I have a rod, a uniform rod, and the rod has mass M and it has length l. And I hit the rod with a brief impulse in a direction perpendicular to the rod. I hit it here. This rod is on a frictionless horizontal table.

Let us assume, if you want to get some numbers, that M equals 4 kilograms and that the length of the rod is 1/2 a meter. And that the magnitude of the impulse j, this is the magnitude now. This is the direction perpendicular to the rod. That that is 2 kilogram meters per second. So let this be the center of the rod, which is o. And so the length of the rod is l. And I hit the rod at a distance d above o. Bang.

What will happen? This rod is going to move towards the right. The translation of the center of mass, since there's no friction that motion will continue all the time. But in addition, it will start to rotate. So you're going to get a translation of the center of mass. And we're going to calculate the velocity of that center of mass. In addition, we're going to have a rotation about the center of mass.

Let us see how that would work out with a ruler, which is now not particularly a rod. And of course, we have friction. So if I give it a kick here, then you will see that it moves toward the right and it rotates. However, you will not see that that motion continues all the time. Because there is fiction, and so it will come to a halt. Ready?

So I will give it a kick here. Notice that the center of mass has moved to this point. So it was starting off with a certain velocity and notice that indeed, there is a rotation. And in the absence of friction this would not have come to a halt.

Now we want to calculate the velocity of the center of mass, and we want to calculate the angular velocity about the center of mass.

Let's first do the velocity of the center of mass. The impulse j, which is given here perpendicular to the rod is the momentum of the rod after the hit. The hit is given in a very brief amount of time. Minus the momentum before it hit. And this is 0. This is the momentum of the center of mass, center of mass. And it's not moving in the beginning. Therefore, if we look at the magnitude, I will leave the arrows out, so this means the magnitude of the impulse equals the magnitude of the momentum, which will be of course, in the same direction as the impulse. Notice the vectors are in the same direction. And that therefore is the mass of the rod times the velocity of the center of mass. And so it follows immediately that the velocity of the center of mass, which is this point o equals j divided by M. And in our case, would be about 0.5 meters per second. And notice that this velocity of this center of mass shortly after it is kicked, and it will remain all the time because there's no friction. That velocity of the center of mass is 1/2 a meter per second. In addition, the rod will start to rotate with angular velocity omega about the center of mass. And that's what I want to calculate next.

So what now is omega? I'm going to choose o as my origin. And I remember that the torque, which is a vector relative to point o-- that's the torque that is experience due to this impulse. And the torque will last some time. That equals dL dt about that point o. So the angular momentum measured relative to point o. It is the angular momentum change. That is the position vector relative to point o crossed times the force F. There is a force F here, which acts during a certain amount of time. And I have cross product of r o towards that point. So this would be my r o. Notice that the angle between the force and r o is 90 degrees. So I don't have to think too much about this cross.

Now, I will multiply each side with dt all these quantities. And so I will get dL relative to point o equals the position vector cross F dt and that equals-- I'm going to do an integral now. From t1 before the hit to t2 after the hit. t1 before the hit to t2 after the hit. And I recognize that the integral-- let's remove the equal sign-- t1 to t2 of F dt, that is the definition of our impulse. And so I can write now for the integral of dL about point o, L at time t2, which is L after the impact. Minus L at time t1, which is L before. And that now equals r0 cross j and r0 cross j-- as you see here, there are 90 degrees angle between the Two. It's simply d times j. So this equals d times j.

I watched this portion of the tape and I noticed a little bit of a sloppy notation. Sorry for that. L after minus L before equals dj. A vector notation.

If d is positive, then the direction of the angular momentum after into the paper. This one is 0. There is no angular momentum before about point o. And so, this equals dj. Now this may give you the impression that the direction of this vector is the same as j. But that's not true because we know j is in this direction perpendicular to the rod. What this really is of course, is the cross product between two vectors. The one from o2 the impulse, and the cross product with the impulse itself.

