# Kinetic Energy, Work, and Power

# Forms of Energy

- kinetic energy
- gravitational energy
- elastic energy
- thermal energy
- electrical energy
- chemical energy
- electromagnetic energy
- nuclear energy
- mass energy

# Energy Transformations

- Falling water releases stored ‘gravitational potential energy’ turning into a ‘kinetic energy’ of motion.
- Human beings transform the stored chemical energy of food into catabolic energy
- Burning gasoline in car engines converts ‘chemical energy’ stored in the atomic bonds of the constituent atoms of gasoline into heat
- Stretching or compressing a spring stores ‘elastic potential energy’ that can be released as kinetic energy

# Energy Conservation

- Energy is always conserved

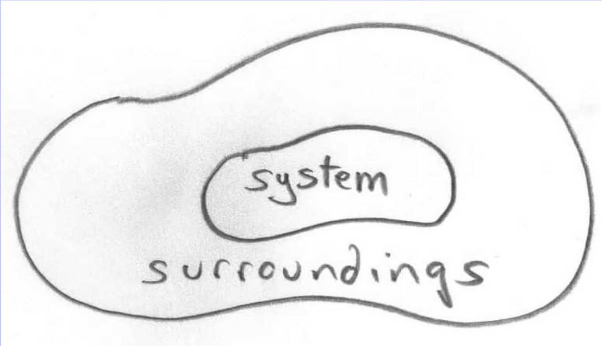
$$\boxed{\sum_{i=1}^{N} \Delta E_i = \Delta E_1 + \Delta E_2 + \ldots = 0}$$

- It is converted from one form into another, from an "initial state" to a "final state"

$$\Delta E \equiv E_{\text{final}} - E_{\text{initial}}$$

- Energy can also be transferred from a system to its surroundings

$$\Delta E_{\text{system}} + \Delta E_{\text{surroundings}} = 0$$



# Kinetic Energy

- Scalar quantity (reference frame dependent)

$$\boxed{K = \frac{1}{2}mv^2 \geq 0}$$

- SI unit is joule:

$$1\,\mathrm{J} \equiv 1\,\mathrm{kg}\cdot\mathrm{m}^2/\mathrm{s}^2$$

- Change in kinetic energy:

$$\boxed{\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}m(v_{x,f}^2 + v_{y,f}^2 + v_{z,f}^2) - \frac{1}{2}m(v_{x,0}^2 + v_{y,0}^2 + v_{z,0}^2) \geq 0}$$

# Kinematics: Recall Integration Formula

The *x*-component of the acceleration of an object is the derivative of the *x*-component of the velocity

$$a_x \equiv \frac{dv_x}{dt}$$

Therefore the integral of *x*-component of the acceleration with respect to time, is the *x*-component of the velocity

$$\int_{t_0}^{t_f} a_x dt = \int_{t_0}^{t_f} \frac{dv_x}{dt} dt = \int_{v_{x,0}}^{v_{x,f}} dv_x = v_{x,f} - v_{x,0}$$

# Kinematics: An Integral Theorem for One Dimensional Motion

The integral of *x*-component of the acceleration with respect to the displacement of an object, is given by

$$\int_{x_0}^{x_f} a_x dx = \int_{x_0}^{x_f} \frac{dv_x}{dt} dx = \int_{x_0}^{x_f} dv_x \frac{dx}{dt} = \int_{v_{x,0}}^{v_{x,f}} v_x dv_x$$

$$\int_{x_0}^{x_f} a_x dx = \int_{v_{x,0}}^{v_{x,f}} d\left((1/2)v_x{}^2\right) = \frac{1}{2}(v_{x,f}^2 - v_{x,0}^2)$$

Multiply both sides by the mass of the object

$$\int_{x_0}^{x_f} ma_x dx = \int_{v_{x,0}}^{v_{x,f}} d\left((1/2)v_x{}^2\right) = \frac{1}{2}mv_{x,f}^2 - \frac{1}{2}mv_{x,0}^2 = \Delta K$$

