Momentum and Impulse

Photo courtesy of sombraala on Flickr

Momentum and Impulse

- Obeys a conservation law
- Simplifies complicated motions
- Describes collisions
- Basis of rocket propulsion & space travel

Quantity of Motion

DEFINITION II Newton’s Principia

The quantity of motion is the measure of the same, arising from the velocity and quantity of matter conjointly.

The motion of the whole is the sum of the motions of all the parts; and therefore in a body double in quantity, with equal velocity, the motion is double; with twice the velocity, it is quadruple.

Momentum and Impulse: Single Particle

- Momentum

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

  SI units

$$[\mathrm{kg}\cdot\mathrm{m}\cdot\mathrm{s}^{-1}] = [\mathrm{N}\cdot\mathrm{s}]$$

- change in momentum

$$\Delta\vec{\mathbf{p}} = m\Delta\vec{\mathbf{v}}$$

- average force

$$\vec{\mathbf{F}}_{ave} = \Delta\vec{\mathbf{p}} / \Delta t$$

- impulse

  SI units

$$\vec{\mathbf{I}} \equiv \vec{\mathbf{F}}_{ave}\Delta t \qquad \vec{\mathbf{I}} \equiv \int_{t_i}^{t_f} \vec{\mathbf{F}}dt$$

$$[\mathrm{N}\cdot\mathrm{s}]$$

Checkpoint Problem

Consider two carts, of masses *m* and 2*m*, at rest on an air track. If you push first one cart for 3 s and then the other for the same length of time, exerting equal force on each, the momentum of the light cart is

1. four times
2. twice
3. equal to
4. one-half
5. one-quarter the momentum of the heavy cart.

Newton's Second Law

"The change of motion is proportional to the motive force impresses, and is made in the direction of the right line in which that force is impressed",

$$\vec{\mathbf{F}}^{\text{total}} = \frac{d\vec{\mathbf{p}}}{dt} = m\frac{d\vec{\mathbf{v}}}{dt} = m\vec{\mathbf{a}}$$

$$\vec{\mathbf{I}} \equiv \int_{t_i}^{t_f} \vec{\mathbf{F}}^{\text{total}} dt = \Delta\vec{\mathbf{p}} \equiv \vec{\mathbf{p}}(t_{\text{f}}) - \vec{\mathbf{p}}(t_i)$$

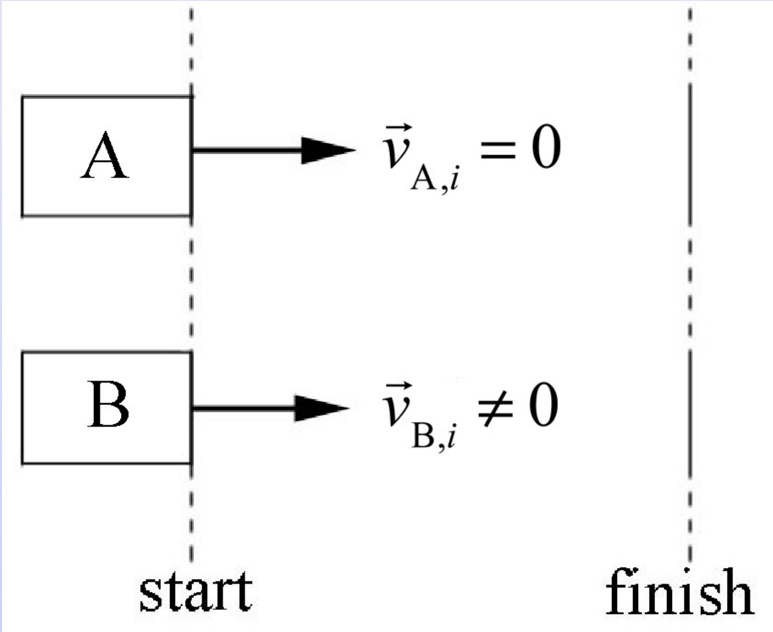
When $\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt} = \vec{\mathbf{0}}$

