Application of Newton’s Second Law

Newton's Second Law

The change of motion is proportional to the motive force impresses, and is made in the direction of the right line in which that force is impressed,

$$\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}.$$

When multiple forces are acting,

$$\sum_{i=1}^{N} \vec{\mathbf{F}}_i = m\,\vec{\mathbf{a}}.$$

In Cartesian coordinates:

$$\sum_{i=1}^{N} F_{x,i} = m\,a_x, \quad \sum_{i=1}^{N} F_{y,i} = m\,a_y, \quad \sum_{i=1}^{N} F_{z,i} = m\,a_z.$$

Concept of System: Reduction

Modeling complicated interaction of objects by isolated a subset (possible one object) of the objects as the system

Treat each object in the system as a point-like object

Identify all forces that act on that object

Model – Point Mass with Forces

Newton's Laws of Motion:

- Forces replace rest of universe, animism
- If $\sum \boldsymbol{F} = 0$ then $\boldsymbol{a} = \mathbf{0}$ ***inertial coordinate system***
- $m\boldsymbol{a} = \sum \boldsymbol{F}$
- Forces generated in pairs by interactions

Intrinsically a 3-D Model:

- Any object is subject to forces

Ex: Planet, automobile, book on table, bridge member

- Provides "explanation" of all (classical) motion

Model: Newton's Laws of Motion

System: Point mass with applied force

Description of System:

- Objects: Point Mass
- State Variables: m, $a(t)$, $r(t)$
- Agents: real forces on object

Multiple Representations;

- Words, Force Diagrams, Equations

Interactions:

- Force Laws: contact, spring, universal gravity, uniform gravity, drag.

Law of Motion:

$\sum \mathbf{F} = m\mathbf{a}$

- Origin and Type of forces, Vectors

Free Body Diagram

1. Represent each force that is acting on the object by an arrow on a **free body force diagram** that indicates the direction of the force

$$\vec{\mathbf{F}}^T = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \cdots$$

2. Choose set of independent unit vectors and draw them on free body diagram.

3. Decompose each force $\vec{\mathbf{F}}_i$ in terms of vector components.

$$\vec{\mathbf{F}}_i = F_{i,x}\,\hat{\mathbf{i}} + F_{i,y}\,\hat{\mathbf{j}} + F_{i,z}\,\hat{\mathbf{k}}$$

4. Add vector components to find vector decomposition of the total force

$$F_x^{\ T} = F_{1,x}^{\ \ T} + F_{2,x}^{\ \ T} + \cdots$$

$$F_y^{\ T} = F_{1,y}^{\ \ T} + F_{2,y}^{\ \ T} + \cdots$$

$$F_z^{\ T} = F_{1,z}^{\ \ T} + F_{2,z}^{\ \ T} + \cdots$$

Newton’s Second Law: Strategy

Treat each object in the system as a point-like object

Identify all forces that act on that object, draw a free body diagram

Apply Newton’s Second Law to each body

Find relevant constraint equations

Solve system of equations for quantities of interest

Methodology for Newton's 2nd Law

I. Understand – get a conceptual grasp of the problem

Sketch the system at some time when the system is in motion.

Draw free body diagrams for each body or composite bodies:

Each force is represented by an arrow indicating the direction of the force

Choose an appropriate symbol for the force

II. Devise a Plan

Choose a coordinate system:

- Identify the position function of all objects and unit vectors.
- Include the set of unit vectors on free body force diagram.

Apply vector decomposition to each force in the free body diagram:

$$\vec{\mathbf{F}}_i = (F_x)_i\ \hat{\mathbf{i}} + (F_y)_i\ \hat{\mathbf{j}} + (F_z)_i\ \hat{\mathbf{k}}$$

Apply superposition principle to find total force in each direction:

$$\hat{\mathbf{i}}:\ F_x^{\text{total}} = (F_x)_1 + (F_x)_2 + \cdots$$

$$\hat{\mathbf{j}}:\ F_y^{\text{total}} = (F_y)_1 + (F_y)_2 + \cdots$$

$$\hat{\mathbf{k}}:\ F_z^{\text{total}} = (F_z)_1 + (F_z)_2 + \cdots$$

II. Devise a Plan: Equations of Motion

- Application of Newton's Second Law

$$\vec{\mathbf{F}}^{\text{total}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \cdots = m\,\vec{\mathbf{a}}.$$

