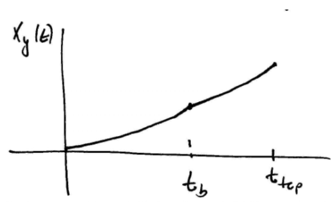
d) How high did the rocket go up? (This part will tax your integration and algebra skills so please be careful.)

Problem 4 Solutions:

Motion Diagram and Graphical representation:







Model: The rocket undergoes two stages of one-dimensional motion. The first stage has a time dependent acceleration and the second stage is free fall with constant acceleration. We can determine the velocity and position for the first stage by integrating the acceleration. This stage ends when the fuel burns out at $t = t_b$. We need to calculate the velocity and position at this time because those values are the initial conditions for the free fall stage. We note that the parameter α must be larger than a certain value or the rocket will already have reached its highest point during the burning stage. We now use the free fall one dimensional equation for velocity to find the time that it takes the rocket to reach its heist point. We then use the position equation to find how high the rocket went.

Strategy:

1) Find an expression for the y-component of the velocity $v_y(t)$ valid at all times in the interval $0 < t < t_b$, and in particular find the value of the y-component of the velocity $v_y(t = t_b)$ at the end of the interval ($t = t_b$) when the fuel burns out.

2) Find an expression for the height $y(t)$ valid at all times in the interval $0 < t < t_b$ and in particular find the value $y(t = t_b)$ at the end of the interval ($t = t_b$) when the fuel burns out.

3) Now use the final conditions at the end of the burning stage as the initial conditions at the beginning of the free fall stage to find an expression for the y-component of the velocity $v_y(t)$ valid for the time between burnout, t_b, and the time the rocket returns to the ground.

4) Since the y-component of the velocity $v_y(t = t_{top}) = 0$ is zero when the rocket reaches its maximum height, us this condition to find the time it takes for the rocket to reach its maximum height $t = t_{top}$.

5) Now use the final conditions at the end of the burning stage as the initial conditions at

the beginning of the free fall stage to find an expression for the height $y(t)$ valid for the time between burnout, t_b, and the time the rocket returns to the ground. In particular, use your result for the time it takes to reach the top, to determine how high the rocket went up?

Solutions:

(1) Find an expression for the velocity $v_y(t)$ valid at all times in the interval $0 < t < t_b$.

$$v_y(t) - v_y(t=0) = \int_0^{t_b} a_y dt = \int_0^{t_b} (-g + \alpha g - \beta t^2) dt = (\alpha - 1)gt - \frac{1}{3}\beta t^3 \qquad (4.3)$$

Since $v_{y,0} = 0$,

$$v_y(t) = (\alpha - 1)gt - \frac{1}{3}\beta t^3 \qquad (4.4)$$

At time $t = t_b$, the velocity is

$$v_y(t = t_b) = (\alpha - 1)gt_b - \frac{1}{3}\beta t_b{}^3 \qquad (4.5)$$

(2) Find an expression for the height $y(t)$ valid at all times in the interval $0 < t < t_b$.

$$y(t) - y_0 = \int_0^{t_b} v_y dt = \int_0^{t_b} ((\alpha - 1)gt - \frac{1}{3}\beta t^3) dt = \frac{1}{2}(\alpha - 1)gt^2 - \frac{1}{12}\beta t^4 \qquad (4.6)$$

Since we choose $y_0 = 0$,

$$y(t) = \frac{1}{2}(\alpha - 1)gt^2 - \frac{1}{12}\beta t^4 \qquad (4.7)$$

At time $t = t_b$, the position is

$$y(t = t_b) = \frac{1}{2}(\alpha - 1)gt_b{}^2 - \frac{1}{12}\beta t_b{}^4 \qquad (4.8)$$

(3) Find an expression for the velocity $v_y(t)$ valid for the time between burnout, t_b, and the time the rocket returns to the ground.

$$v_y(t) - v_y(t = t_b) = \int_{t_b}^{t} a_y dt = \int_{t_b}^{t} (-g)dt = -g(t - t_b) \qquad (4.9)$$

