Work and Kinetic Energy
Concept Questions

Question 1 Work and Kinetic Energy

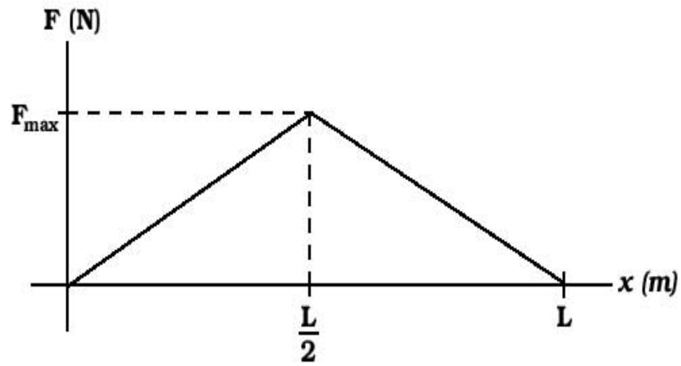
Compared to the amount of energy required to accelerate a car from rest to 10 miles per hour, the amount of energy required to accelerate the same car from 10 mph to 20 mph is

1. the same
2. twice as much
3. three times as much
4. four times as much
5. unsure.

Answer: 3. The energy increase in going from zero speed to speed v is $(1/2)mv^2$. To go from v to $2v$ is $(1/2)m(2v)^2 - (1/2)mv^2 = (3/2)mv^2$, so the amount of energy required is three times as much.

Question 2 Work and variable force

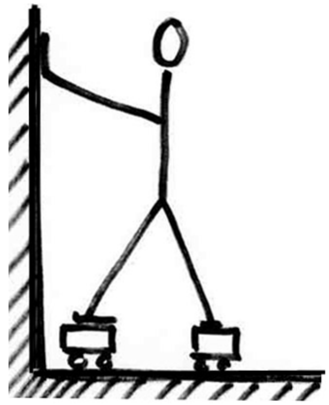
A particle starts from rest at $x = 0$ and moves to $x = L$ under the action of a variable force $F(x)$, which is shown in the figure. What is the particle's kinetic energy at $x = L/2$ and at $x = L$?



1. $F_{\max} L/2, F_{\max} L$
2. $F_{\max} L/4, 0$
3. $F_{\max} L, 0$
4. $F_{\max} L/4, F_{\max} L/2$
5. $F_{\max} L/2, F_{\max} L/4$

Answer: 4. The work is equal the area under the graph of force vs. displacement. The work is also equal to the kinetic energy. At $x = L/2$, the area is $F_{\max} L/4$. At $x = L$ the area is $F_{\max} L/2$.

Question 3: Pushing Against a Wall



The work done by the contact force of a wall on a person as the person moves is

1. positive.
2. Negative.
3. zero.
4. Impossible to determine from the information given in the question and the figure.

Answer 3. Because the person's hand is at rest with respect to the wall, the displacement of the point of contact of the force is zero, hence the contact force does zero work on the person.

Question 4

You lift a 10 kg weight that was resting on the ground to a height of 2 m above the ground, and then hold it there at rest. How much work do you do on the weight in moving it (take $g = 10 \text{ m} \cdot \text{s}^{-2}$)?

1) -200 N m

2) greater than -200 N m but less than 0 because the work changes sign part way through.

3) You do no work because the weight begins and ends at rest.

4) greater than 0 but less than 200 N m because the work changes sign part way through

5) 200 N m

6) None of the above.

Solution 5. There are two forces acting on the weight, that of the person and that of gravity. Since the weight starts and ends at rest, the work done by the sum of the two forces is 0. The force of gravity is a constant at 100 Newtons acting downward. Since the motion is upward, in the opposite direction to gravity, gravity does -200 Newton Meters of work. Since the total work is zero, the person must have done 200 Newton Meters of work.

Comment: The force exerted by the person is not constant during the lift. It is first a bit greater than 100 Newtons in order to accelerate the weight upward. It must then drop to less than 100 Newtons so that the weight is able to decelerate back to zero velocity. However, the integral over distance of the force exerted by the person must equal 200 Newton Meters.

Question 5 The same horizontal force, of magnitude F, is applied to two different blocks, of mass m and $3m$ respectively. The blocks move on a frictionless surface and both blocks begin from rest.

(a) If the force is applied for the same time to each block, which one of the following sentences is true?

(i) The heavier block acquires 9 times as much kinetic energy as the lighter block.

(ii) The heavier block acquires 3 times as much kinetic energy as the lighter block.

(iii) The two blocks acquire the same kinetic energy.

(iv) The lighter block acquires 3 times as much kinetic energy as the heavier block.

(v) The lighter block acquires 9 times as much kinetic energy as the heavier block.

Solution: With the same force, the heavier block will accelerate more slowly by a factor of 3. Since $v = at$ and t is the same for both blocks, the final speed v_h of the heavier block will be related to the speed v_l of the lighter block by $v_h = v_l / 3$. So, the final kinetic energy of the heavier block is

$$K_h = \frac{1}{2}(3m)v_h^2 = \frac{1}{2}(3m)(v_l / 3)^2 = \frac{1}{3}\left(\frac{1}{2}mv_l^2\right) = \frac{1}{3}K_l$$

Question 6 The same horizontal force, of magnitude F, is applied to two different blocks, of mass m and $3m$ respectively. The blocks move on a frictionless surface and both blocks begin from rest. If each block moves the same distance as the force is applied, which one of the following sentences is true?

(i) The heavier block acquires 9 times as much kinetic energy as the lighter block.

