# One Dimensional Kinematics
## Challenge Problem Solutions

**Problem 1: One-Dimensional Kinematics:**

Two stones are released from rest at a certain height, one after the other.

a) Will the difference between their speeds increase, decrease, or stay the same?

b) Will their separation distance increase, decrease, or stay the same?

c) Will the time interval between the instants at which they hit the ground be smaller than, equal to, or larger than the time interval between the instants of their release?

d) Plot the speed vs. time for both balls in the same plot.

e) Plot the position vs. time of the two balls in the same plot.

**Problem 1 Solutions:**

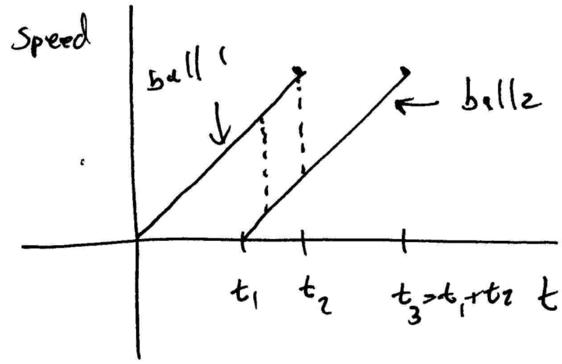
a) Neglecting air friction and other forces, both stones will accelerate with the same constant magnitude of acceleration, the acceleration due to gravity. In any time interval, the stones will increase their speeds by the same amount; the difference between their speeds will stay the same.

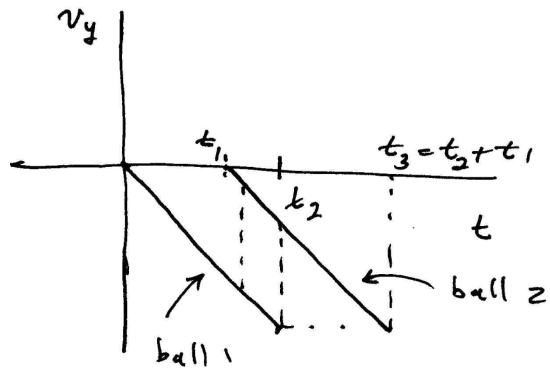
b) While the difference between the speeds will be a constant, the speeds will not be the same at any time; the stone released first will be always moving faster, and hence the separation between the stones will increase.

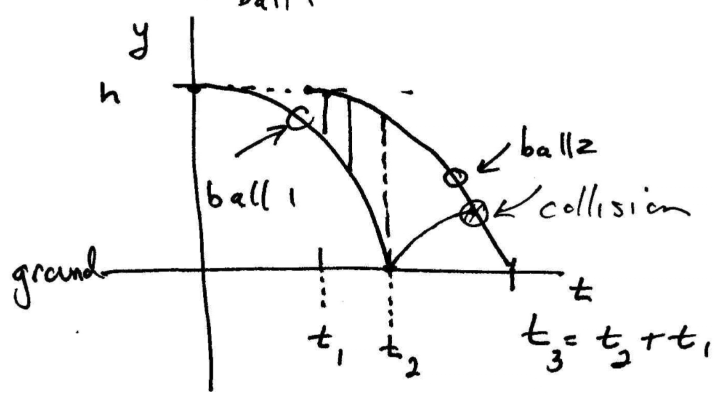
c) The time for either stone to hit the ground is determined solely by the height from which it is dropped, and has nothing to do with the other stone. The time interval between the release times will be the same as the time interval between the times when they hit.

d) We must first introduce some symbols to express various physical quantities. Suppose both balls are dropped from a height $h$. Suppose ball one is dropped at time $t = 0$ and ball two is dropped after a time interval $t_1$ has elapsed. Denote the instant ball one hits the ground by $t_2$ . Provided that the balls do not collide, denote the instant ball two hits the ground by $t_3 = t_2 + t_1$. In the figures below, plots of the graphs of speed vs. time, y-component of velocity vs. time, and position vs. time are shown.

e)




Note a possible collision is shown on the last plot of position vs. time shown above.

From the speed vs. time plot, we can see that when both balls are in the air, the difference between their speeds stay the same.

From the position vs. time plot, we can see that the distance between the balls increases while both balls are in the air.

Again from the position vs. time plot, we can see that the time interval between when the balls hit the ground is the same as the time interval between when the balls were released.

**Problem 2: Bus stop**

A bus leaves a stop at MIT and accelerates at a constant rate for 5 seconds. During this time the bus traveled 25 meters. Then the bus traveled at a constant speed for 15 seconds. Then the driver noticed a red light 18 meters ahead and slams on the brakes. Assume the bus decelerates at a constant rate and comes to a stop some time later just at the light.

a) What was the initial acceleration of the bus?

b) What was the velocity at the bus after 5 seconds?

c) What was the braking acceleration of the bus? Is it positive or negative?

d) How long did the bus brake?

e) What was the distance from the bus stop to the light?

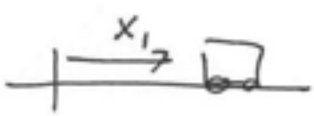
f) Make a graph of the position vs. time for the entire trip.

g) Make a graph of the velocity vs. time for the entire trip.

h) Make a graph of the acceleration vs. time for the entire trip.

