Concept of Force and Newton’s Laws of Motion

Concept of Force

Force is a vector quantity $\vec{\mathbf{F}}^{total} \equiv m\vec{\mathbf{a}}$

The magnitude of the total force is defined to be

F = (mass) x (magnitude of the acceleration)

The direction of the total force on a body is the same as the direction of the acceleration.

The SI units for force are newtons (N):

$$1\ \text{N} = 1\text{kg} \cdot \text{m/s}^2$$

Superposition Principle

Apply two forces $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ on a body,

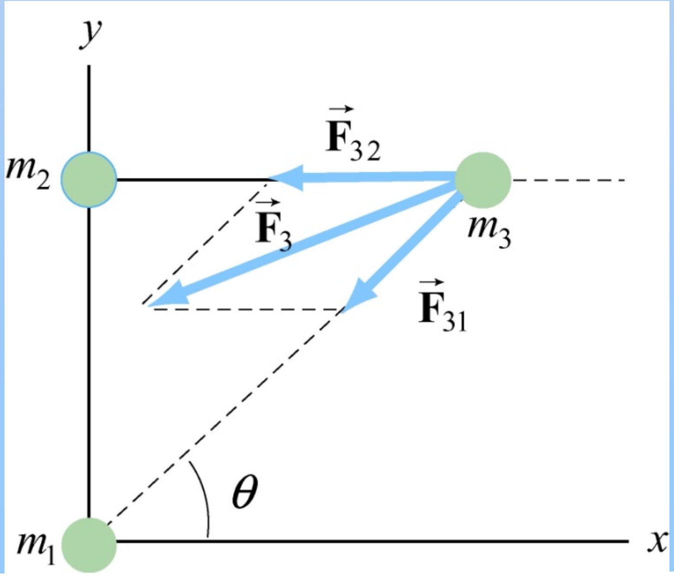
the total force is the vector sum of the two forces:

$$\vec{\mathbf{F}}^{\text{total}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$$

Notation: The force acting on body 1 due to the interaction between body 1 and body 2 is denoted by $\vec{\mathbf{F}}_{12}$

Example: The total force exerted on m_3 by m_1 and m_2 is:

$$\vec{\mathbf{F}}_3^{\,total} = \vec{\mathbf{F}}_{31} + \vec{\mathbf{F}}_{32}$$



Forces

Gravitation

Electric and magnetic forces

Elastic forces (Hooke’s Law)

Frictional forces: static and kinetic friction, fluid resistance

Contact forces: normal forces and static friction

Tension and compression

Free Body Diagram

1. Represent each force that is acting on the object by an arrow on a **free body force diagram** that indicates the direction of the force

$$\vec{\mathbf{F}}^{T} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \cdots$$

2. Choose set of independent unit vectors and draw them on free body diagram.

3. Decompose each force $\vec{\mathbf{F}}_i$ in terms of vector components.

$$\vec{\mathbf{F}}_i = F_{i,x}\,\hat{\mathbf{i}} + F_{i,y}\,\hat{\mathbf{j}} + F_{i,z}\,\hat{\mathbf{k}}$$

4. Add vector components to find vector decomposition of the total force

$$F_x{}^{T} = F_{1,x}{}^{T} + F_{2,x}{}^{T} + \cdots$$

$$F_y{}^{T} = F_{1,y}{}^{T} + F_{2,y}{}^{T} + \cdots$$

$$F_z{}^{T} = F_{1,z}{}^{T} + F_{2,z}{}^{T} + \cdots$$

Newton's First Law

Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it.

$$\sum \vec{\mathbf{F}}_i = \vec{\mathbf{0}} \quad \Rightarrow \quad \vec{\mathbf{v}} = \text{constant}$$

Reference Systems

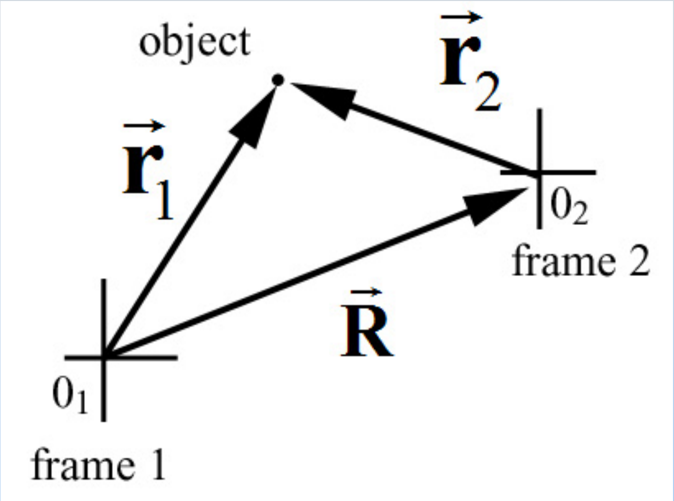
Use coordinate system as a ‘*reference frame*’ to describe the position, velocity, and acceleration of objects.

