Systems and Conservation of Momentum

Conservation of Momentum: System of Particles

The change in momentum of a system and its surroundings is zero,

$$\Delta\vec{\mathbf{p}}_{\text{system}} + \Delta\vec{\mathbf{p}}_{\text{surroundings}} = 0$$

The total momentum of a system remains constant unless the system is acted on by an ***external*** *force*

Checkpoint Problem: Choice of system

Drop a stone from the top of a high cliff. Consider the earth and the stone as a system. As the stone falls, the momentum of the system

1. increases in the downward direction.
2. decreases in the downward direction.
3. stays the same.
4. not enough information to decide.

Position and Velocity of Center of Mass

Total mass for discrete or continuous body (mass density *ρ)*

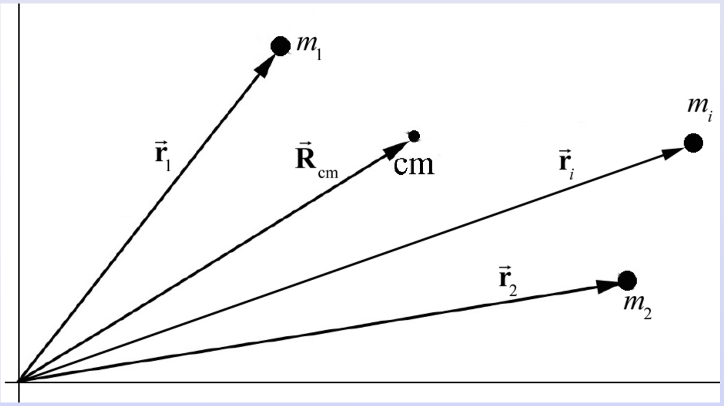
$$m_{\text{sys}} = \sum_{i=1}^{i=N} m_i = \int_{body} \rho dV$$

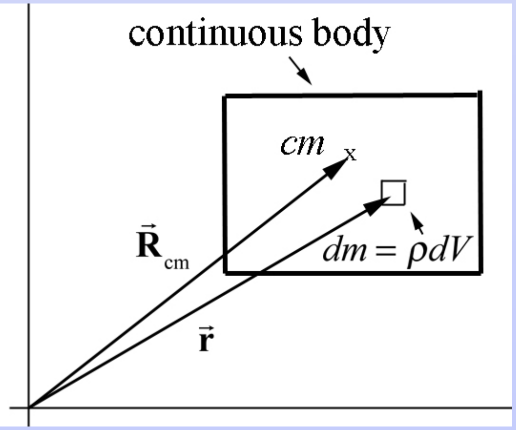
Position of center of mass

$$\vec{\mathbf{R}}_{cm} = \frac{1}{m_{\text{sys}}} \sum_{i=1}^{i=N} m_i \vec{\mathbf{r}}_i = \frac{1}{m_{\text{sys}}} \int_{body} \vec{\mathbf{r}} \rho dV$$

Velocity of center of mass

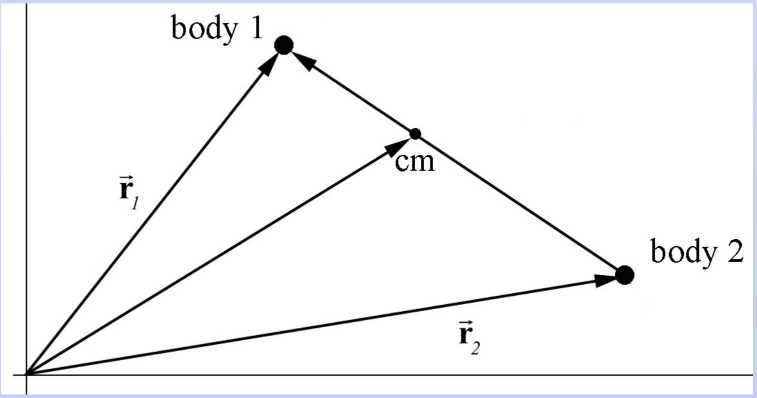
$$\vec{\mathbf{V}}_{cm} = \frac{1}{m_{\text{sys}}} \sum_{i=1}^{i=N} m_i \vec{\mathbf{v}}_i = \frac{1}{m_{\text{sys}}} \int_{body} \vec{\mathbf{v}} \rho dV = \frac{\vec{\mathbf{p}}_{\text{sys}}}{m_{\text{sys}}}$$





Checkpoint Problem: Center of Mass of Earth-Moon System

The mean distance from the center of the earth to the center of the moon is $r_{em} = 3.84 \times 10^8$ m. The mass of the earth is $m_e = 5.98 \times 10^{24}$ kg and the mass of the moon is $m_m = 7.34 \times 10^{22}$ kg . The mean radius of the earth is $r_e = 6.38 \times 10^6$ m. The mean radius of the moon is $r_m = 1.74 \times 10^6$ m. Where is the location of the center of mass of the earth-moon system? Is it inside the earth's radius or outside?