then $\vec{\mathbf{p}} = m\vec{\mathbf{v}} = \text{constant (vector)}$

Checkpoint Problem: Pushing Identical carts

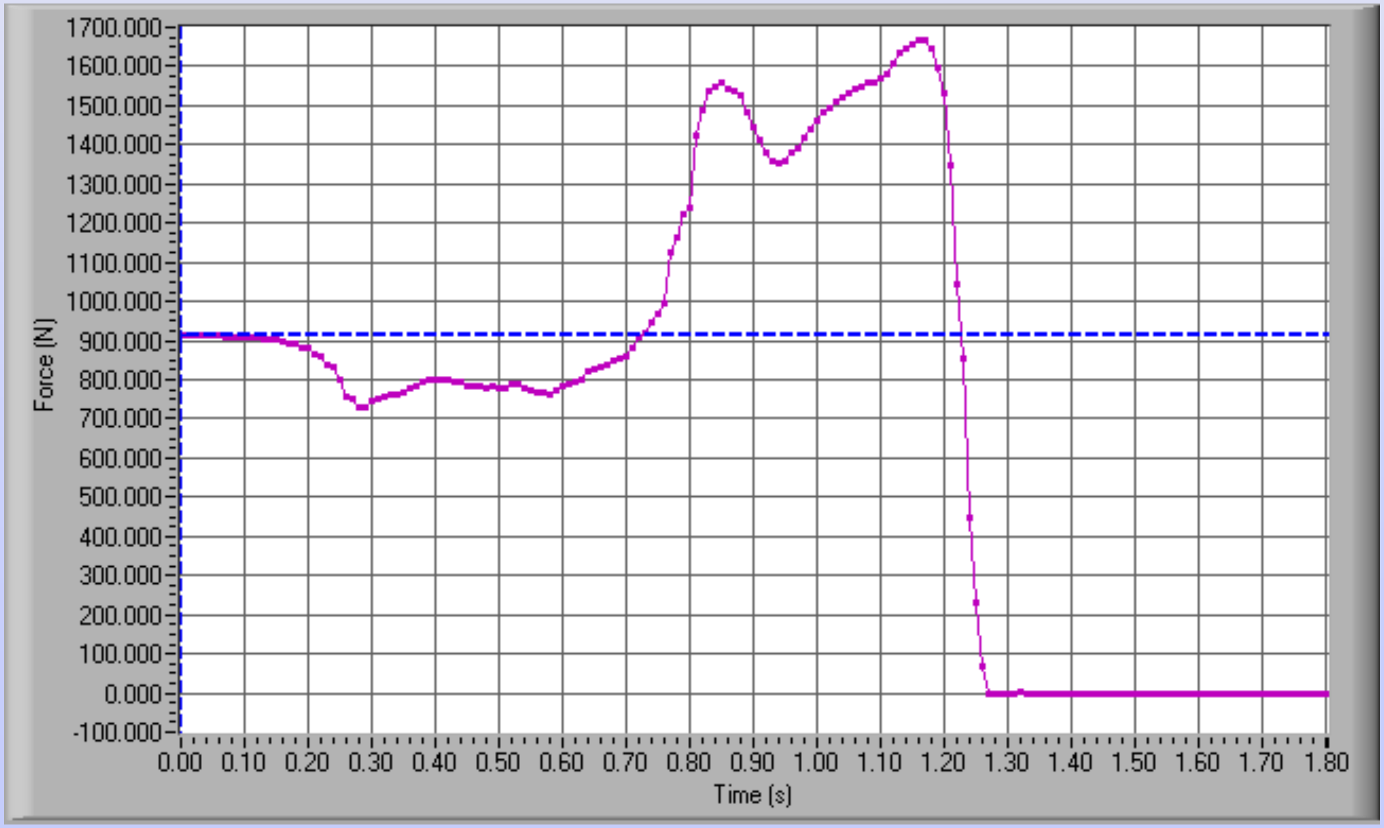
Identical constant forces push two identical objects A and B continuously from a starting line to a finish line. If A is initially at rest and B is initially moving to the right,

1. Object A has the larger change in momentum.
2. Object B has the larger change in momentum.
3. Both objects have the same change in momentum
4. Not enough information is given to decide.