**Problem 2 Solutions:**

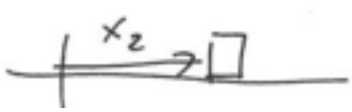
a)




$$x_1 = \frac{1}{2}a_1 t_1^2 \Rightarrow a_1 = \frac{2K_1}{t_1^2}$$

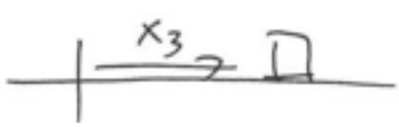
$$a_1 = \frac{(21(25m))}{(5s)^2} = 2\,{}^{m}\!/_{s} \qquad v_1 = a_1 t_1 = \left(\frac{2m}{s^2}\right)(5s) = 10\frac{m}{s}$$

b) use t=0:




$$x_2 = v_1 t_2 = \left(10\frac{m}{s}\right)(15s) = 150m$$

c+d) reset t=0




$$x_3 = v_1 t_3 + \frac{1}{2}a_3 t_3^2$$
$$v_3 = a_3 t_3 + v_1$$

at end $v_3=0$ $\Rightarrow$

$$v_1 + a_3 t_3 = 0 \Rightarrow t_3 = \frac{-v_1}{a_3}$$

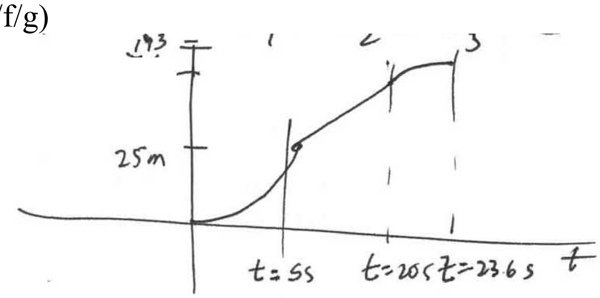
$$x_3 = v_1\left(\frac{-v_1}{a_3}\right) + \frac{1}{2}a_3\left(\frac{-v_1}{a_3}\right)^2 = -\frac{1}{2}\frac{v_1^2}{a_3}$$

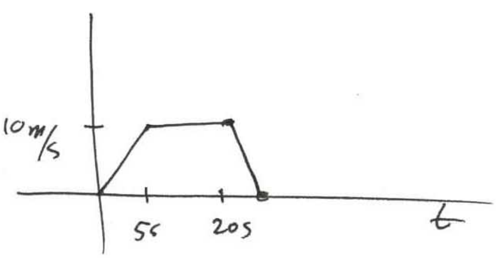
$$a_3 = -\frac{1}{2}\frac{v_1^2}{x_3} = \left(-\frac{1}{2}\right)\left(\frac{\left(10\,{}^{m}\!/_{s}\right)^2}{18m}\right) = -2.8\,\frac{m}{s^2}$$

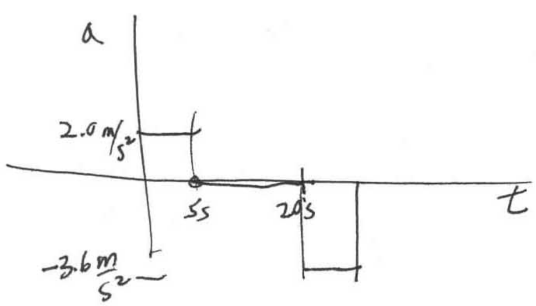
$$t_3 = \frac{-v_1}{a_3} = \frac{-10\,m/s}{-2.8\,m/s^2} = 3.6s$$

$$x^{total} = x_1 + x_2 + x_3 = (25m) + (150m) + (18m) = 193m$$

e/f/g)

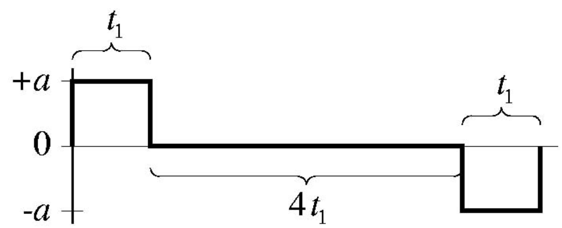


**Problem 3**

A person of given mass $m$ is standing on a scale in an elevator in Building 24. Initially the elevator is at rest. The elevator then begins to ascend to the sixth floor, which is a given distance $h$ above the starting point. The elevator undergoes an unknown constant acceleration $a$ for a given time interval $t_1$. Then the elevator moves at a constant velocity for a time interval $\Delta t_2 = 4t_1$. Finally the elevator brakes with a deceleration of the same magnitude as the initial acceleration for a time interval $\Delta t_3 = t_1$ until stopping at the sixth floor. Assume the gravitational constant is given as $g$. Find the magnitude of the acceleration. Briefly explain how you intend to model this problem and write down your strategy for solving it. Estimate the height $h$ and the time interval $t_1$ and check if your answer makes sense.




**Problem 3 Solution:**

There are three stages of motion, corresponding to the three parts of the graph given in the problem. The initial and final stages are at constant acceleration and the intermediate stage is at constant velocity. The key link between the stages is that the final speed after the first stage is the constant speed during the intermediate stage, and the initial speed for the final stage. I can write down the equations for displacement for each stage noting that the final speed after the first stage is the constant speed during the intermediate stage, and is the initial speed for the final stage.

After the time interval $t_1$, the elevator has an upward speed and displacement given by

$$\begin{aligned} y_1 &= y(t_1) = (1/2)\,a t_1^2, \\ v_1 &= v(t_1) = a t_1 \end{aligned} \qquad \textbf{(4.3.1)}$$

where $a$ is the unknown acceleration.

During the second stage, the elevator travels at a constant speed $v_1 = a t_1$ for a time interval $t_2 = 4t_1$, traveling a distance $y_2 = v_1 t_2 = 4 a t_1^2$ .

For the third stage, the initial speed is $v_1 = a t_1$ and the constant acceleration is $-a$ , and the distance traveled is during the time interval $\Delta t_3 = t_1$ is

$$y_3 = \left(at_1\right)t_1 - (1/2)a t_1^2 = (1/2)a t_1^2 . \qquad \textbf{(4.3.2)}$$

The total distance is the sum of the distances traveled in the three stages, and is equal to the height $h$ of the building,

$$h = y_1 + y_2 + y_3 = (1/2)a t_1^2 + 4 a t_1^2 + (1/2)a t_1^2 = 5 a t_1^2 . \qquad \textbf{(4.3.3)}$$

Solving for the acceleration gives

$$a = \frac{h}{5t_1^2} . \qquad \textbf{(4.3.4)}$$

Let's assume that the sixth floor of building 24 is about $h \simeq 25\text{ m}$ above the ground. This happens to be the slowest elevator at MIT taking approximately 30 s to reach the top so $t_1 \simeq 5\text{ s}$. Therefore the acceleration is

$$a \simeq \frac{25\text{ m}}{(5)(5\text{ s})^2} = 0.2\text{ m s}^{-2} \qquad \textbf{(4.3.5)}$$

This number is reasonable, I barely notice that the elevator is accelerating when I ride in the elevator. Notice that as longer as the magnitudes of the initial and final accelerations are equal, the elevator will always accelerate for 2.5 m no matter how long the journey takes.