Substitute Eq. (4.5) into Eq. (4.9) yielding

$$v_y(t) = -g(t - t_b) + v_y(t = t_b) = -g(t - t_b) + (\alpha - 1)gt_b - \frac{1}{3}\beta t_b^3 \qquad (4.10)$$

Use the fact that $t_b^3 = t_b^2 t_b = \dfrac{\alpha g}{\beta} t_b$ in Eq. (4.10) to get

$$v_y(t) = -gt + \alpha g t_b - \frac{1}{3}\beta t_b^2 t_b = -gt + \alpha g t_b - \frac{1}{3}\beta \frac{\alpha g}{\beta} t_b = \frac{2}{3}\alpha g t_b - gt \qquad (4.11)$$

(4) Find the time it takes for the rocket to reach its maximum height. Express your answer as a dimensionless factor times t_b.

$$0 = v_y(t = t_{top}) = \frac{2}{3}\alpha g t_b - g t_{top} \qquad (4.12)$$

$$t_{top} = \frac{2}{3}\alpha t_b \qquad (4.13)$$

e) How high did the rocket go up?

$$y(t) - y(t = t_b) = \int_{t_b}^{t_{top}} v_y dt = \int_{t_b}^{t_{top}} \left(\frac{2}{3}\alpha g t_b - gt \right) dt = \frac{2}{3}\alpha g t_b (t_{top} - t_b) - \frac{1}{2} g (t_{top}{}^2 - t_b{}^2) \qquad (4.14)$$

Substitute Eq. (4.13) for t_{top} in Eq. (4.14):

$$y(t) = y(t = t_b) + \frac{2}{3}\alpha g t_b (\frac{2}{3}\alpha - 1)t_b) - \frac{1}{2} g (\frac{4}{9}\alpha^2 - 1)t_b^2 \qquad (4.15)$$

Substitute Eq. (4.8) for $y(t = t_b)$ in Eq. (4.15) using $t_b^4 = \dfrac{\alpha g}{\beta} t_b^2$ yielding

$$y(t) = \frac{1}{2}(\alpha - 1)g t_b^2 - \frac{1}{12}\beta \frac{\alpha g}{\beta} t_b^2 + g(\frac{2}{9}\alpha^2 - \frac{2}{3}\alpha + \frac{1}{2})t_b^2 \qquad (4.16)$$

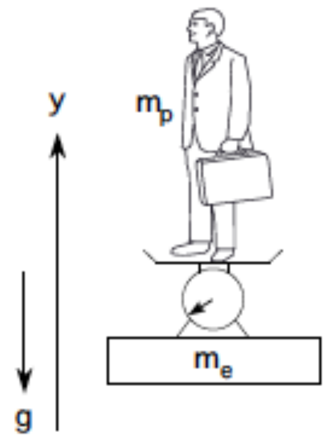
After a little bit of algebra we get

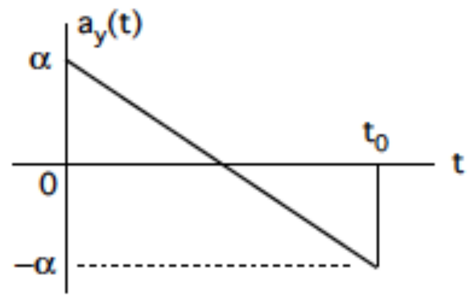
$$y(t) = g(\frac{2}{9}\alpha^2 - \frac{1}{4}\alpha)t_b^2 \qquad (4.17)$$

Now substitute $t_b^2 = \dfrac{\alpha g}{\beta}$ in Eq. (4.17) to finally get how high the rocket traveled.

$$y(t) = (\frac{2}{9}\alpha - \frac{1}{4})\frac{\alpha^2 g^2}{\beta} \qquad (4.18)$$

Problem 5: Elevator





A person of mass m_p stands on a scale in an elevator of mass m_p. The scale reads the magnitude of the force F exerted on it from above in a downward direction. Starting at rest at $t = 0$ the elevator moves upward, coming to rest again at time $t = t_0$. The downward acceleration of gravity is g. The acceleration of the elevator during this period is shown graphically above and is given analytically by

$$a_y(t) = \alpha - \frac{2\alpha}{t_0}t. \tag{5.1}$$

a) Find the maximum speed of the elevator.

b) Find the total distance traveled by the elevator.