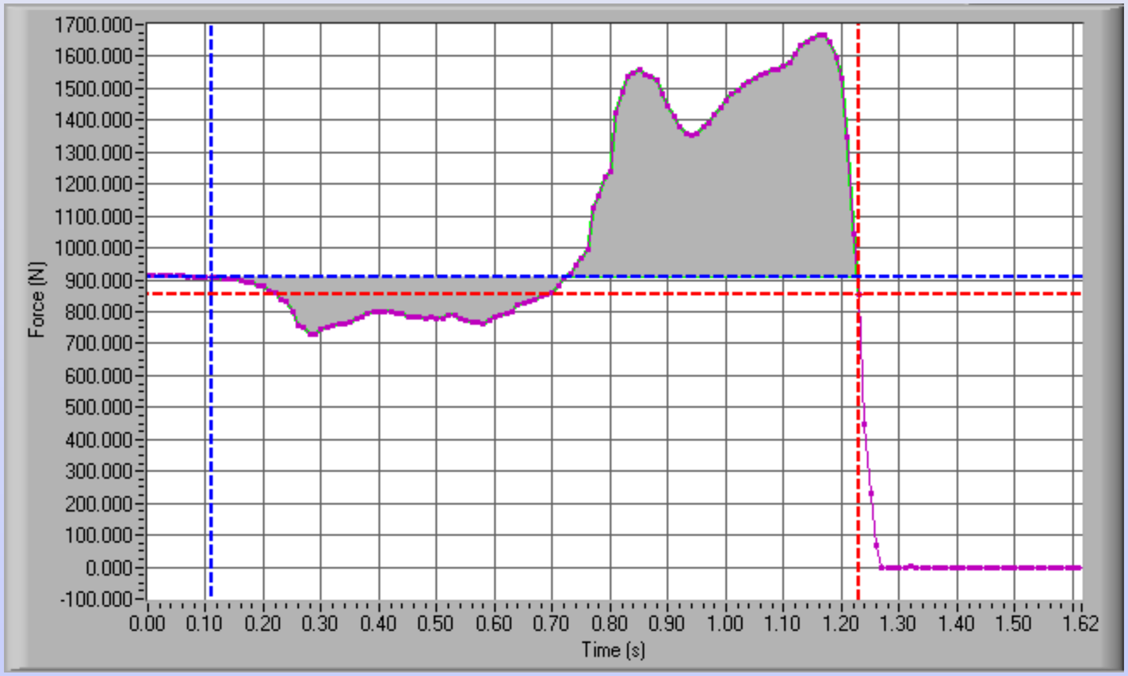
Jumping: Non-Constant Force

- Plot of total external force vs. time for Andy jumping off the floor. Weight of Andy is 911 N.



Jumping: Impulse

- Shaded area represents impulse of total force acting on Andy as he jumps off the floor



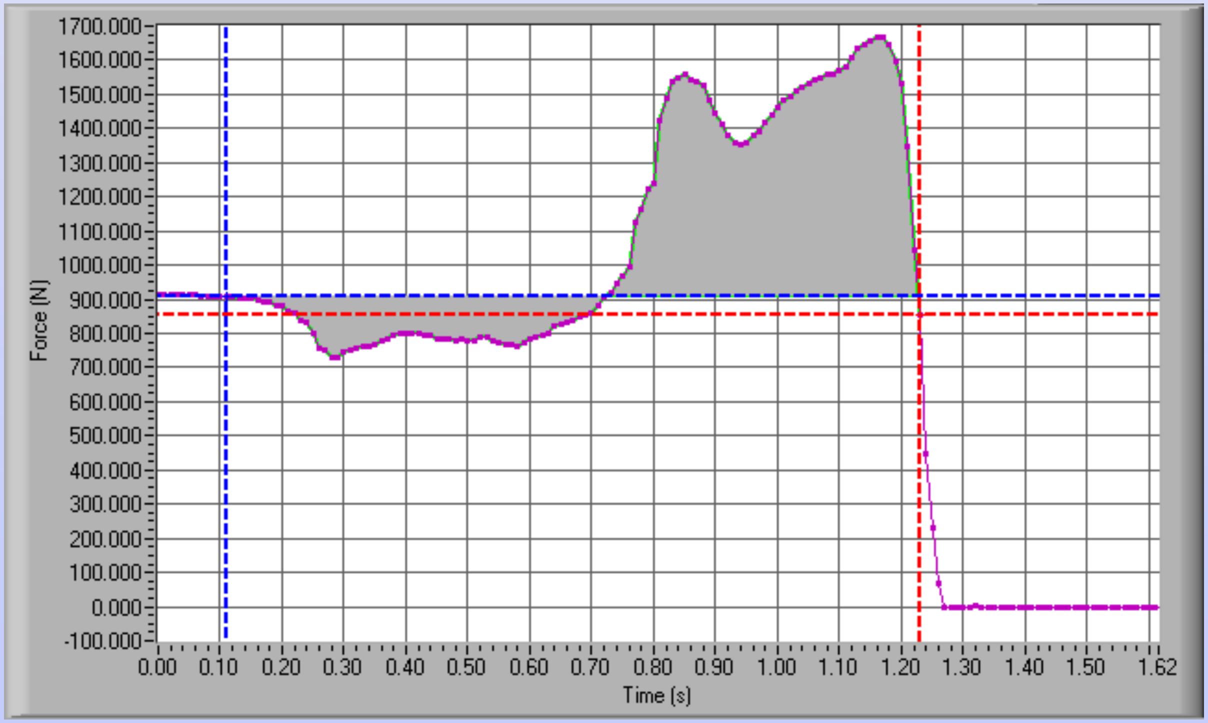
$$\vec{\mathbf{F}}^{\text{total}}(t) = \vec{\mathbf{N}}(t) + \vec{\mathbf{F}}_{grav}$$