**Problem 4**

A ball is released from rest at a height $h$ above the ground. The ball collides with the ground and bounces up at 75% of the impact speed the ball had with the ground. The collision with the ground is nearly instantaneously. A second ball is released above the first ball from the same height the instant the first ball loses contact with the ground. Calculate the time when the two balls collide and show where the collision occurs on the graph.

**Problem 4 Solution:**

Suppose ball one is dropped at time $t = 0$. Denote the instant ball one hits the ground by $t_2$.

We first need to calculate how long it takes for the first ball to hit the ground. Then we can calculate the y- component of the velocity of the first ball when it hit the ground. Since the ball rebounds with 75% of the speed it hit the ground, we know the y-component of the velocity of the first ball the instant it starts back upward. We also need to calculate the position and y- component of the velocity of the second ball the instant the first ball hits the ground. At this point we can restart our clock and using our kinematic equations for both balls, to determine the instant that they collide, and hence the height above the ground.

The equation for the position of the first ball is given by

$$y(t) = h - \frac{1}{2}gt^2 \qquad \textbf{(4.4.1)}$$

When the ball hits the ground at time $t_2$, $y(t_2) = 0$, so Eq. (4.4.1) becomes $0 = h - \frac{1}{2}gt_2^2$ which we can solve for $t_2$:

$$t_2 = \sqrt{2h / g} \qquad \textbf{(4.4.2)}$$

The equation for the y-component of the velocity of the first ball is given by

$$v_y(t) = -gt \qquad \textbf{(4.4.3)}$$

So the y-component of the velocity of the first ball when it hits the ground is

$$v_y(t_2) = -g\sqrt{2h / g} = -\sqrt{2hg} \qquad \textbf{(4.4.4)}$$

Immediately after the first ball rebounds from the ground, the y-component of the velocity is

$$v_{y10} = \frac{3}{4}\sqrt{2hg} \qquad \textbf{(4.4.5)}$$

Throughout the remainder of the problem we shall neglect the time interval that the first ball collided with the ground. We base this assumption on the fact that the collision occurred very rapidly on the order of milliseconds.

The second ball is released the instant the first ball loses contact with the ground. We shall now restart our clock (set $t = 0$) the instant the first ball loses contact with the ground. At that instant, the position of the second ball is

$$y_{20} = h \qquad \textbf{(4.4.6)}$$

and the y-component of the velocity of the second ball is

$$v_{2y0} = 0 \qquad \textbf{(4.4.7)}$$

The equation for the position of the first ball is

$$y_1(t) = v_{y10}\, t - \frac{1}{2}gt^2 \qquad \textbf{(4.4.8)}$$

Equation (4.4.8) becomes after substituting of Eq. (4.4.5)

$$y_1(t) = \frac{3}{4}\sqrt{2hg}\, t - \frac{1}{2}gt^2 \qquad \textbf{(4.4.9)}$$

The equation for the position of the second ball is

$$y_2(t) = y_{20} + v_{y20}t - \frac{1}{2}gt^2 \qquad \textbf{(4.4.10)}$$

**which becomes after substituting Equations (4.4.6) and (4.4.7)**

$$y_2(t) = h - \frac{1}{2}gt^2 \qquad \textbf{(4.4.11)}$$

Denote the instant the two balls hit by $t_h$. Then

$$y_2(t_h) = y_1(t_h) \qquad \textbf{(4.4.12)}$$

We now substitute Eq. (4.4.11) and Eq. (4.4.9) into Eq. (4.4.12) to get

$$h - \frac{1}{2} g t_h^2 = \frac{3}{4}\sqrt{2hg}\, t_h - \frac{1}{2} g t_h^2 \qquad \textbf{(4.4.13)}$$

which simplifies to

$$h = \frac{3}{4}\sqrt{2hg}\, t_h \qquad \textbf{(4.4.14)}$$

This equation can be solved for time $t_h$

$$t_h = \frac{\sqrt{8h/g}}{3} \qquad \textbf{(4.4.15)}$$

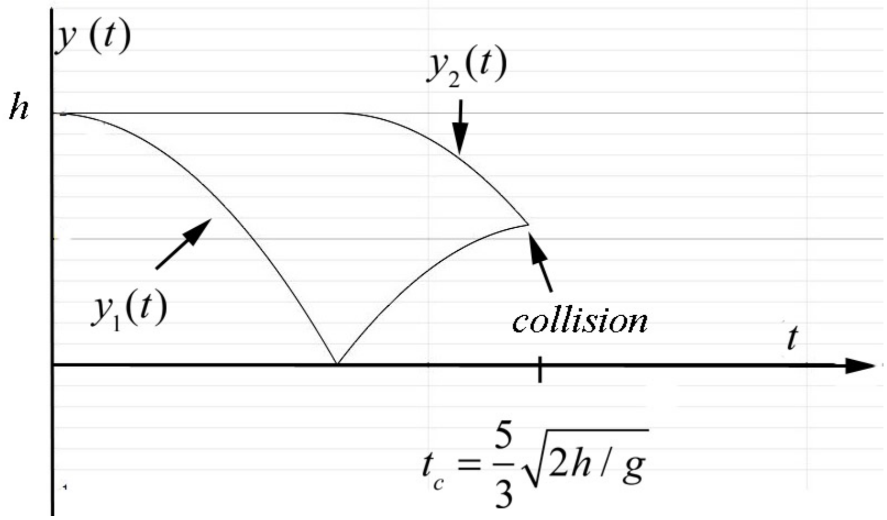
**The time elapsed from the instant the first ball was dropped is then**

$$t_c = t_2 + t_h = \sqrt{2h/g} + \frac{\sqrt{8h/g}}{3} = \frac{5}{3}\sqrt{2h/g} \qquad \textbf{(4.4.16)}$$

Although the problem did not specifically ask for this, we note that the collision took place at a height

$$y_2(t_h) = h - \frac{1}{2} g t_h^2 = h - \frac{1}{2} g \frac{8h}{9g} = \frac{5}{9} h \qquad \textbf{(4.4.17)}$$

The position vs. time for each ball is plotted in the figure below. The point of intersection of the two plots corresponds to the collision.