Problem 5 Solutions:

Part a)

The velocity is the integral of the acceleration. Inspection of the graph shows the integral increases until $t = t_0 = 2$ where the velocity reaches its maximum value. That value is given by the area of the triangle above the axis.

$$v_{y,\max} = (1/2)\alpha(t_0/2) = \frac{\alpha}{4}t_0. \tag{5.2}$$

Part b)

$$v_y(t) - v_{y0} = \int_0^t a_y(t)dt = \int_0^t \left(\alpha - \frac{2\alpha}{t_0} t \right) dt = \alpha t - \frac{\alpha}{t_0} t^2 . \tag{5.3}$$

Because $v_{y0} = 0$, the y-component of the velocity is

$$v_y(t) = \alpha t - \frac{\alpha}{t_0} t^2 . \tag{5.4}$$

Integrating again to find the position

$$y(t) - y_0 = \int_0^t v_y(t)dt = \int_0^t \left(\alpha t - \frac{\alpha}{t_0} t^2 \right) dt = \frac{\alpha}{2} t^2 - \frac{\alpha}{3t_0} t^3 . \tag{5.5}$$

Because we choose $y_0 = 0$,

$$y(t) = \frac{\alpha}{2} t^2 - \frac{\alpha}{3t_0} t^3 . \tag{5.6}$$

The total distance traveled by the elevator is the position at time $t = t_0$

$$y(t_0) = \frac{\alpha}{2} t_0^{\,2} - \frac{\alpha}{3t_0} t_0^{\,3} = \frac{\alpha}{6} t_0^{\,2} . \tag{5.7}$$

Problem 6: Periodic Motion

The motion of an object moving in one dimension is given by the function

$$x(t) = A\cos(2\pi t / T) \tag{6.1}$$

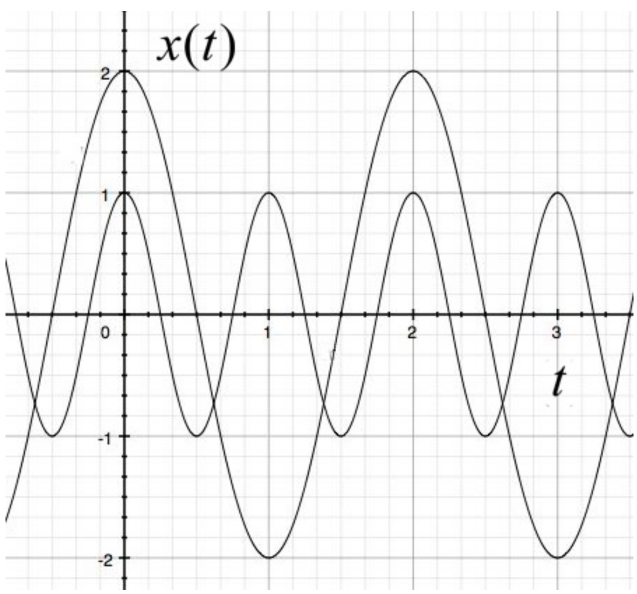
a) In your own words, describe the meaning of the constants T and A that appears in the above equation.

b) Find the velocity and acceleration of the object as functions of time.

c) Graph the position, velocity, and acceleration as functions of time. Be sure to indicate clearly on your graph the constants T and A.

Problem 6 Solutions:

a) The figure below shows two plots of the expression, one scaled to $A = 1$, $T = 1$, the other with $A = 2$, $T = 2$.