# Newton’s Second Law: An Integral Theorem for One Dimensional Motion

Newton’s Second Law If *F* is not constant, then (in one dimension),

$$F_x = ma_x$$

Therefore the integral theorem coupled with Newton’s Second Law becomes

$$\int_{x_0}^{x_f} ma_x dx = \int_{x_0}^{x_f} F_x dx = \Delta K$$

# Work Done by a Constant Force for One Dimensional Motion

**Definition:**

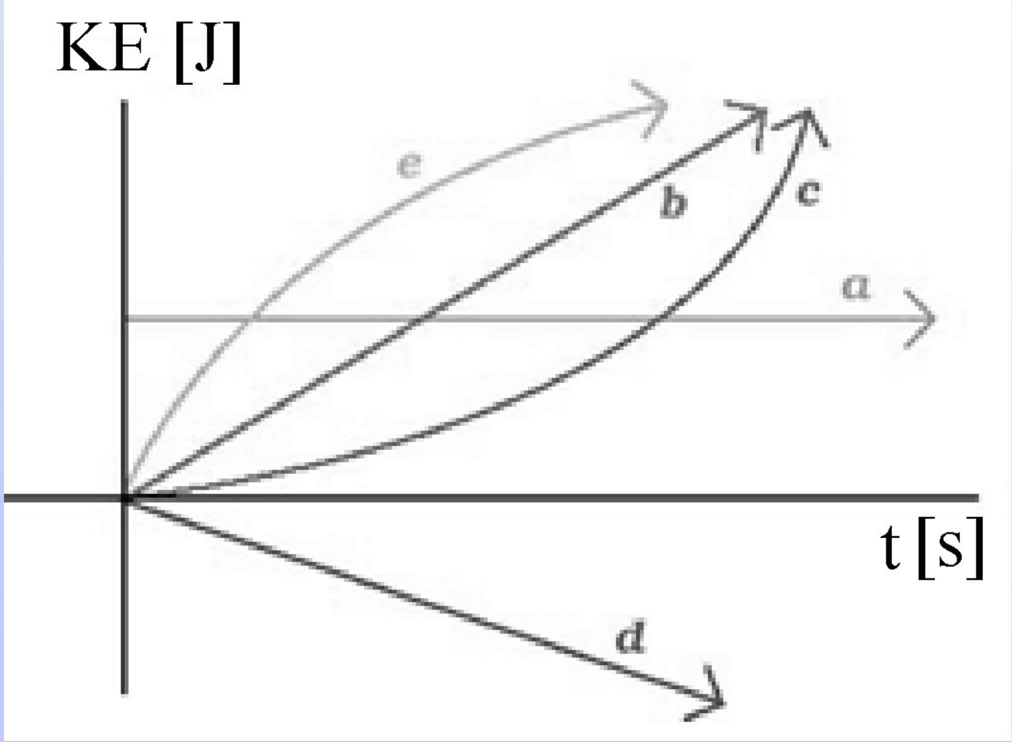
The work $W$ done by a constant force with an $x$-component, $F_x$, in displacing an object by $\Delta x$ is equal to the $x$-component of the force times the displacement:

$$\boxed{W = F_x \Delta x}$$

# Checkpoint Problem: Work-Energy

An object is dropped to the earth from a height of 10m. Which of the following sketches best represent the kinetic energy of the object as it approaches the earth (neglect friction)?