Now the magnitude of these two is dj because the angle between them is 90 degrees. So I should not have written just the arrow over the j, but maybe I should have written two arrows with a cross product and then I can completely live with this because then this cross product is also into the paper. So let's now continue the problem where we left it off.

And so, if I now want to know what the angular momentum is after the impact, write down L after. And I leave the vector off because I now deal only with the magnitude. That is d times the magnitude of j. This d is the distance between o and the point where I hit. But the angular momentum about point o-- this is all about point o-- is the moment of inertia about that point o, rotation about point o times the angular velocity after I have hit. This always holds. Angular momentum at any moment in time of a rotating object about the center of mass is always the moment of inertia for rotation about that center of mass times omega at any moment in time.

Now in the case that we have no fiction, of course, this is not going to change. But this equation hold in general.

Now for a uniform rod, the moment of inertia for rotation about the center of mass equals 1/12 times M l squared. In our case, for the numbers that I have given you, I believe it is about 0.0833 kilogram meters squared. So I find that omega after I hit it equals 12 times the distance to the hit d times j, the magnitude of the impulse, divided by M l squared. And if I put in the numbers, then I find 24 times d radians per second. And notice that now the angular velocity here is dependent on d. Whereas, the velocity of the center of mass did not depend on d.

The fact that omega depends on d is, of course, very intuitive. Because if I take this ruler, I put a ruler down here. And I hit the ruler right at the center, you wouldn't expect any rotation would you? But if I hit it a little higher, you would expect rotation. And if I hit it even higher, you would expect really more rotation; a larger angular velocity.

If d is positive, that means I'm above o. I expect a rotation in this direction. And I expect the center of mass to translate with the velocity that we calculated.

If I hit it right at the center, I expect only a translation, no rotation at all. d equals 0. No angular velocity.

If d is negative, that means I'm below o. Then omega becomes negative and what does that mean? Negative omega means it will rotate in this direction. But still the displacement of the center of mass with the velocity vC center of mass will be the same. In all cases, if my impulse is not different in all cases, will the motion of the center of mass, the velocity of the center of mass be the same? But the angular velocity will be different.

Now I will try to show these to you qualitatively. First of all, this sheet is not frictionless. So the motion will not continue forever and the thing will come to a halt. Not only is the sheet not frictionless, but the friction here may be a little larger than there. So it may stick a little. But I will try any how to make you see qualitatively, the effect of the translation motion of the center of mass, which is near the 6 on the ruler. You can't see the 6, but this is the middle of the ruler. And you will see then if I hit here that it will rotate and the center will move along a straight line. Let's try this. There we go. Notice the center of mass translated from here to here came to a halt because of friction, and it rotated.

If I would hit it with the same hit, which is hard for me to do, a little higher, I expect more rotation.

If I hit it lower, I expect less rotation, which is exactly what you see. Friction makes it come to a halt.

If I'm lucky and I could hit it right at the center of mass, which of course, is very difficult. Then you would only sees this motion. Now I hope you realize that there's no way I can do that. I will always be close to the center of mass, but not exactly at the center of mass. So let me try that.

Now you see, I was very close. There was almost no rotation. And so d was very close to 0.

Now I will clearly go for a negative value for d and so the rotation will be like so. I will hit it below d, and you see exactly the center of mass moved. Translation and the rotation was like this: counterclockwise. And if I hit it very low, then I get a larger rotation in the same amount of time. Because the angular velocity now is larger.

Well even though these demonstrations were rather qualitative, I hope they were useful and gave you some insight in the physics, what's happening.

If d equals 0 it is intuitive that omega equals 0. It's obvious.

If d, for instance, if I put in a number-- were 0.1 meters. Then omega would be 2.4 radians per second.

And if d were the maximum value possible, 0.25 meters, then omega max in our case would be 6 radians per second.

And if d were negative, that means I hit it below the center of mass. So I would hit it here. Then omega will have a different sign. Omega will be negative and I will get a counter clockwise rotation. Omega will be in this direction.