- This is a vector equality; the two sides are equal in magnitude and direction.

$$\hat{\mathbf{i}} : (F_x)_1 + (F_x)_2 + \cdots = m\,a_x$$

$$\hat{\mathbf{j}} : \left(F_y\right)_1 + \left(F_y\right)_2 + \cdots = m\,a_y$$

$$\hat{\mathbf{k}} : (F_z)_1 + (F_z)_2 + \cdots = m\,a_z$$

II. Devise a Plan (cont’d)

Analyze whether you can solve the system of equations

- Common problems and missing conditions.
- Constraint conditions between the components of the acceleration.
- Action-reaction pairs.
- Different bodies are not distinguished.

Design a strategy for solving the system of equations.

III. Carry Out your Plan

Hints:

Use all your equations. Avoid thinking that one equation alone will contain your answer!

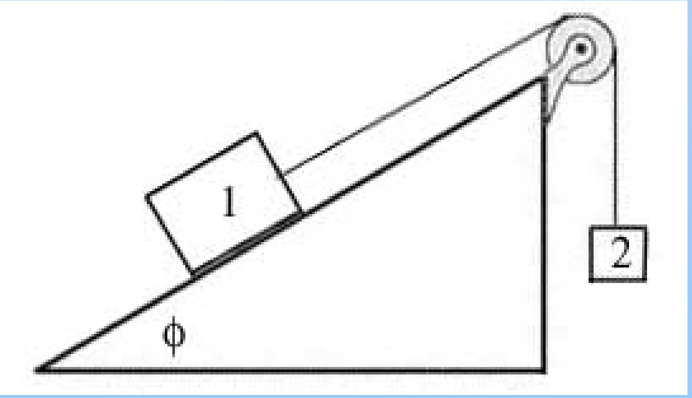
Solve your equations for the components of the individual forces.

IV. Look Back

- **Check your algebra**
- **Substitute in numbers**
- **Check your result**
- **Think about the result:** Solved problems become models for thinking about new problems.

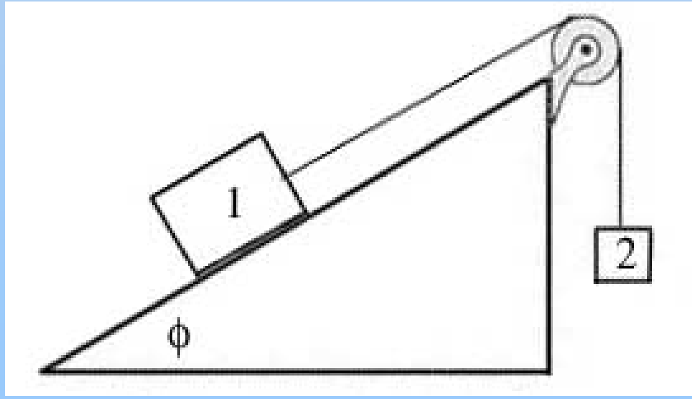
Worked Example: Pulley and Inclined Plane 1

A block of mass m_1, constrained to move along a plane inclined at angle φ to the horizontal, is connected via a massless inextensible rope that passes over a massless pulley to a bucket to which sand is slowly added. The coefficient of static friction is μ_s. Assume the gravitational constant is g. What is mass of the bucket and sand just before the block slips upward?