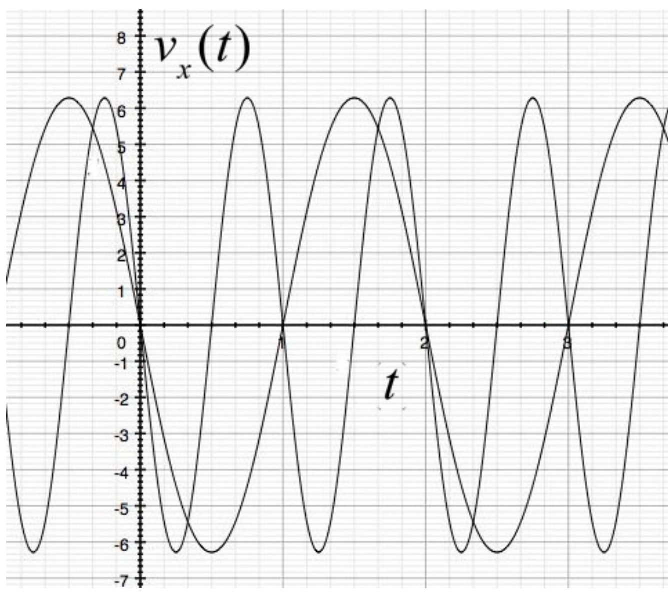
The multiplying factor A is known as the *amplitude* (the amplitude is, strictly speaking, is $|A|$). The function $x(t)$ assumes values between $-|A|$ and $|A|$. The constant T has dimensions of time, and is known at the *period* of the oscillation. For the purposes of this problem (and many similar problems), a function $x(t)$ has *periodicity* T if

$$x(t) = x(t + T) \tag{6.2}$$

a) & c) Let $u(t) = 2\pi t / T$, so that $x(t) = A\cos(u(t))$. The velocity of the object is

$$
\begin{aligned}
v_x(t) &= \frac{dx(t)}{dt} = \frac{d}{dt}\left(A\cos(u(t))\right) = -A\sin(u(t))\frac{d}{dt}u(t) = -A\sin(u(t))\frac{2\pi}{T} \\
&= -\frac{2\pi}{T}A\sin(2\pi t / T)
\end{aligned}
\tag{6.3}
$$

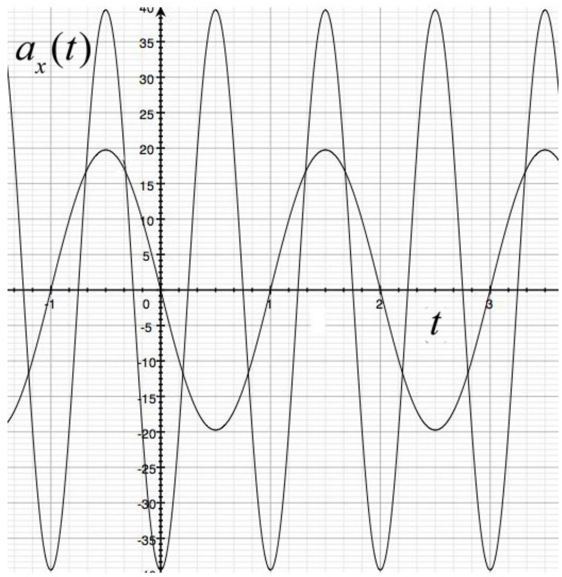
Plots of the x-component of the velocity corresponding to the expressions for $v_x(t)$ are shown below. Note that the maximum and minimum of both plots are the same, since for the chosen scaling the ration A / T is the same for both plots.



b) & c), continued: Another differentiation of the expression in (6.3) gives

$$
\begin{aligned}
a_x(t) &= \frac{dv_x(t)}{dt} = \frac{d}{dt}\left(-A\sin(u(t))\frac{2\pi}{T}\right) = -\frac{2\pi}{T}A\cos(u(t))\frac{d}{dt}u(t) = -\left(\frac{2\pi}{T}\right)^2 A\cos(u(t))\frac{2\pi}{T} \\
&= -\left(\frac{2\pi}{T}\right)^2 A\cos\sin(2\pi t / T)
\end{aligned}
\tag{6.4}
$$

The corresponding plots are shown below. Note that the maxima of the curve with period $T = 1$ are larger than those of the curve with the longer period.



$a_x(t)$
t