$y\,(t)$
$h$
$y_2(t)$
$y_1(t)$
collision
$t$
$t_c = \frac{5}{3}\sqrt{2h/g}$

**Problem 5**

A person starts running with a constant velocity trying to catch a streetcar that is initially $2.0\times10^{1}\text{ m}$ away from the person and has just started to accelerate from rest with a constant acceleration of $0.9\text{ m}\cdot\text{s}^{-2}$. The person runs just fast enough to catch the streetcar and hop on.

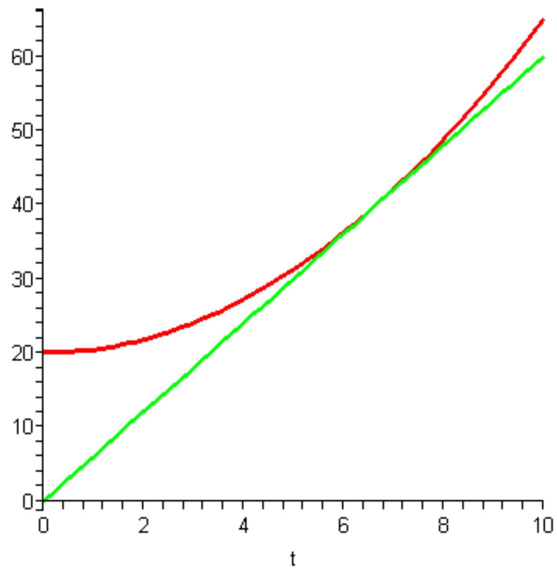
a) Describe the strategy you have chosen for solving this problem. You may want to consider the following issues. What does a sketch of the problem look like? What type of coordinate system will you choose? What information can you deduce from a plot of distance *vs.* time for both the person and the streetcar? What conditions must be satisfied when the person just catches up to the streetcar?

b) Now show all your work in answering the following three questions.
   i) How long did the person run?
   ii) What is the speed of the person when they just caught up to the streetcar?
   iii) How far did the person run?

**Problem 5 Solutions:**

a) This is a one-dimensional situation, with only one useful coordinate. Take the direction of the streetcar and the person to be the positive direction, with origin at the position where the person just starts to run. In order to satisfy the conditions of the problem, the person must be at the same position of the streetcar at the same time and running at the same speed as that of the streetcar.

In the figure below, the parabolic curve (the red curve if you're seeing this in color) represents the position of the streetcar as a function of time and the straight line (green) represents the position of the running person. The straight line is drawn from the origin and tangent to the parabola. Where the curves are tangent they have the same slope, and hence the streetcar and the running person have the same velocity. From the curve, it is seen that the time when this happens is between six and eight seconds.




b) Treating the parameters symbolically might give further insight into this and other similar problems. Specifically, let the original position of the streetcar be $x_{0,\mathrm{car}}$ and the constant speed of the person be $v_{\mathrm{person}}$, and let the streetcar's constant acceleration be $a$. Taking $t$ to be the time since the person has started to run, the positions of the streetcar and the person, and the speed of the streetcar are

$$\begin{aligned} x_{\mathrm{car}} &= x_{0,\mathrm{car}} + \frac{1}{2} a t^2 \\ v_{\mathrm{bus}} &= a t \\ x_{\mathrm{person}} &= v_{\mathrm{person}}\, t. \end{aligned} \tag{4.5.1}$$

For the person to just catch the streetcar, there must be some time $t_{\mathrm{catch}}$ at which $x_{\mathrm{person}} = x_{\mathrm{car}}$ and $v_{\mathrm{person}} = v_{\mathrm{car}}$. From the equations, these conditions are expressed algebraically as

$$\begin{aligned} v_{\mathrm{person}}\, t_{\mathrm{catch}} &= x_{0,\mathrm{car}} + \frac{1}{2} a t_{\mathrm{catch}}^2 \\ a t_{\mathrm{catch}} &= v_{\mathrm{person}}. \end{aligned} \tag{4.5.2}$$

The second of the equations is readily solved for $t_{\mathrm{catch}} = v_{\mathrm{person}} / a$, and substitution into the first yields

$$\begin{aligned}
&v_{\text{person}}\frac{v_{\text{person}}}{a} = x_{0,\text{car}} + \frac{1}{2}a\left(\frac{v_{\text{person}}}{a}\right)^2 \\
&\frac{1}{2}\frac{v_{\text{person}}^2}{a} = x_{0,\text{car}} \\
&v_{\text{person}} = \sqrt{2a\,x_{0,\text{car}}}\,.
\end{aligned} \tag{4.5.3}$$

We can now answer the questions asked in part b):

i) Substituting the last equation into $t_{\text{catch}} = v_{\text{bus}} / a$ gives

$$t_{\text{catch}} = \frac{v_{\text{person}}}{a} = \frac{\sqrt{2a\,x_{0,\text{car}}}}{a} = \sqrt{\frac{2\,x_{0,\text{car}}}{a}}\,. \tag{4.5.4}$$

Inserting the numerical values gives

$$t_{\text{catch}} = \sqrt{\frac{2\left(2.0\times 10^{1}\ \text{m}\right)}{0.9\ \text{m}\cdot\text{s}^{-2}}} = 6.7\,\text{s}\,. \tag{4.5.5}$$

ii) The speed of the person is found from the last equation

$$v_{\text{person}} = \sqrt{2\left(0.9\ \text{m}\cdot\text{s}^{-2}\right)\left(2.0\times 10^{1}\ \text{m}\right)} = 6.0\,\text{m}\cdot\text{s}^{-1}\,. \tag{4.5.6}$$

Of course, using $v_{\text{person}} = a\,t_{\text{catch}}$ gives the same result.

iii)

$$\begin{aligned}
x_{\text{catch}} &= v_{\text{person}}\,t_{\text{catch}} \\
&= \sqrt{2a\,x_{0,\text{car}}}\sqrt{\frac{2\,x_{0,\text{car}}}{a}} \\
&= 2\,x_{0,\text{car}} \\
&= 4.0\times 10^{1}\ \text{m}.
\end{aligned} \tag{4.5.7}$$

It should be noted that running $40\,\text{m}$ in under seven seconds is pretty good.

