PROFESSOR: I'd now like to discuss a classic problem of Mary and Peter arguing in the rain.

Mary and Peter want to go home, and it starts to rain all of a sudden. And Peter says, you know, if we don't want to get very wet, we should walk very slowly. Because if we walk very slowly sinCE the rain is coming down vertically, we'll only get wet on our hat and our hat-- our head, is a very small area. Mary says, no, no. No, no. We should go as fast as we possibly can. Because if we run home very fast, then, of course, it takes only a little amount of time, and in a little amount of time, you probably catch much less rain.

Well, the question's always, who's right and who's wrong?

In a situation like this, it's always interesting, always useful to go to an extreme and to discuss the situation whereby the speed of Mary and Peter is effectively zero going home and one speed whereby the speed is huge.

I first want to represent Mary and Peter for that matter by a block. Because we're going to make some qualitative calculations later, and then this is my representation of a person. This is the head, which has sides a, b and the lengths of the person is h.

And so they're going to go over a distance that's the whole idea. d to make it home. This distance is d. The area of their head would be ab and the area of their front would be ah. a times h. And their velocity, let's assume, equals v Peter and Mary.

We will assume that the rain is falling down exactly vertically, hitting the street exactly vertically. And we will get in general the name v rain or v of r, and for practical purposes, if you like, you may later substitute 3 meters per second. But it's not so important right at the start.

Now when Peter and Mary look at the rain, the rain is coming towards them at an angle. Now you can see that easily because the rain is falling down with the velocity v rain. Peter and Mary are going with this velocity to the right, so from their frame of reference, the rain is also coming towards them with that component v Peter and Mary. And so, the actual direction as well as the speed with which the rain approaches Mary and Peter is given by this vector. You can calculate the magnitude of the vector and you can calculate its angle. We'll get into that very shortly.

If this value of the speed at which Peter and Mary go home, if that is zero, it's clear that the rain comes down vertically on their heads. And if, however, the speed of Peter and Mary is gigantic-- huge-- then this resultant velocity with which the rain approaches Peter and Mary as seen from their reference frame will become more and more horizontal. And in case of an absurdly high speed, which I'm sure they could never accomplish-- obviously they can only go so fast. But in the hypothetical case that they could go 10% of the speed of light say, then of course, the rain would approach them this way. This vector would become horizontal and only their front part would be wet.

Now, who is right, Peter or Mary?

Let's first assume that the velocity of Peter and Mary is zero.

If the velocity is zero, then it will take them an infinite amount of time to go home. Because the time for them to go home is the distance, d, divided by their speed. And for the case that vPM equals 0, the time goes to infinity. And during all that time, the rain will fall vertically on their head, and so they will get very, very, very wet.

Now we take the situation that we take an absurdly high speed. Let's say 10% of the speed of light. Of course, there's no way they could go that fast. Just for the sake of argument. So now the velocity of Peter and Mary is 0.1 times the speed of light. And that is about 3 times 10 to the seventh meters per second. Let us takes just as an example, let's take that D equal 1,000 meters so you get some numbers to deal with. Then the time that it takes for Peter and Mary to reach home is the D divided by the speed. And that is approximately 3 times 10 to the minus 5 seconds. So it's approximately 30 microseconds. So they made the trip in 30 microseconds.

How much has the rain fallen vertically in those 30 microseconds assuming that its speed were 3 meters per second? Well, the distance that the rain will have traveled vertically in 30 microseconds is, of course, the 3 meters per second that the rain comes down times the 30 microseconds, and that is approximately 10 to the minus 4 meters, which is about 0.1 millimeters. So in the 30 microseconds for Peter and Mary to reach home, this rain has fallen vertically over an insignificant distance, insignificant compared to the height of Peter are Mary, how tall they are.

So, for all practical purposes, you could treat the problem as if the rain hadn't fallen at all. And if the rain hasn't fallen at all, it's very obvious how much rain Peter and Mary will catch. It is all the rain that is in this box. This box, I will try to give it some color. All the rain in the volume of this box. Because remember, the amount by which the rain has fallen vertically is negligible. So whatever rain is in this box, that's the rain that they will catch, and they go with such an enormous speed that the rain will approach them horizontally. And so their front will get wet. Their front is actually like a broom. And what this broom is doing, it is sweeping up in 30 microseconds all the rain that is in this volume.