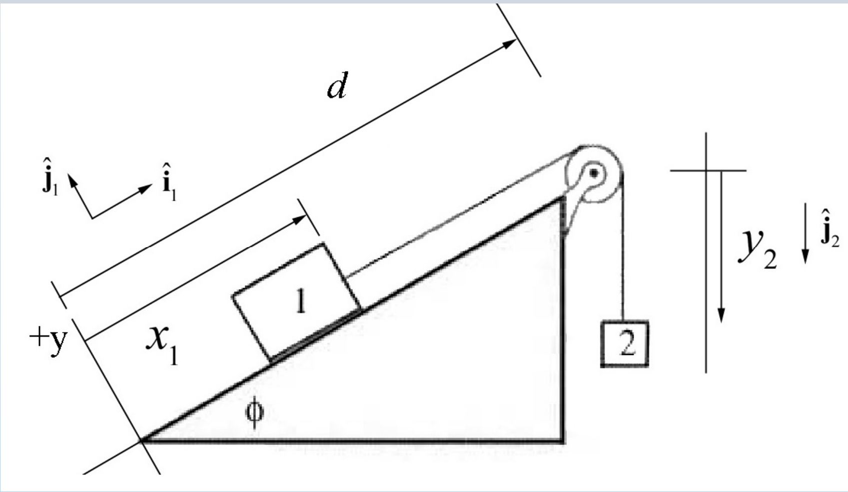


Worked Example: Pulley and Inclined Plane 2

A block of mass m_1, constrained to move along a plane inclined at angle φ to the horizontal, is connected via a massless inextensible rope that passes over a massless pulley to a second block of mass m_2. Assume the block is sliding up the inclined plane. The coefficient of kinetic friction is μ_k. Assume the gravitational constant is g. Calculate the acceleration of the blocks.

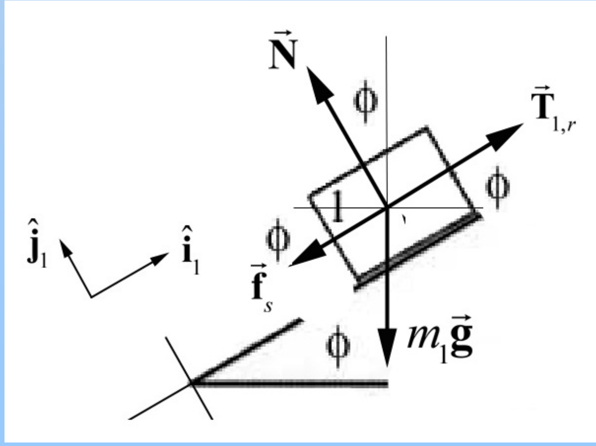


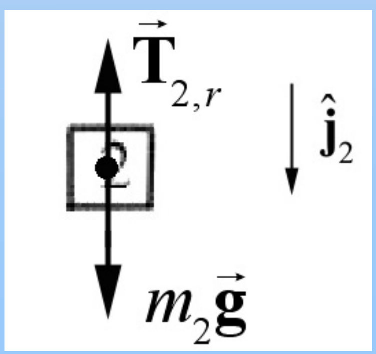
Solution: Pulley and Inclined Plane



Coordinate system

Free body force diagrams





Solution: Pulley and Inclined Plane

Constraint: $a \equiv a_{y,2} = a_{x,1}$

Object on inclined plane: $\hat{\mathbf{i}}_1 : T - m_1 g \sin\phi - f_\mathrm{k} = m_1 a$

$\hat{\mathbf{j}}_1 : N - m_1 g \cos\phi = 0$ $\quad$ $f_\mathrm{k} = \mu_\mathrm{k} N = \mu_\mathrm{k} m_1 g \cos\phi$