**Problem 6**

A motorist traveling with constant speed of 15 m/s passes a school-crossing corner, where the speed limit is 10 m/s. Just as the motorist passes, a police officer on a motorcycle stopped at the corner ($x = 0$) starts off in pursuit. The officer accelerates from rest at $a_x = 2.5$ m/s$^2$ until reaching a speed of 20 m/s. The officer then slows down at a constant rate until coming alongside the car at x = 360 m, traveling with the same speed as the car. a) How long does it take for the officer to catch up with the motorist? b) How long does the officer speed up? c) How far is the officer from the corner and from the car when switching from speeding up to slowing down? d) What is the acceleration of the officer when slowing down? e) Draw an $x$-$t$ graph and a $v_x$-$t$ graph for the two vehicles.

**Problem 6 Solution:**

Part a) of this problem can be very easily solved because the distance from the starting point where the motorcycle comes alongside the car is specified. The problem is more interesting if we do not specify that distance but solve for it. So I will first solve it with the distance $x = 360\text{ m}$ as part of the given information.

A coordinate system has already been chosen in the problem statement with the origin located at the school-crossing corner and the positive x-axis pointing in the direction of motion of the moving car. Introduce a coordinate function for the car $x_c(t)$. The car is not accelerating, so our model for the motion of the car is $a_c = 0$. Two integrations yield

$$v_c(t) = v_{co} \qquad \textbf{(4.6.1)}$$

$$x_c(t) = x_{co} + v_{co}t\,. \qquad \textbf{(4.6.2)}$$

The car started at the origin but had a non-zero initial x-component of the velocity so

$$x_{c10} = 0,\ v_{c10} = 15\text{ m}\cdot\text{s}^{-1}. \qquad \textbf{(4.6.3)}$$

So Eq(4.6.2) becomes

$$x_c(t) = v_{co}t\,. \qquad \textbf{(4.6.4)}$$

At the instant $t = t_f$, $x_{c1}(t_f) = 360\text{ m}$, therefore we can solve Eq. (4.6.4) for $t = t_f$ yielding

$$t_f = x_{c1}(t_f)/v_{c10} = (360\text{ m})/(15\text{ m}\cdot\text{s}^{-1}) = 24\text{ s}\,. \qquad \textbf{(4.6.5)}$$

Part b)

Part b) The problem involves two objects, the motorcycle and the car. The motorcycle has two distinct stages of motion, stage one occurs during the interval

$0 < t < t_1$ where $t_1$ represents the instant the motorcycle started to slow down. Stage two: occurs during the interval $t_1 < t < t_f$. During stage one the motorcycle speeds up at a constant rate i.e. the magnitude of the acceleration is constant and its direction is in the direction of motion. During stage two the motorcycle slows down at a constant rate which also implies constant acceleration in the opposite direction.

We begin by considering stage one. Introduce a coordinate function for the motorcycle $x_{m1}(t)$.

We model the acceleration of the motorcycle by noting that the problem statement states the acceleration of the motorcycle $a_{m1} = 2.5\ \text{m} \cdot \text{s}^{-2}$ and hence is constant, therefore by two integrations we can get both the velocity and position component functions for the motorcycle,

$$v_{m1}(t) = v_{m10} + a_{m1}t \tag{4.6.6}$$

$$x_{m1}(t) = x_{m10} + v_{m10}t + \tfrac{1}{2}a_{m1}t^2 \tag{4.6.7}$$

From the given information we note that since the motorcycle started at the origin form rest

$$x_{m10} = 0,\ \ v_{m10} = 0 \tag{4.6.8}$$

Therefore Eqs. (4.6.6) and (4.6.7), become considerably simpler

$$v_{m1}(t) = a_{m1}t \tag{4.6.9}$$

$$x_{m1}(t) = \tfrac{1}{2}a_{m1}t^2 \tag{4.6.10}$$

At the end of the first stage of motion, at the instant $t = t_1$, we are informed that the motorcycle has a x-component of the velocity given as $v_{m1}(t_1) = 20\ \text{m} \cdot \text{s}^{-1}$. Therefore we can solve Eq. (4.6.9) for the time $t_1$

$$t_1 = v_{m1}(t_1) / a_{m1} = (20\ \text{m} \cdot \text{s}^{-1})/(2.5\ \text{m} \cdot \text{s}^{-2}) = 8\ \text{s}\,. \tag{4.6.11}$$

part c) Substituting $t = t_1$ into Eq. (4.6.10) and using Eq. (4.6.11) we can determine how far the officer traveled while speeding up

$$x_{m1}(t_1) = \tfrac{1}{2}a_{m1}t_1^2 = \tfrac{1}{2}(2.5\ \text{m} \cdot \text{s}^{-2})(8\ \text{s})^2 = 80\ \text{m} \tag{4.6.12}$$

Similarly, the car has traveled at distance

$$x_c(t_1) = v_{co}t_1 = (15\text{ m}\cdot\text{s}^{-1})(8\text{ s}) = 120\text{ m} \tag{4.6.13}$$

So the officer is a distance

$$d = x_c(t_1) - x_{m1}(t_1) = 120\text{ m - }80\text{ m} = 40\text{ m}. \tag{4.6.14}$$

