Units and Dimensional Analysis
Challenge Problems

Problem 1: An ideal (non-viscous) liquid with a density of ρ is poured into a cylindrical vessel with a cross-sectional area of A_1 to a level at a height h from the bottom. The bottom has an opening with a cross-sectional area A_2. Find the time it takes the k=liquid to flow out.

Problem 2: Non-Uniform Acceleration: *Terminal Velocity of Raindrop*

A raindrop of initial mass m_0 starts falling from rest under the influence of gravity. If we assume the air resistance is proportional to the square of the velocity, the resulting acceleration is given by the equation

$$\frac{dv}{dt} = g - k\,v^2 \tag{2.1}$$

where $g = 9.8\ \mathrm{m \cdot s^{-2}}$ and k is a constant. Note that the positive direction is downward.

a) What are the dimensions of k?

b) What is the terminal velocity of the raindrop? Make an order of magnitude estimate for the constant k based on your experience. Do all raindrops fall at the same terminal velocity? On which quantities do you think the terminal velocity may depend? A nice simulation of falling raindrops can be found at raindrop terminal velocity.

Problem 3: Dimensional Analysis Solution

The speed of a sail-boat or any other craft that does not plane is limited by the wave it makes – it can't climb uphill over the front of the wave. What is the maximum speed you'd expect?

Hint: relevant quantities might be the length l of the boat, the density ρ of the water, and the gravitational acceleration g:

$$v_{\text{boat}} \sim l^X \rho^Y g^Z \tag{3.1}$$