Suspended Object: $T - m_1 g \sin\phi - \mu_\mathrm{k} m_1 g \cos\phi = m_1 a$

Solution: $\hat{\mathbf{j}}_2 : m_2 g - T = m_2 a$ $\quad$ $T = m_2 g - m_2 a$

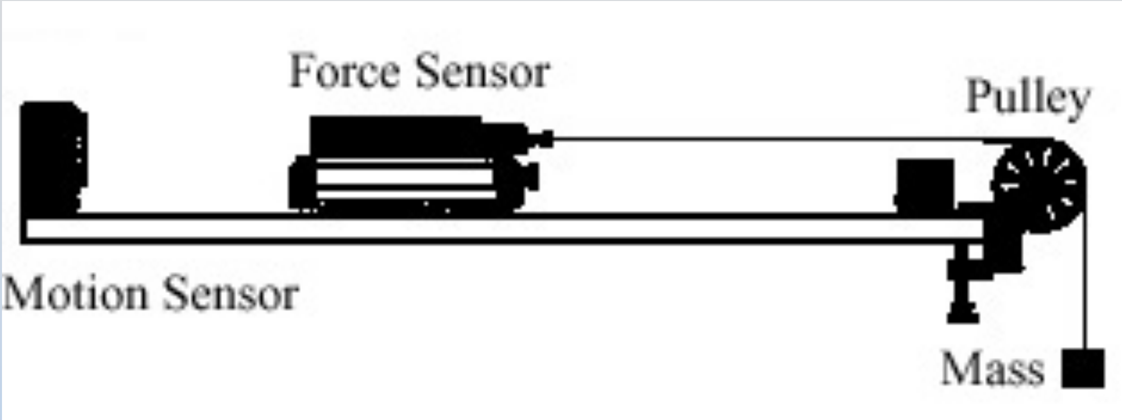
$$m_2 g - m_2 a - m_1 g \sin\phi - \mu_\mathrm{k} m_1 g \cos\phi = m_1 a$$

$$a = \frac{(m_2 - m_1(\sin\phi + \mu_\mathrm{k}\cos\phi))}{m_1 + m_2} g$$

Lecture Demo: Block with Pulley and Weight on Incline B24

http://scripts.mit.edu/~tsg/www/index.php?page=demo.php?letnum=B%24&show=0

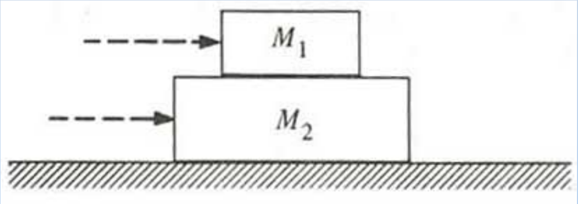
Checkpoint Problem: Tension



Consider a track with a pulley located at one end. The force sensor and cart have total mass m_1. They are connected by an inextensible rope of length l (passing over the pulley) to a block of mass m_2. You may ignore the small mass of the rope and pulley. You may also assume that all friction effects are negligible. What is the tension in the string both before the cart is released and while the cart is in motion?

Checkpoint Problem: Pushing Textbooks

Consider two textbooks that are resting one on top of the other. The lower book has M_2 and is resting on a nearly frictionless surface. The upper book has mass $M_1 < M_2$. Suppose the coefficient of static friction between the books is μ_s.

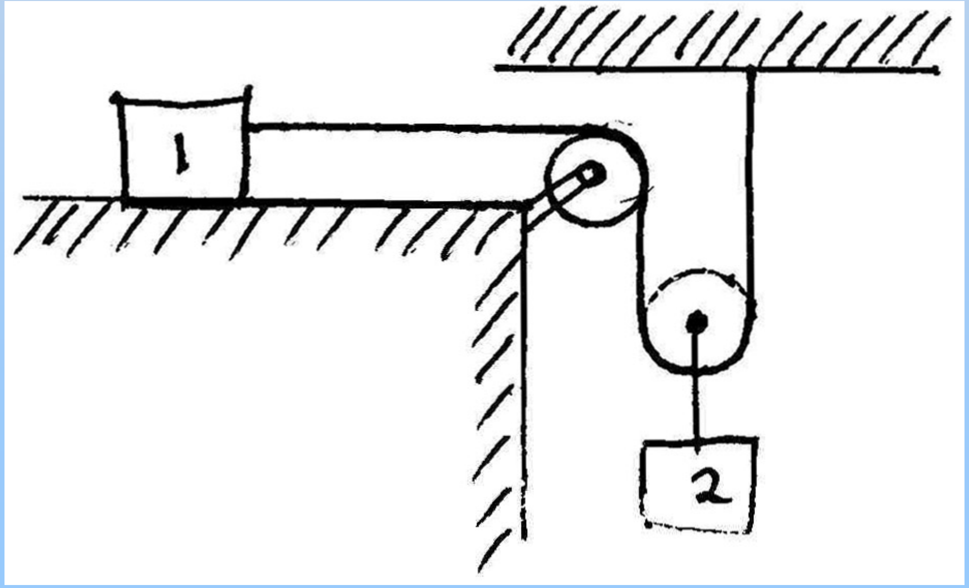


a) What is the maximum force with which the upper book can be pushed horizontally so that the two books move together without slipping? Identify all action-reaction pairs of forces in this problem.

b) What is the maximum force with which the lower book can be pushed horizontally so that the two books move together without slipping? Identify all action-reaction pairs of forces in this problem.