(ii) The heavier block acquires 3 times as much kinetic energy as the lighter block.

(iii) The two blocks acquire the same kinetic energy.

(iv) The lighter block acquires 3 times as much kinetic energy as the heavier block.

(v) The lighter block acquires 9 times as much kinetic energy as the heavier block.

Solution: This is a simple application of the work-energy theorem. For a constant force in the same direction of the motion, $W = force \times distance$, so the same amount of work is done on the two blocks. Since the work done is equal to the change in kinetic energy, both blocks acquire the same kinetic energy.

Question 7 The same horizontal force, of magnitude F, is applied to two different blocks, of mass m and $3m$ respectively. The blocks move on a frictionless surface and both blocks begin from rest. If the force is applied to each block until they reach the same speed v_0, which one of the following sentences is true?

(i) The force is applied to the heavier block 9 times longer than the lighter block.

(ii) The force is applied to the heavier block 3 times longer than the lighter block.

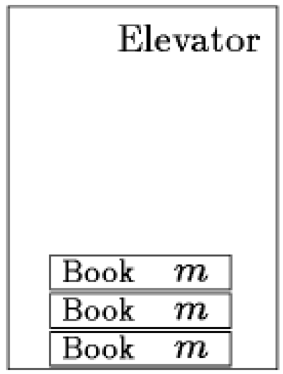
(iii) The force is applied to the two blocks for the same amount of time.

(iv) The force is applied to the lighter block 3 times longer than the heavier block.

(v) The force is applied to the lighter block 9 times longer than the heavier block.

Solution: Again the heavier block experiences 1/3 of the acceleration of the lighter block. Since $v_0 = at$, the heavier block must accelerate for 3 times as long.

Question 8 Three books, each of mass *m*, rest on the floor of an elevator. The elevator starts at the first floor and rises to the sixth floor. It travels at a constant speed between the second and fifth floors, as it rises by a total distance *h*. This problem focuses on the work *W* done on the **middle book** by all forces (conservative or non-conservative) during the passage from the second to the fifth floors. In the following display, circle the correct entry in each row:



Total work $W =$	$-3mgh$	$-2mgh$	$-mgh$	$\boxed{0}$	mgh	$2mgh$	$3mgh$
Work done by gravity $=$	$-3mgh$	$-2mgh$	$\boxed{-mgh}$	0	mgh	$2mgh$	$3mgh$
Work done by elevator floor $=$	$-3mgh$	$-2mgh$	$-mgh$	$\boxed{0}$	mgh	$2mgh$	$3mgh$
Work done by book below $=$	$-3mgh$	$-2mgh$	$-mgh$	0	mgh	$\boxed{2mgh}$	$3mgh$
Work done by book above $=$	$-3mgh$	$-2mgh$	$\boxed{-mgh}$	0	mgh	$2mgh$	$3mgh$
Work done by book on itself $=$	$-3mgh$	$-2mgh$	$-mgh$	$\boxed{0}$	mgh	$2mgh$	$3mgh$

Solution: With all the forces on the middle book determined, the work done by each force becomes a simple application of the definition of work. When the force is constant, $W = force \times distance$. Since the displacement is upward by a distance h, I can write $W = Fh$, where F is the upward component of the force on the middle book. The total work is zero, since the total force on the middle book is zero. The work done by gravity is $-mgh$, since the force of gravity is $F_g = -mg$ (i.e., magnitude mg, directed downward). The elevator floor exerts no direct force on the middle book, since it does not touch the middle book, so it does no work. The book below exerts a normal force $N_{below} = 2mg$, so the work is $2mgh$. The book above exert a normal force downward of magnitude mg, so the upward component is $F_{Nabove} = -N_{above} = -mg$, so the work done by this force is $-mgh$. Finally, the book exerts no net force on itself, so it does no work on itself. A rigid body never does work on itself, a though a body that changes shape can do work on itself.

Question 9 When a person walks, the force of friction between the floor and the person's feet accelerates the person forward. The floor does

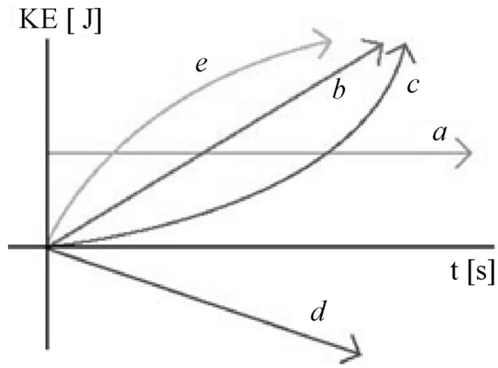
1. Positive work on the person.

2. Negative work on the person.

3. No work on the person.

Answer: 3. The friction is static and there is no displacement of the foot on the floor, $d\vec{\mathbf{r}} = 0$, when the force is applied $\vec{\mathbf{F}}$. So the contribution to the work $dW \equiv \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = 0$. Keep in mind that a human being is not a rigid body. The correct energy transformation is chemical energy is transformed into the motion of the muscles which is transformed into kinetic energy of the center of mass.

Question 10 An object is dropped to the earth from a height of 10m. Which of the following graphs of kinetic energy vs. time best represent the kinetic energy of the object as it approaches the earth (neglect friction).



1. a
2. b
3. c
4. d
5. e

Answer: 3. The velocity increases linearly with time (for constant acceleration from rest, $v_y = -gt$), and kinetic energy is proportional to v_y^2 so K is proportional to t^2.

8.01SC Physics I: Classical Mechanics