Now let us wait 10 seconds and ask ourselves the question, what is the motion of this object in 10 seconds? Well the center of mass has moved. In terms of translation that means move with constant velocity to the right 0.5 meters per second for 10 seconds. So that is 5 meters. The object has rotated about the center of mass. Let's call that over an angle theta. So the rod is first like this, and a little later in time, the rod is like this. The center of mass has moved over a certain distance and the rod has moved over an angle. Let's call that angle theta. Then theta equals omega t. This is the center o, center of mass. And we know omega.

If we take, for instance, the case that it is-- that d equals-- oh no, we haven't taken d yet. Omega is 24d, so we have 240d. In radians.

And so, if you took d equals 0, notice that theta equals 0. That's immediately obvious. Because if I hit it right at the center, then of course, there is no angular velocity. And if there's no angular velocity, then theta will remain 0. There will only be a translational velocity. So theta is 0.

If d were 0.1 meters, then theta would be 24 radians. And 24 radians is about 3.82 full rotations.

2 pi r-- no. 2 pi. 2 pi radians is one rotation. And 24 radians is 3.82 full rotations.

If we want to know what the kinetic energy is, the kinetic energy at any moment in time equals 1/2 M v center of mass squared. That is the translational kinetic energy plus the rotational kinetic energy, which is 1/2 I about the center of mass, moment of inertia, times omega squared. And for the case that we, for the number that we have chosen, we get 0.5 plus 24 times d squared.

Now the kinetic energy does depends on d. That's obvious because omega depends on d. But whether you take for d plus 0.1 or whether you take minus 0.1 meters, you would find the same kinetic energy. And the reason is obvious because the kinetic energy is proportional to omega squared doesn't care about the sign. It's proportional to d squared. Whether d is positive or negative makes no difference.

If d is positive, the object d is plus 0.1 would move like this and rotate like this. The kinetic energy would be the same if we were to hit it 0.1 meters below the center. It would then still move to the right, but it would rotate counter clockwise. There would be no change in the kinetic energy.

Now I don't want to confuse you, but I want you to appreciate that if you were to calculate the angular momentum relative to any point on this line, that means not relative to point o as we have just done, but relative to any point on this line, then the angular momentum is 0 before it hits and it is 0 after it hits. And the reason why that is the case, that if you take any point on this line and you take the cross product, r cross F, then you will see that the position vector and F are either in the same direction. Or if you take the point here in the opposite direction, so the sine of the angle between the two is always 0. So a remarkable thing is that the angular momentum relative to any point chosen on this line is conserved. Not only is it 0 before and after, but it doesn't change. Whereas, the angular momentum relative to this point does change. This is always a difficulty with the idea of angular momentum. We don't have a very good feeling for angular momentum. Because angular momentum depends on which point you choose. Well if you want to know what the period of rotation is, of course, that is not so difficult now. The period of 1 full rotation would be 2 pi divided by omega as we calculated. That was the angular velocity, which in this case, equals to the angular frequency. Because the object is rotating with a constant angular velocity. Then the angular velocity and the angular frequency are the same. The angular velocity equals d theta dt. And the two are the same. And I would invite you to visit a separate section that I have in which I discuss the angular velocity and the angular frequency.

If d equals 0, then I would find that omega equals 0. And that means the period would go to infinity, which is immediately obvious because if there is no rotation, the object takes infinitely long to make one full rotation.

If d were either plus or minus 0.1 meters, then I would find that the periods for 1 rotation-- either clockwise if it is a plus sign or counter clockwise if it is a minus sign-- would be about 2.62 seconds for the numbers that we have chosen. And that means if I wait 10 seconds, then in 10 seconds it will have made, remember, 3.82 rotations. We calculated that just a few minutes ago. And therefore, if I wait 10 seconds and 1 rotation takes 2.62 seconds, then indeed, I find exactly the number that we had before. That is equivalent to 3.82. So in 10 seconds it makes 3.82 rotations because 1 rotation if I hit it at a distance 0.1 meter from the center, it takes 2.62 seconds for a full rotation.