Relatively Inertial Reference Frames

Two reference frames.

Origins need not coincide.

One moving object has different position vectors in different frames



$$\vec{\mathbf{r}}_1 = \vec{\mathbf{R}} + \vec{\mathbf{r}}_2$$

Relative velocity between the two reference frames

$$\vec{\mathbf{V}} = d\vec{\mathbf{R}} \Big/ dt$$

is constant since the relative acceleration is zero

$$\vec{\mathbf{A}} = d\vec{\mathbf{V}} \Big/ dt = \vec{\mathbf{0}}$$

Newton’s First Law

Newton’s First Law in relatively inertial reference frames: If there is no net force impressed on an object at rest in Frame 2, then there is also no net force impressed on the object in Frame 1.

An object that is at rest in Frame 2 is moving at a constant velocity in reference Frame 1.

Newton's Second Law

The change of motion is proportional to the motive force impresses, and is made in the direction of the right line in which that force is impressed,

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}}.$$

When multiple forces are acting,

$$\sum_{i=1}^{N} \vec{\mathbf{F}}_i = m\vec{\mathbf{a}}.$$

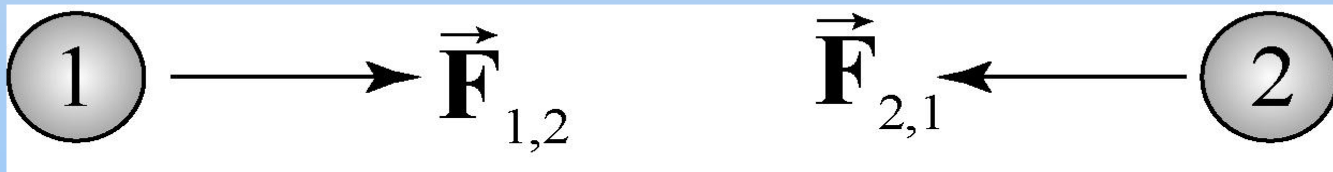
In Cartesian coordinates:

$$\sum_{i=1}^{N} F_{x,i} = m\,a_x, \quad \sum_{i=1}^{N} F_{y,i} = m\,a_y, \quad \sum_{i=1}^{N} F_{z,i} = m\,a_z.$$

Newton's Third Law

To every action there is always opposed an equal reaction: or, the mutual action of two bodies upon each other are always equal, and directed to contrary parts.

$$\boxed{\vec{\mathbf{F}}_{1,2} = -\vec{\mathbf{F}}_{2,1}}$$



Action-reaction pair of forces cannot act on same body; they act on different bodies.

Force Law: Newtonian Induction

- Definition of force has no predictive content.
- Need to measure the acceleration and the mass in order to define the force.
- **Force Law:** Discover experimental relation between force exerted on object and change in properties of object.
- **Induction:** Extend force law from finite measurements to all cases within some range creating a **model.**
- **Second Law can now be used to predict motion!**
- If prediction disagrees with measurement adjust model.

Force Law: Gravitational Force near the Surface of the Earth

Near the surface of the earth, the gravitational interaction between a body and the earth is mutually attractive and has a magnitude of

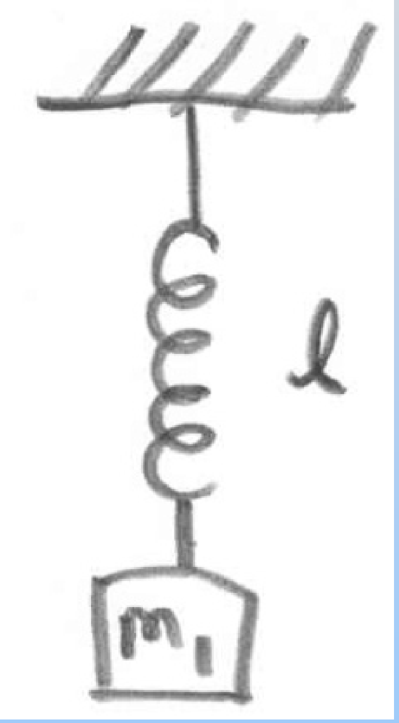
$$\left|\vec{\mathbf{F}}_{\text{grav}}\right| = m_{\text{grav}}\, g$$

where m_{grav} is the *gravitational mass* of the body and g is a positive constant.