Now it's clear that that amount of rain is finite. The volume of rain that they catch-- they always get it, of course, in front. That volume is this surface area, which is a times h times D. That's the volume in cubic meters. If we multiply that by the density, how many kilograms per cubic meter rain there are, then we multiply it by row, and this tells us then how many kilograms of rain Peter and Mary will encounter-- will catch in front. That is always less than an infinite number of amount-- infinite number of rain that will fall on their head in the case that their speed was zero.

Now let's try to be quantitative and solve this problem in its generality. We all ready agreed that Mary was right and Peter was wrong. But let's now do it in its generality.

And we will assume that the velocity of the rain is v rain. No numbers for now. The velocity of Peter and Mary is vPM. The density of the rain is rho kilograms per cubic meter. And the distance that they travel equals D.

We can calculate the angle alpha, which is the angle at which the rain approaches Peter and Mary is the vertical. This is that velocity v and the magnitude of that velocity equals the square root of v Peter and Mary squared plus v rain squared. This is v Peter Mary and this is the rain. That's the magnitude. And this angle of alpha, which is the angle at which the rain hits them relative to the vertical, that angle of alpha is easy to calculate. The tangent of alpha equals v Peter Mary divided by v rain. So we know the magnitude. We know the speed with which the rain approaches them, and we know this angle. Let's now look at their head only.

It is the lengths b and this is the h, which I will not draw. The length b that is projected onto the direction of the rain becomes shortened. The rain comes in like this, the angle is alpha, which we know, and so the projected lengths of b perpendicular to the direction of the rain, this length equals b cosine alpha. This is also alpha and this is b. So how much rain are they catching now when they go home over a distance, D, with this velocity? If this is the velocity of the rain vertically down and if rho is the number of kilograms of water per cubic meter.

First, we take the area of their head, but we take the area projected perpendicular to the direction of the rain. That's all that matters. So we're going to get ab cosine alpha. That's the projected area of their head. We now have to multiply that by the speed of the rain. So we multiply that by the square root of v Peter Mary squared plus v rain. This is in square meters. This is in meters per second. So what you see here is how many cubic meters of volume they encounter every second.

Now I multiply by the density rho. This is in kilograms per cubic meter. So this now is how many kilograms of water they catch every second. Now I calculate by the time that they travel, which is D divided by vPM. They're going in this direction. And now so this is the time. So now we have kilograms. This was kilograms per second. We multiplied by time. So now this whole thing is now simply in terms of kilograms water. Kilograms rain. That is how much they will catch on their head.

And you can do something similar for their fronts. You can make a similar calculation of the front area, which you project now in the direction perpendicular to the direction of the rain. And once you have done that, well, I've done almost all the rest for you. The area, the from area projected onto the direction of the rain-- and I will leave you with that-- equals ah sine alpha. Be careful, what was a cosine here now becomes a sine.

And so if this is the projected area in the direction perpendicular to the rain, you multiply that by the velocity, by the speed of the rain, by the density of the rain and number of kilograms per cubic meter by the time that it takes them to reach home. And then you know how many kilograms of water reaches them on front.

Maybe you noticed what I noticed. There is a square missing. Look here. Obviously, this has to be v rain squared. I'm sure you caught that. Sorry for that. And the total amount of water is of course, the sum of the two.

Now I would like to give it a little twist to this problem. Let's suppose this is the velocity vPM and this is the velocity v rain as you will see it fall where you're standing still on the street. Then clearly, the rain approaches Peter and Mary under this angle alpha with the vertical.

If now Peter and Mary were to tilt a little and walk at this angle with velocity vPM, but they would be bending over, so they're not spending straight. They go like this. Then of course, only water can hit their head. There's no cosine alpha now because the velocity vector hits their head at 90 degree angles. So the area of their heads is no ab.

And it's clear that when they do this that the amount of rain that they will catch is less than what I just calculated because nothing will catch them on front. And you can easily work this out. If we make the assumption that the rain is coming down at 3 meters per second, and if for instance, we would ask Peter and Mary to run home with a speed of 3 meters per second-- I'm sure they can do that. 3 meters per second is about 7 miles per hour. Most of us could do that provided we don't do it for too long. Then the angle alpha would be 45 degrees. Because this will be 3 meters per second and this would be 3, so the angle alpha would be 45 degrees. And so in all practical purposes, if they could run as fast as the rain is coming down, and they would tilt at an angle of 45 degrees, I think they would catch the least amount of rain of all, and it would hit only their head.

Well, I think we've hit this problem hard enough, and I want you to think about it. It's a classic problem, and the bottom line is, of course, Mary is right. Women are always right.