1. a
2. b
3. c
4. d
5. e



# Work done by Non-Constant Force: One Dimensional Motion

(Infinitesimal) work is a scalar

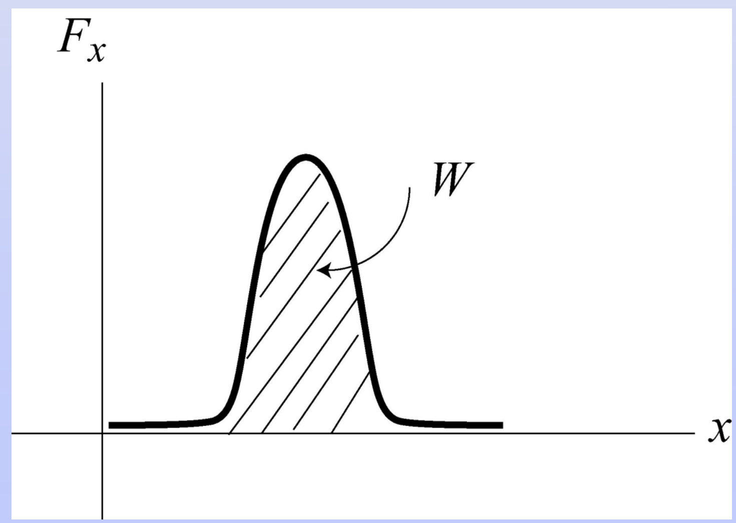
$$\Delta W_i = (F_x)_i \Delta x_i$$

Add up these scalar quantities to get the total work as area under graph of $F_x$ vs $x$ :

$$W = \sum_{i=1}^{i=N} \Delta W_i = \sum_{i=1}^{i=N} (F_x)_i \Delta x_i$$

$$\text{As } N \to \infty \text{ and } |\Delta x_i| \to 0$$

$$W = \lim_{\substack{N\to\infty \\ \Delta x_i \to 0}} \sum_{i=1}^{i=N} (F_x)_i \Delta x_i = \int_{x=x_0}^{x=x_f} F_x dx$$



# Checkpoint Problem: Work Done by Gravity Near the Surface of the Earth

Consider an object of mass  near the surface of the earth falling directly towards the center of the earth. The gravitational force between the object and the earth is nearly constant. Suppose the object starts from an initial point that is a distance $y_0$ from the surface of the earth and moves to a final point a distance $y_f$ from the surface of the earth.

How much work does the gravitational force do on the object as it falls?

# Work-Kinetic Energy Theorem for One Dimensional Motion

Work-energy theorem is the statement that the work done by a force in displacing an object is equal to the change in kinetic energy of the object

$$W = \Delta K$$

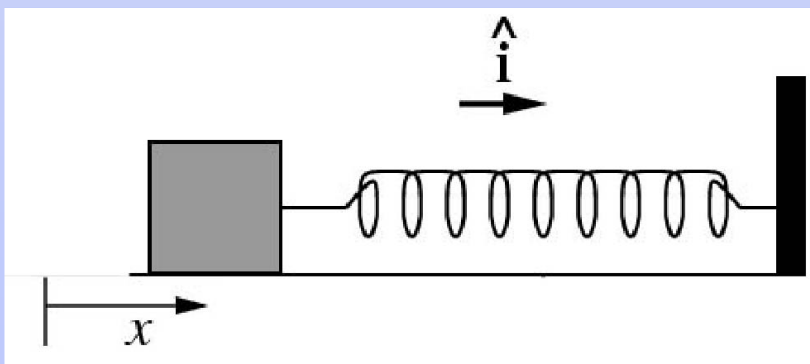
# Checkpoint Problem: Work Done by the Spring Force

Connect one end of a spring of length $l_0$ with spring constant $k$ to an object resting on a smooth table and fix the other end of the spring to a wall. Stretch the spring until it has length $l$ and release the object.

How much work does the spring do on the object as a function of $x = l - l_0$, the distance the spring has been stretched or compressed?

# Checkpoint Problem: Work Done by a Several Forces

A block of mass m slides along a horizontal table with speed $v_0$. At $x = 0$ it hits a spring with spring constant $k$ and begins to experience a friction force. The coefficient of kinetic friction is given by $\mu$. How far did the spring compress when the block first momentarily comes to rest?



# Power

- The average power of an applied force is the rate of doing work

$$\overline{P} = \frac{\Delta W}{\Delta t} = \frac{F_{\text{applied},x}\,\Delta x}{\Delta t} = F_{\text{applied},x}\overline{v}_x$$

- SI units of power: Watts

$$1\,\mathrm{W} \equiv 1\,\mathrm{J/s} = 1\,\mathrm{kg}\cdot\mathrm{m}^2/\mathrm{s}^3$$

- Instantaneous power

$$P = \lim_{\Delta t \to 0}\frac{\Delta W}{\Delta t} = F_{\text{applied},x}\left(\lim_{\Delta t \to 0}\frac{\Delta x}{\Delta t}\right) = F_{\text{applied},x}\,v_x$$