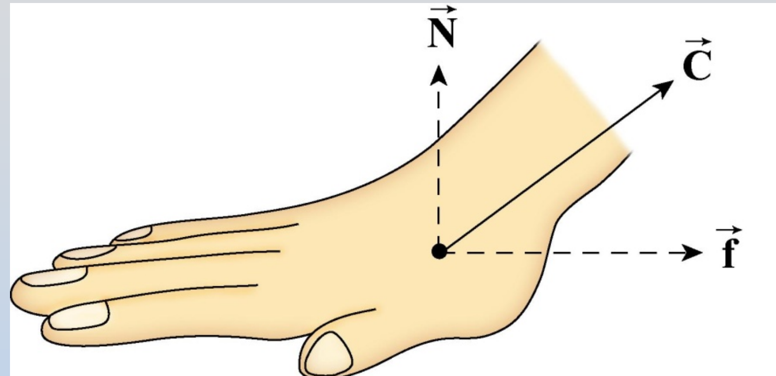
$$g = 9.80665\ \text{m} \cdot \text{s}^{-2}$$

Checkpoint Problem: Hooke's Law

Consider a spring with negligible mass that has an unstretched length 8.8 cm. A body with mass 150 g is suspended from one end of the spring. The other end (the upper end) of the spring is fixed. After a series of oscillations has died down, the new stretched length of the spring is 9.8 cm. Assume that the spring satisfies Hooke's Law when stretched. What is the spring constant?



Force Laws: Contact Forces Between Surfaces



The contact force between two surfaces is denoted by the vector

$$\vec{\mathbf{F}}^{total}_{surface,hand} \equiv \vec{\mathbf{C}}$$

Normal Force: Component of the contact force perpendicular to surface and is denoted by

$$\vec{\mathbf{F}}^{normal}_{surface,hand} \equiv \vec{\mathbf{N}}$$

$$\vec{\mathbf{C}} \equiv \vec{\mathbf{N}} + \vec{\mathbf{f}}$$

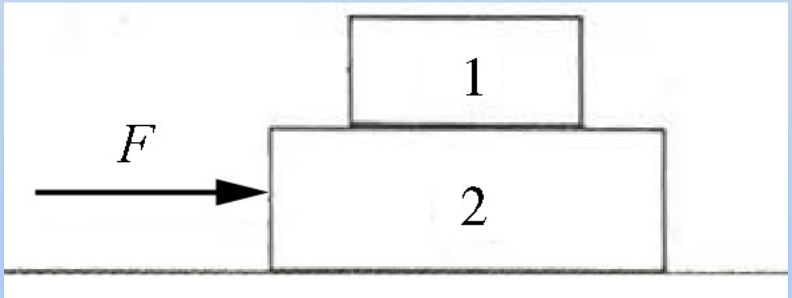
Friction Force: Component of the contact force tangent to the surface and is denoted by

$$\vec{\mathbf{F}}^{tangent}_{surface,hand} \equiv \vec{\mathbf{f}}$$

Therefore the contact force can be modeled as a vector sum

Checkpoint Problem: Pushing Textbooks

Consider two textbooks that are resting one on top of the other. The lower book has M_2 and is resting on a nearly frictionless surface. The upper book has mass $M_1 < M_2$. Suppose the coefficient of static friction between the books is μ_s.



A horizontal force of magnitude F is applied to the lower book so that the two books move together without slipping. Identify all action-reaction pairs of forces in this problem and draw free-body force diagrams on each object.

Kinetic Friction

The kinetic frictional force f_k is proportional to the normal force, but independent of surface area of contact and the velocity.

The magnitude of f_k is

$$\boxed{f_k = \mu_k N}$$

where μ_k is the coefficients of friction.