For stage two of the motion we will reset our clock to $t = 0$ , noting that the final conditions of stage one are the intial conditions of stage two. Introduce new coordinate functions for the motorcycle $x_{m2}(t)$ and the car $x_{c2}(t)$. We model the acceleration of the motorcycle by noting that the problem statement states the motorcycle is slowing down at a constant rate, therefore the acceleration of the motorcycle $a_{m2} < 0$ and is constant. The initial conditions for the motorcycle at stage two are

$$x_{m20} = 80\text{ m},\ v_{m20} = 20\text{ m}\cdot\text{s}^{-1} \tag{4.6.15}$$

By two integrations we again can get the velocity and position for the motorcycle,

$$v_{m2}(t) = v_{m20} + a_{m2}t \tag{4.6.16}$$

$$x_{m2}(t) = x_{m20} + v_{m20}t + \tfrac{1}{2}a_{m2}t^2 \tag{4.6.17}$$

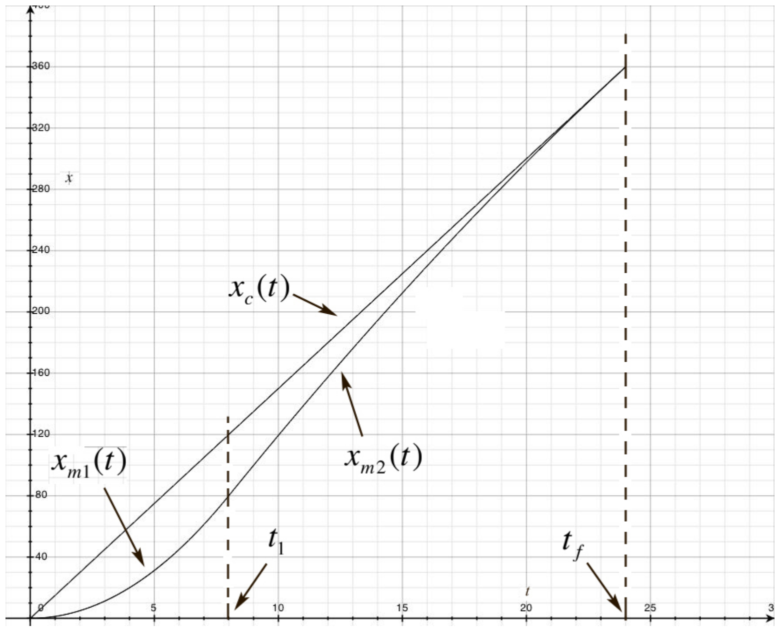
We note that stage two takes place during an interval $0 < t < t_2 = t_f - t_1 = 16\text{ s}$. We know the the final position of the motorcycle $x_{m2}(t_2) = 360\text{ m}$ so we can apply Eq. (4.6.17) with $t = t_2 = 16\text{ s}$ and solve for the acceleration of the motorcycle

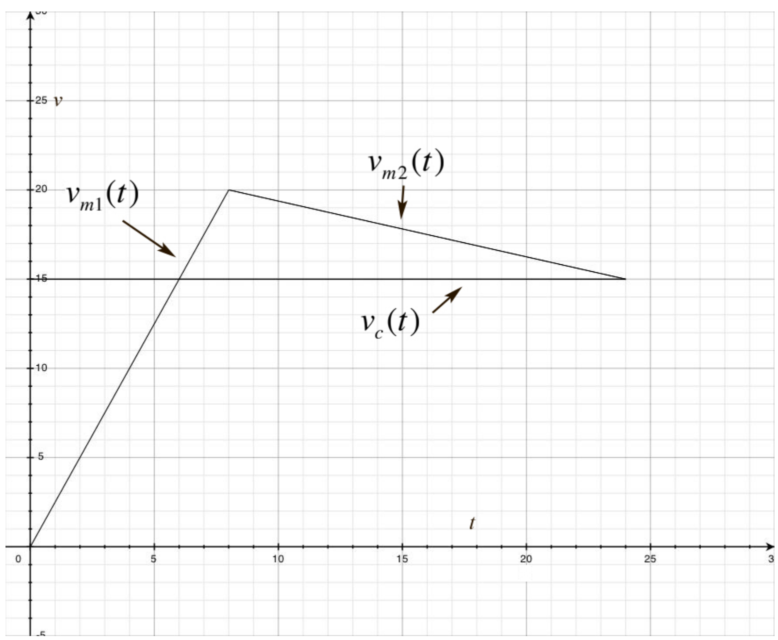
$$\begin{aligned} a_{m2} &= \left(\frac{2\big(x_{m2}(t_2) - x_{m20} - v_{m20}t_2\big)}{t_2^2}\right) \\ &= \left(\frac{2\big((360\text{ m}) - (80\text{ m}) - (20\text{ m}\cdot\text{s}^{-1})(16\text{ s})\big)}{(16\text{ s})^2}\right) = -0.3125\text{ m}\cdot\text{s}^{-2} \end{aligned} \tag{4.6.18}$$

Equivalently, we were told that the motorcycle and the car have the same speed at $t = t_2 = 16\text{ s}$, therefore Eq. (4.6.16) can be solved for the acceleration of the motorcycle

$$a_{m2} = \frac{v_{m2}(t_2) - v_{m20}}{t_2} = \frac{(15\text{ m}\cdot\text{s}^{-1}) - (20\text{ m}\cdot\text{s}^{-1})}{(16\text{ s})} = -0.3125\text{ m}\cdot\text{s}^{-2} \tag{4.6.19}$$

A $v_x(t)$ vs. $t$ and $x(t)$ vs. $t$ for both objects for the interval $0 < t < 24\text{ s}$ are shown below.



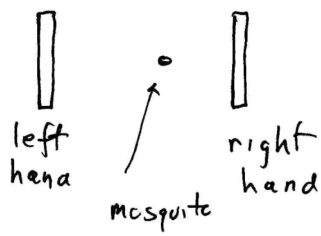



**Problem 7**

The scene opens with a mosquito on the line between two approaching hands that are initially a distance $D$ apart. The mosquito is a distance $s$ from the left hand which moves toward the right hand at a speed $v_L$. The right hand moves towards the left hand at speed $v_R$. If the mosquito flows towards the left hand at speed $v_M$, then reverses direction back toward the right hand at the last instant to avoid being struck by the left hand, how far does the mosquito fly before it is killed?

Be sure to clearly outline your strategy so that someone else can apply it to solve the problem.