Checkpoint Problem: Center of Mass of Rod:

A thin rod has length *L* and total mass *M*.

a) Suppose the rod is uniform. Find the position of the center of mass with respect to the left end of the rod.

b) Now suppose the rod is not uniform but has a linear mass density that varies with the distance from the left end according to

$$\lambda(x) = \frac{\lambda_0}{L^2} x^2$$

where λ_0 is a constant and has SI units [kg-m^{-1}]. Find λ_0 and the position of the center of mass with respect to the left end of the rod.

Internal Force on a System of N Particles

- The total internal force on the *i*th particle is sum of the interaction forces with all the other particles

$$\vec{\mathbf{F}}_{\text{int},i} = \sum_{\substack{j=1 \\ i \neq j}}^{j=N} \vec{\mathbf{F}}_{i,j}$$

- The total internal force is the sum of the total internal force on each particle

$$\vec{\mathbf{F}}_{\text{int}}^{\text{total}} = \sum_{i=1}^{i=1} \vec{\mathbf{F}}_{\text{int},i} = \sum_{\substack{j=1 \\ i \neq j}}^{j=N} \vec{\mathbf{F}}_{i,j}$$

- Newton's Third Law: internal forces cancel in pairs

$$\vec{\mathbf{F}}_{i,j} = -\vec{\mathbf{F}}_{j,i}$$

- So the total internal force is zero

$$\vec{\mathbf{F}}_{\text{int}}^{\text{total}} = \vec{\mathbf{0}}$$

Total Force on a System of N Particles is the External Force

The total force on a system of particles is the sum of the total external and total internal forces. Since the total internal force is zero

$$\vec{\mathbf{F}}^{\text{total}} = \vec{\mathbf{F}}^{\text{total}}_{\text{ext}} + \vec{\mathbf{F}}^{\text{total}}_{\text{int}} = \vec{\mathbf{F}}^{\text{total}}_{\text{ext}}$$

External Force and Momentum Change

The total momentum of a system of N particles is defined as the sum of the individual momenta of the particles

$$\vec{\mathbf{p}}_{\text{sys}} \equiv \sum_{i=1}^{i=N} \vec{\mathbf{p}}_i$$

Total force changes the momentum of the system

$$\vec{\mathbf{F}}^{\text{total}} = \sum_{i=1}^{i=N} \vec{\mathbf{F}}_i^{\text{total}} = \sum_{i=1}^{i=N} \frac{d\vec{\mathbf{p}}_i}{dt} \equiv \frac{d\vec{\mathbf{p}}_{\text{sys}}}{dt}$$

Total force equals total external force

$$\vec{\mathbf{F}}^{\text{total}} = \vec{\mathbf{F}}_{\text{ext}}^{\text{total}}$$

Newton's Second and Third Laws for a system of particles: The total external force is equal to the change in momentum of the system

$$\vec{\mathbf{F}}_{\text{ext}}^{\text{total}} = \frac{d\vec{\mathbf{p}}_{\text{sys}}}{dt}$$

Translational Motion of the Center of Mass of Rigid Body

- Momentum of system $\vec{\mathbf{p}}_{\mathrm{sys}} = m_{\mathrm{sys}} \vec{\mathbf{V}}_{cm}$

- System can be treated as point mass located at center of mass. External force accelerates center of mass

$$\vec{\mathbf{F}}_{\mathrm{ext}}^{\mathrm{total}} = \frac{d\vec{\mathbf{p}}_{\mathrm{sys}}}{dt} = m_{\mathrm{sys}} \frac{d\vec{\mathbf{V}}_{cm}}{dt} = m_{\mathrm{sys}} \vec{\mathbf{A}}_{cm}$$

- Impulse changes center of mass momentum

$$\vec{\mathbf{I}} \equiv \int_{t_i}^{t_f} \vec{\mathbf{F}}_{\mathrm{ext}}^{\mathrm{total}} dt = \Delta \vec{\mathbf{p}}_{\mathrm{sys}} = m_{\mathrm{sys}} (\vec{\mathbf{V}}_{cm}(t_{\mathrm{f}}) - \vec{\mathbf{V}}_{cm}(t_i))$$

Demo : Center of Mass trajectory B78

http://tsgphysics.mit.edu/front/index.php?page=demo.php?letnum=B%2078&show=0

Odd-shaped objects with their centers of mass marked are thrown. The centers of mass travel in a smooth parabola. The objects consist of: a squash racket, a 16” diameter disk weighted at one point on its outer rim, and two balls connected with a rod. This demonstration is shown with UV light.

External Forces and Change of Momentum Vector

- The external force may be zero in one direction but not others

- The component of the system momentum is constant in the direction that the external force is zero

- The component of system momentum is not constant in a direction in which external force is not zero

Strategy: Momentum of a system

1. Choose system

2. Identify initial and final states

3. Identify any external forces in order to determine whether any component of the momentum of the system is constant or not

i) If there is a non-zero total external force:

$$\vec{\mathbf{F}}_{ext}^{total} = \frac{d\vec{\mathbf{p}}_{sys}}{dt}$$

ii) If the total external force is zero then momentum is constant

$$\vec{\mathbf{p}}_{sys,0} = \vec{\mathbf{p}}_{sys,f}$$

Modeling: Instantaneous Interactions

- Decide whether or not an interaction is instantaneous.
- External impulse changes the momentum of the system.

$$\vec{\mathbf{I}}[t, t+\Delta t_{col}] = \int_{t}^{t+\Delta t_{col}} \vec{\mathbf{F}}_{ext} dt = (\vec{\mathbf{F}}_{ext})_{ave} \Delta t_{col} = \Delta \vec{\mathbf{p}}_{sys}$$

- If the collision time is approximately zero,

$$\Delta t_{col} \quad 0$$

then the change in momentum is approximately zero.