Direction of f_k: opposes motion

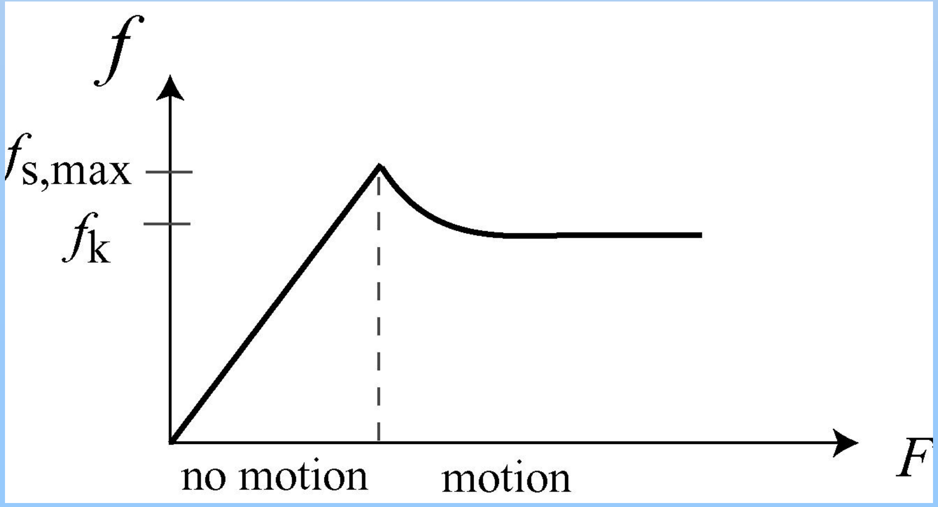
Static Friction

Varies in direction and magnitude depending on applied forces:

$$0 \le f_s \le f_{s,\max} = \mu_s N$$

Static friction is equal to it's maximum value

$$f_{s,\max} = \mu_s N$$



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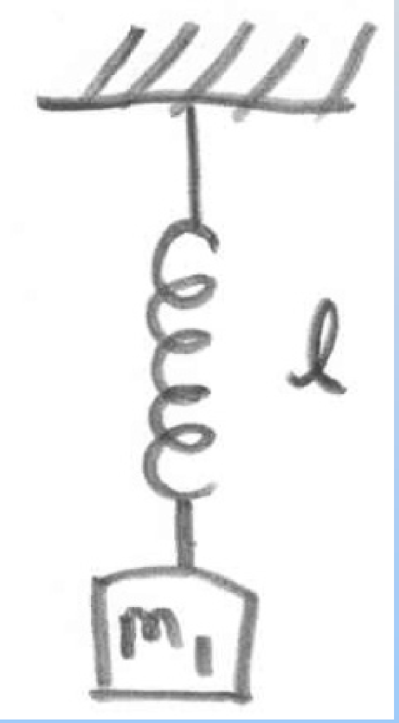
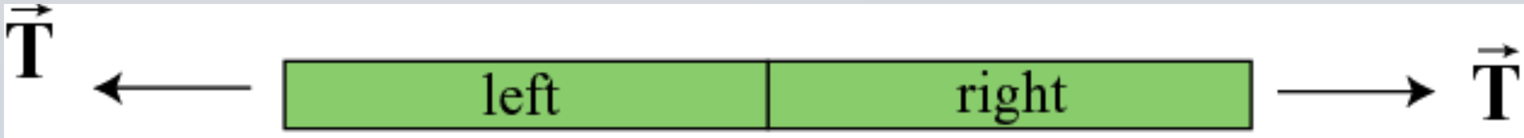
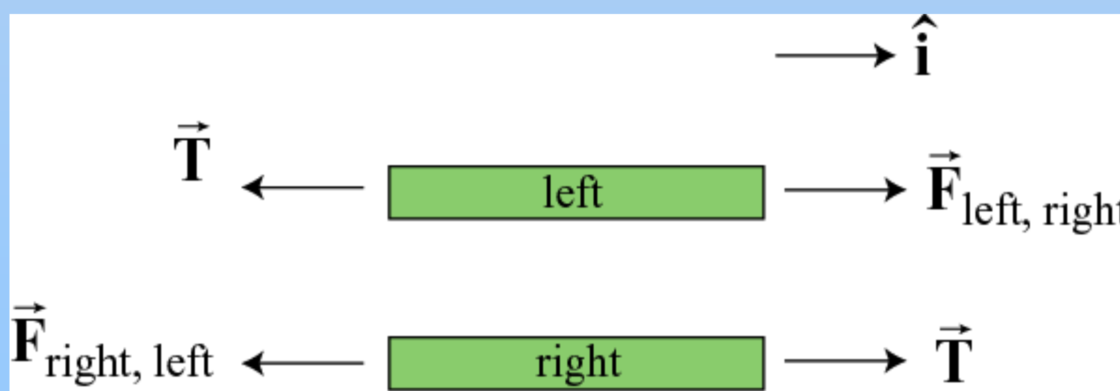