**Problem 7 Solution:** We begin by assuming that the mosquito is killed when it just reaches the right hand. (This collision will probably not kill the mosquito but we will assume so anyway.)




We will divide the motion of the hands and the mosquito into two stages (Figure 2.1 and 2.2), when the mosquito travels towards the left hand and when the mosquito travels towards the right hand.

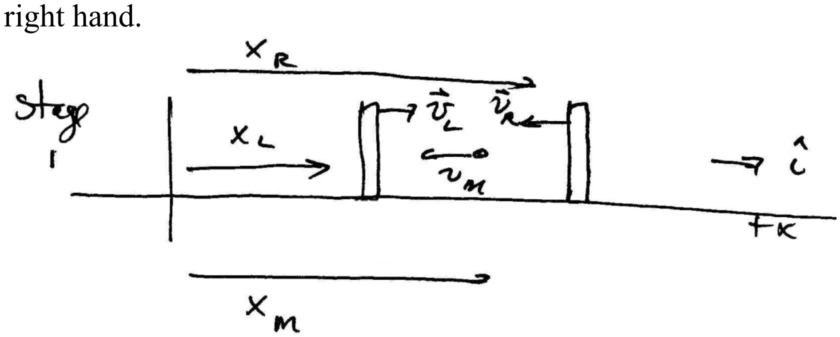



Figure 2.1

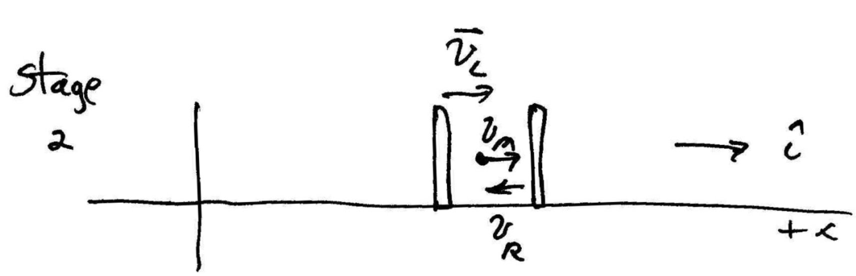


Figure 2.2

Let $t_1$ denote the time the mosquito just reaches the left hand. Let $t_2$ denote the time the mosquito just reaches the right hand; stage one: $0 \le t \le t_1$, stage two $t_1 \le t \le t_2$ . We will model the motion of all three objects by noting that their accelerations are all zero. By choosing an origin at the location of the left hand at time t =0 and positive x-direction to the right as shown in the Figure 2.1, we can use our kinematic equations to describe all three objects. We must take some care to get the initial conditions correct for each stage, noting that the final conditions for stage one are the initial conditions for stage two. We also show a graph of x vs. t for the motion of each object for the combined stages in Figure 2.3.

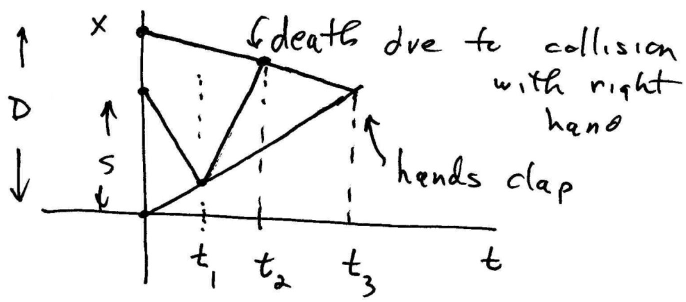



Figure 2.3

Stage one $0 \le t \le t_1$: The initial conditions are

mosquito: $x_{M0} = s$, $v_{x,M0} = -v_M$

left hand: $x_{L0} = 0$, $v_{x,L0} = v_L$

right hand: $x_{R0} = D$, $v_{x,R0} = -v_R$.

So the equations for the position of the objects are:

mosquito: $x_M(t) = x_{M0} + v_{x,M0}t = s - v_M t$

left hand: $x_L(t) = x_{L0} + v_{x,L0}t = v_L t$

right hand: $x_R(t) = x_{R0} + v_{x,R0}t = D - v_R t$

The condition that the mosquito just reaches the left hand at time $t_1$ is

$$x_M(t_1) = x_L(t_1)$$

which becomes

$s - v_M t_1 = v_L t_1$

Solving for time $t_1$ yields:

$$t_1 = \frac{s}{v_M + v_L}.$$

The position of the mosquito and the left hand at time $t_1$ is then

$$x_M(t_1) = x_L(t_1) = v_L t_1 = v_L \frac{s}{v_M + v_L}.$$

The position of the right hand at time $t_1$ is

$$x_R(t_1) = D - v_R t_1 = D - v_R \frac{s}{v_M + v_L}.$$

Note that the mosquito has traveled a distance

$$d_1 = \left|\Delta x_M\right| = \left|x_M(t_1) - x_{M0}\right| = \left|v_L \frac{s}{v_M + v_L} - s\right| = v_M \frac{s}{v_M + v_L}.$$

Stage 2 $t_1 \le t \le t_2$: Let's reset our clocks to $t = 0$ when the stage begins. Let $t_2 - t_1 = t_f$ denote the time it takes for the mosquito to reach the right hand.

The initial conditions are

mosquito: $x_{M0} = v_L \dfrac{s}{v_M + v_L}$, $v_{x,M0} = v_M$

right hand: $x_{R0} = D - v_R \dfrac{s}{v_M + v_L}$, $v_{x,R0} = -v_R$.