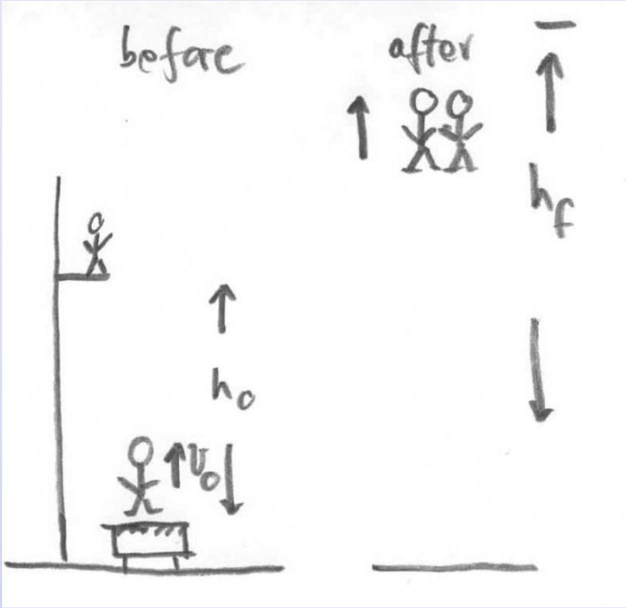
$$\Delta \vec{\mathbf{p}}_{system} \cong \vec{\mathbf{0}}$$

Checkpoint Problem: Landing Plane and Sandbag

A light plane of mass 1000 kg makes an emergency landing on a short runway. With its engine off, it lands on the runway at a speed of 40 ms^{-1}. A hook on the plane snags a cabal attached to a sandbag of mass 120 kg and drags the sandbag along. If the coefficient of friction between the sandbag and the runway is $\mu = 0.4$, and if the plane's brakes give an additional retarding force of magnitude 1400 N, how far does the plane go before it comes to a stop?

Checkpoint Problem: Acrobat and Clown

An acrobat of mass m_A jumps upwards off a trampoline with an initial *y-component* of velocity $v_{y,0} = v_0$. At a height h_0, the acrobat grabs a clown of mass m_B. Assume the acrobat grabs the clown during a time interval that is negligibly small.



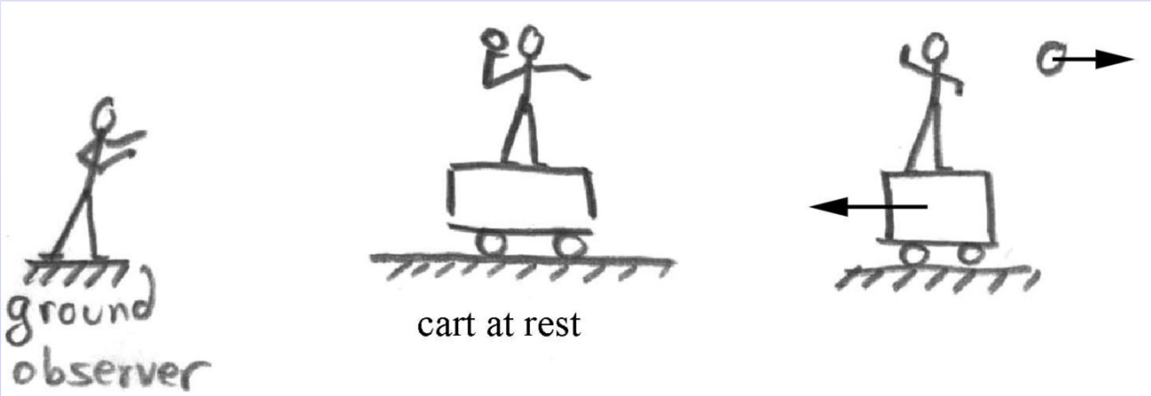
a) What is the velocity of the acrobat immediately before grabbing the clown?

b) What is the velocity of the acrobat immediately after grabbing the clown?

c) How high do the acrobat and clown rise?

Checkpoint Problem: Recoil

A person of mass m_1 is standing on a cart of mass m_2 that is on ice. Assume that the contact between the cart's wheels and the ice is frictionless. The person throws a ball of mass m_3 in the horizontal direction (as determined by the person in the cart). The ball is thrown with a speed u with respect to the cart.



a) What is the final velocity of the ball as seen by an observer fixed to the ground?

b) What is the final velocity of the cart as seen by an observer fixed to the ground?

Momentum Flow Diagram: Recoil