Tension in a Rope



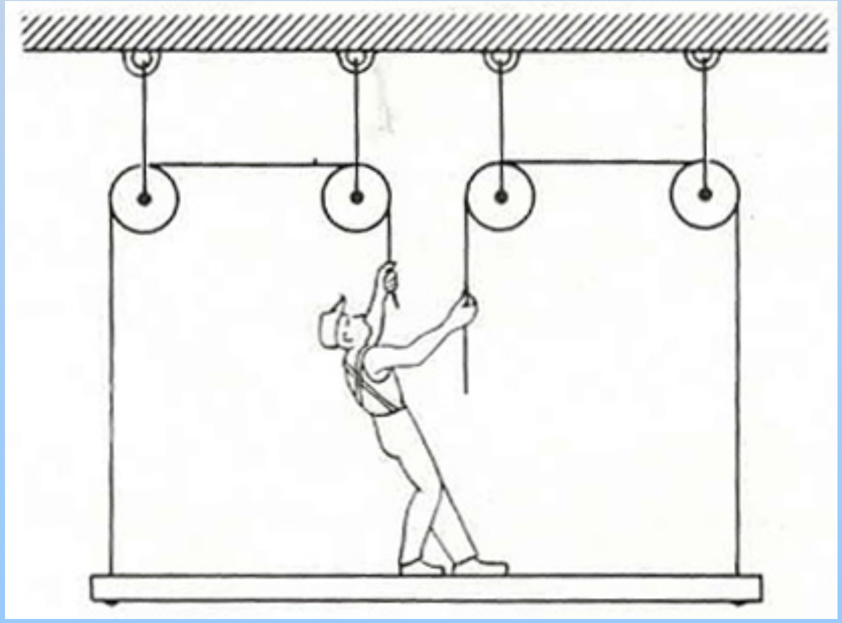
The *tension* in a rope at a distance x from one end of the rope is the magnitude of the action-reaction pair of forces acting at that point ,

$$T(x) = \left|\vec{\mathbf{F}}_{\text{left,right}}(x)\right| = \left|\vec{\mathbf{F}}_{\text{right,left}}(x)\right|$$



Chcekpoint Problem: Painter on Platform

Draw free body force diagrams for the plank and man.



Source: why you need to know physics

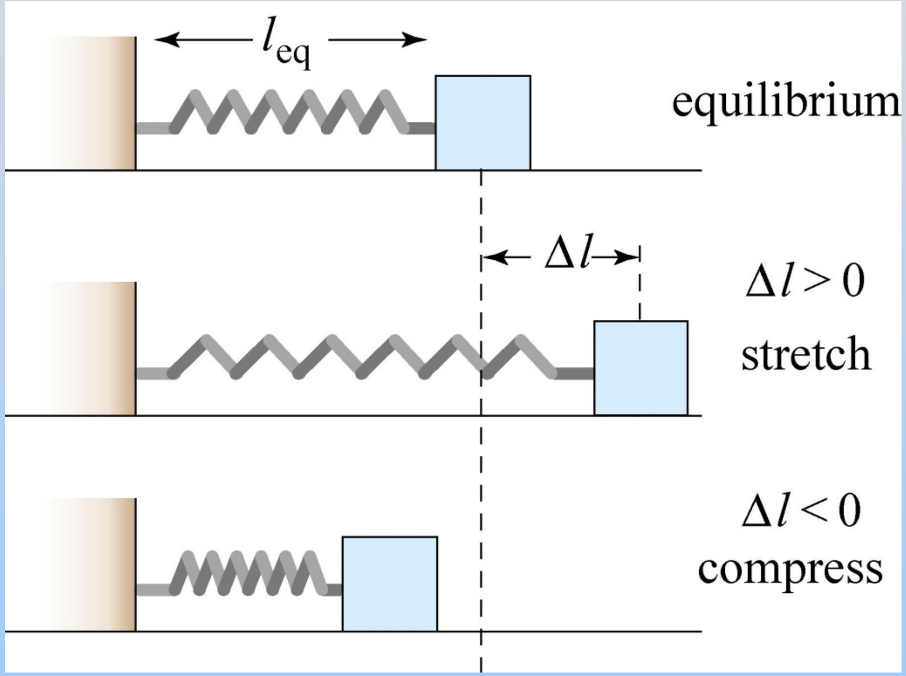
Photo courtesy of Thomas Hawk on Flickr

Checkpoint Problem: Tension

Suppose a rope is tied rather tightly between two trees that are apart 30 m. You grab the middle of the rope and pull on it perpendicular to the line between the trees with as much force as you can. Assume this force is 1000 N, and the point where you are pulling on the rope is 1 m from the line joining the trees. What is the magnitude of the tension in the rope?

Empirical Force Law: Hooke's Law

Consider a mass *m* attached to a spring



Stretch or compress spring by different amounts produces different accelerations

Hooke's law: $|\vec{\mathbf{F}}| = k|\Delta l|$

Direction: restoring spring to equilibrium

Hooke's law holds within some reasonable range of extension or compression

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