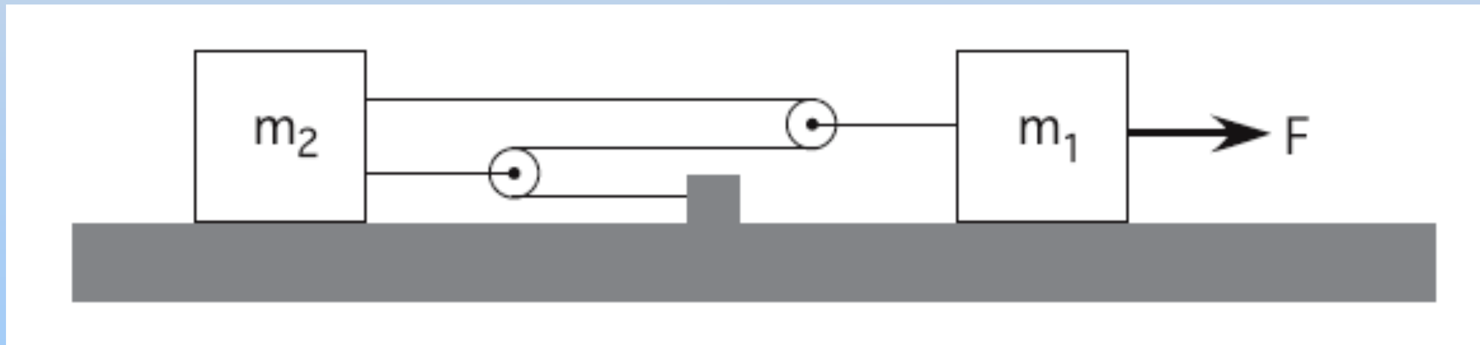
Checkpoint Problem: Two Blocks with Constraint

Two blocks 1 and 2 of mass m_1 and m_2 respectively are attached by a string wrapped around two pulleys as shown in the figure. Block 1 is accelerating to the right on a fricitonless surface. You may assume that the string is massless and inextensible and that the pulleys are massless. Find the accelerations of the blocks and the tension in the string connecting the blocks.



Chcekpoint Problem: Blocks and Pulleys on Table

Two blocks rest on a frictionless horizontal surface. They are connected by 3 massless strings and 2 frictionless, massless pulleys as shown above. A force F is applied to block 1. What is the resulting acceleration of block 1?



Worked Example: velocity dependent force

Consider an object of mass m released at time t = 0 with an initial *x*-component of velocity $v_{x,0} = 0$. A force is acting on the object according to

$$\vec{\mathbf{F}} = m(c_0 - c_1 v_x)\hat{\mathbf{i}}$$

Find the velocity as a function of time.

Worked Example Solution: velocity dependent force

Technique: Separation of Variables: The acceleration is

$$a_x = \frac{dv_x}{dt} = \frac{F_x}{m} = c_0 - c_1 v_x \rightarrow \frac{dv_x}{c_0 - c_1 v_x} = dt$$

$$\int_{v_{x,0}=0}^{v_x(t)} \frac{dv'_x}{c_0 - c_1 v'_x} = \int_{t'=0}^{t'=t} dt' \Big| \rightarrow -\frac{1}{c_1}\ln(c_0 - c_1 v'_x)\Big|_{v_{x,0}=0}^{v_x(t)} = t$$

$$-\frac{1}{c_1}\ln\left(\frac{c_0 - c_1 v_x(t)}{c_0}\right) = t \rightarrow \ln\left(\frac{c_0 - c_1 v_x(t)}{c_0}\right) = -c_1 t$$

$$c_0 - c_1 v_x(t) = c_0 e^{-c_1 t} \rightarrow v_x(t) = \frac{c_0}{c_1}(1 - e^{-c_1 t})$$