$$\vec{\mathbf{I}}[t_i, t_f] = \int_{t_i = 0.11\,\text{s}}^{t_f = 1.23\,\text{s}} \vec{\mathbf{F}}^{\text{total}}(t)\, dt = 199\ \text{N} \cdot \text{s}$$

Checkpoint Problem: Jumping

At the end of the time interval [0.11 s, 1.23 s], what was Andy's center of mass velocity. Assume that at the beginning of the interval Andy was at rest. Andy's weight is 911 N. The impulse is

$$\vec{\mathbf{I}}[t_i, t_f] = \int_{t_i = 0.11\,\text{s}}^{t_f = 1.23\,\text{s}} \vec{\mathbf{F}}^{\text{total}}(t)\, dt = 199\ \text{N} \cdot \text{s}$$

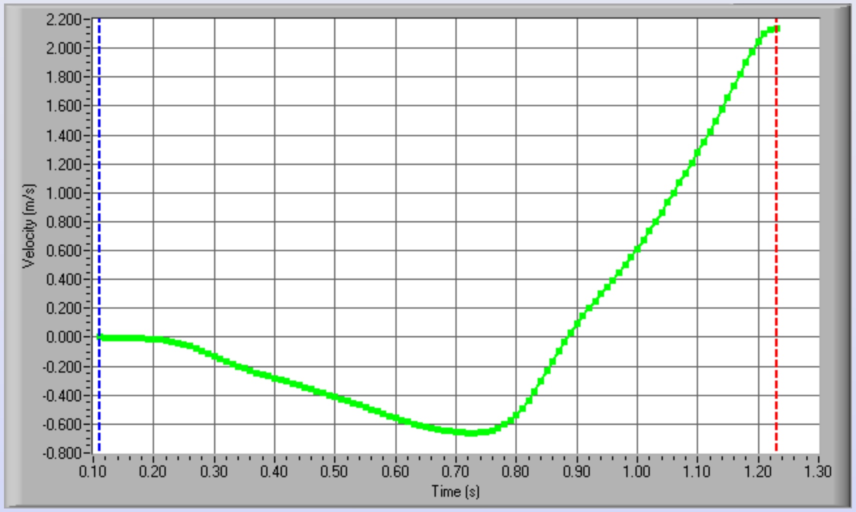


Jumping: Center of Mass Velocity

- When Andy leaves the ground, the impulse is

$$I_y[0.11\ \mathrm{s}, 1.23\ \mathrm{s}] = 199\ \mathrm{N \cdot s}$$

- So the center of mass velocity is



$$V_{cm,y}(t) = I_y[t_i, t] / m = (199\ \mathrm{N \cdot s})(9.80\ \mathrm{m \cdot s^{-2}})/(911\ \mathrm{N}) = 2.14\ \mathrm{m \cdot s^{-1}}$$

What was the magnitude of the displacement of Andy's center of mass after he left the floor?

$$\Delta Y_{cm} = V^2_{cm,y}(t_f) / 2g = (2.14\ \mathrm{m \cdot s^{-1}})^2 / (2)(9.80\ \mathrm{m \cdot s^{-2}}) = 0.23\ \mathrm{m}$$

Checkpoint Problem: Pushing Non-identical Carts

Consider two carts, of masses *m* and 2*m*, at rest on an air track. If you push one cart for 3 s and then the other for the same length of time, exerting equal force on each, what is the ratio of the kinetic energies of the light cart to the heavy cart

Checkpoint Problem: Same Momentum, Different Masses

Suppose a ping-pong ball and a bowling ball are rolling toward you. Both have the same momentum, and you exert the same force to stop each. How do the distances needed to stop them compare?

1. It takes a shorter distance to stop the ping-pong ball.

2. Both take the same distance.

3. It takes a longer distance to stop the ping-pong ball.