So the equations for the position of the mosquito and the right hand are:

mosquito: $x_M(t) = x_{M0} + v_{x,M0} t = v_L \dfrac{s}{v_M + v_L} + v_M t$

right hand: $x_R(t) = x_{R0} + v_{x,R0} t = D - v_R \dfrac{s}{v_M + v_L} - v_R t$

The condition that the mosquito just reaches the right hand at time $t_f$ is

$$x_M(t_f) = x_R(t_f)$$

which becomes

$$v_L \frac{s}{v_M + v_L} + v_M t_f = D - v_R \frac{s}{v_M + v_L} - v_R t_f$$

Solving for time $t_f$ yields:

$$t_f = \frac{1}{v_M + v_R}\left( D - v_R \frac{s}{v_M + v_L} - v_L \frac{s}{v_M + v_L} \right).$$

During stage two the mosquito traveled

$$d_2 = x_M(t_f) - x_{M0} = v_M t_f = \frac{v_M}{v_M + v_R}\left( D - \frac{(v_R + v_L)s}{v_M + v_L} \right).$$

So the total distance the mosquito traveled is equal to

$$d = d_1 + d_2 = v_M \frac{s}{v_M + v_L} + \frac{v_M}{v_M + v_R}\left( D - \frac{(v_R + v_L)s}{v_M + v_L} \right).$$

**Problem 8**

During a track event two runners, Bob and Jim, round the last turn and head into the final stretch with Bob $2.0\,\mathrm{m}$ in front of Jim. They are both running with the same speed $8.0\,\mathrm{m}\cdot\mathrm{s}^{-1}$. When the finish line is $4.8\times10^{1}\,\mathrm{m}$ away from Jim, Jim accelerates at $1.0\,\mathrm{m}\cdot\mathrm{s}^{-2}$ until he catches up to Bob. Jim then continues at a constant speed until he reaches the finish line.

a) Describe the strategy you have chosen for solving this problem

b) How long did it take Jim to catch Bob?

c) How far did Jim still have to run when he just caught up to Bob?

d) How long did Jim take to reach the finish line after he just caught up to Bob?

Bob starts to accelerate when Jim just catches up to him, and accelerates all the way to the finish line and crosses the line exactly when Jim does. Assume Bob's acceleration is constant.

e) What is Bob's acceleration?

f) What is Bob's velocity at the finish line? Who is running faster?

**Problem 8 Solutions:**

Choose t=0, when Jim starts to accelerate. Choose origin at Jim. Then

Jim Accelerates: $x_J = v_{0,J}t + \frac{1}{2}a_J t^2$

Bob constant velocity: $x_B = x_{B,0} + v_{0,B}t$

With: $x_{B,0} = d$

Jim catches Bob when at $t = t_1$, $x_J = x_B$

$$v_{O,J}\cdot t_1 + \frac{1}{2}a_J t_1^2 = d + v_{O,B}t_1 \qquad \textbf{(4.8.1)}$$

$$\frac{1}{2}a_J t_1^2 + \left(v_{O,J} - v_{O,B}\right)t_1 - d = 0 \qquad \textbf{(4.8.2)}$$

$$t_1^2 + \frac{2}{a_J}\left(v_{O,J} - v_{1,B}\right)t_1 - \frac{2d}{a_J} = 0 \tag{4.8.3}$$

Solve for $t_1$:

$$t_1 = \frac{-2}{a_J}\frac{\left(v_{O,J} - v_{O,B}\right) \pm \left(\frac{2}{a_J}\left(v_{O,J} - v_{O,B}\right)^2 + \frac{8d}{a_J}\right)^{1/2}}{2} \tag{4.8.4}$$

Choose the positive square root so that $t_1 > 0$

$$t_1 = \frac{-2}{a_J}\frac{\left(v_{O,J} - v_{O,B}\right) + \left(\frac{2}{a_J}\left(v_{O,J} - v_{O,B}\right)^2 + \frac{8d}{a_J}\right)^{1/2}}{2} \tag{4.8.5}$$

Calculate: $x_B = d + v_{O,B}t_1 = x_J\left(t_1\right)$

The finish line is distance $s - x_B(t_1)$. Jim is running at a constant velocity:

$$v_J = v_{J,O} + a_J t_1 \tag{4.8.6}$$

So it takes Jim a time

$$t_2 = \frac{s - x_B(t_1)}{v_J} = \frac{s - d - v_{O,B}t_1}{v_{J,O} + a_J t_1} \tag{4.8.7}$$

to reach the finish line. Now Bob accelerates and reaches the finish line in the time $t_2$.

$$s - x_B(t_1) = v_B t_2 + \frac{1}{2} a_B t_2^2 \tag{4.8.8}$$

Solve

$$a_B = \frac{2}{t_2^2}\left(s - x_B(t_1) - v_B t_2\right) \tag{4.8.9}$$

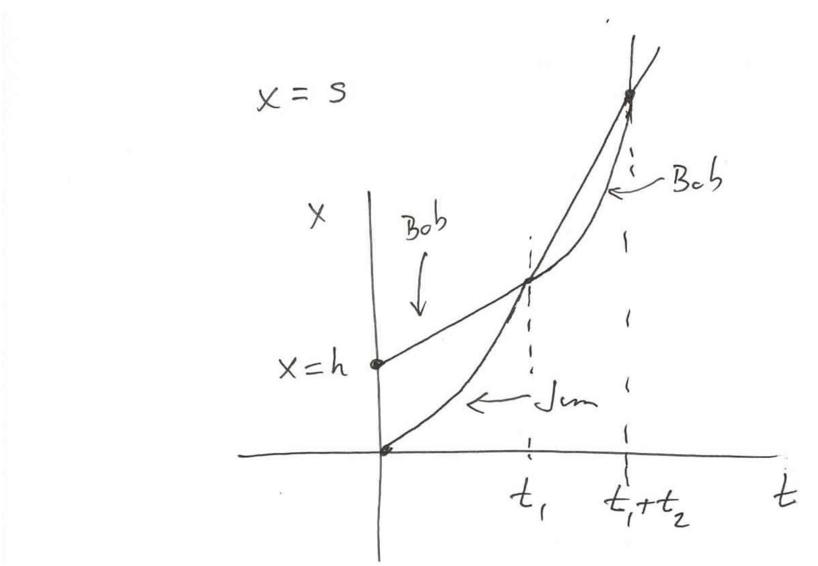
where

$$x_B(t_1) = d + v_{O,B}t_1 \tag{4.8.10}$$

$t_1$ and $t_2$ are above

The final velocity of Bob is

$$v_B(\text{final}) = v_{B,O} + a_B t_2 \tag{4.8.11}$$




From the graph, the slope of Bob's position function at the finish line is greater than Jim's. Bob is running faster at the finish.