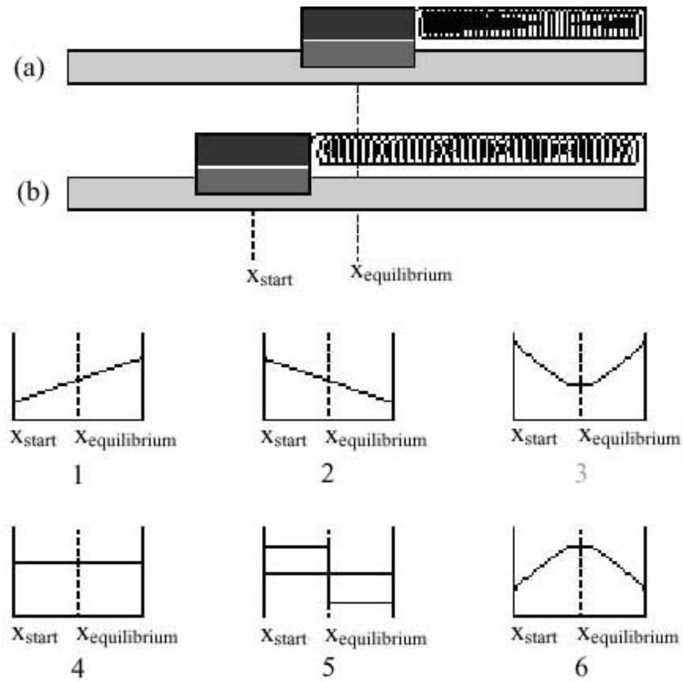
Potential Energy and Energy Diagrams
Concept Questions

Question 1 You lift a ball at constant velocity from a height h_i to a greater height h_f. Considering the ball and the earth together as the system, which of the following statements is true?

1. The potential energy of the system increases.
2. The kinetic energy of the system decreases.
3. The earth does negative work on the system.
4. You do negative work on the system.
5. The source energy of the ball increases.
6. Two of the above.
7. None of the above.

Answer: 1. Here the positive work that you do produces an increase in the system's potential energy. Now the earth is in the system so it cannot do work on the system.

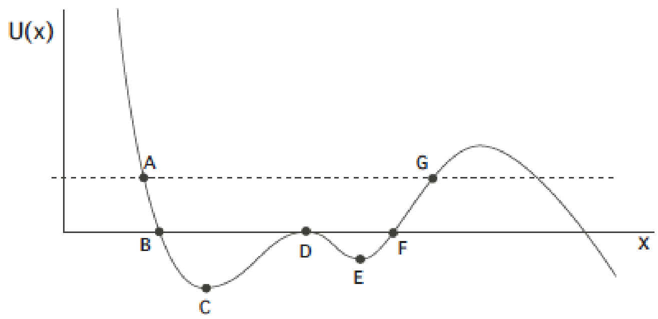
Question 2: In part (a) of the figure, an air track cart attached to a spring rests on the track at the position $x_{equilibrium}$ and the spring is relaxed. In (b), the cart is pulled to the position x_{start} and released. It then oscillates about $x_{equilibrium}$. Which graph correctly represents the potential energy of the spring as a function of the position of the cart?



Answer: 3. The cart starts at x_{start} with no kinetic energy, and so the spring's potential energy is a maximum. Once released, the cart accelerates to the right and its kinetic energy increases as the potential energy of the spring is converted into kinetic energy of the cart. As the cart passes the equilibrium position, its kinetic energy is a maximum and so the spring's potential energy is a minimum. Once to the right of $x_{equilibrium}$, the cart starts to compress the spring and it slows down as its kinetic energy is converted back to potential energy of the recompressed spring. At the rightmost point it reaches, the cart reverses its direction of travel. At that instant, it has no kinetic energy and the spring again has maximum potential energy.

Question 3

The figure below shows a graph of potential energy $U(x)$ verses position x for a particle executing one dimensional motion along the x axis. The total mechanical energy of the system is indicated by the dashed line. At $t=0$ the particle is somewhere between points A and G. For later times, answer the following questions.

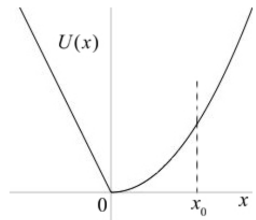


a) At which point will the magnitude of the force be a maximum?
b) At which point will the kinetic energy be a maximum?
c) At how many of the labeled points will the velocity be zero?
d) At how many of the labeled points will the force be zero?
e) Can the motion be accurately approximated by simple harmonic motion (Y/N)?

Question 3 Solution

a) A. The force is the negative of the slope of $U(x)$. The magnitude of the slope is largest at A.
b) C. The kinetic energy is the difference between the total mechanical energy and $U(x)$. That difference is greatest at C.
c) 2. The velocity is zero where the kinetic energy goes to zero. That occurs at 2 points, A and G.
d) 3. The force will be zero where the slope of $U(x)$ is zero. That occurs at 3 points, C, D and E.
e) The motion is periodic, but it is not harmonic since the potential cannot be approximated by a parabola over the range of the x axis which is accessed by the particle.

Question 4: A particle is released from rest at the position $x = x_0$ in the potential described below.



$$U(x) = \begin{cases} -ax\ ; & x < 0 \\ bx^2\ \ ; & x > 0 \end{cases}$$

Determine whether the following statements are true or false. Briefly explain your reasoning.

True False The subsequent motion is periodic.

Answer True: The motion is periodic since the particle is bound by the potential i.e. the energy is constant and so the particle will move back and forth between a maximum negative position and the position $x = x_0$ where in both instances the energy is equal to the potential energy.

True False The velocity is a continuous function of time.

Answer True: The kinetic energy of the particle is continuous because The potential energy is continuous and hence $K(x) = E - U(x)$, the kinetic energy of the particle, is also continuous. Therefore the velocity is continuous.

True False The acceleration is a continuous function of time.

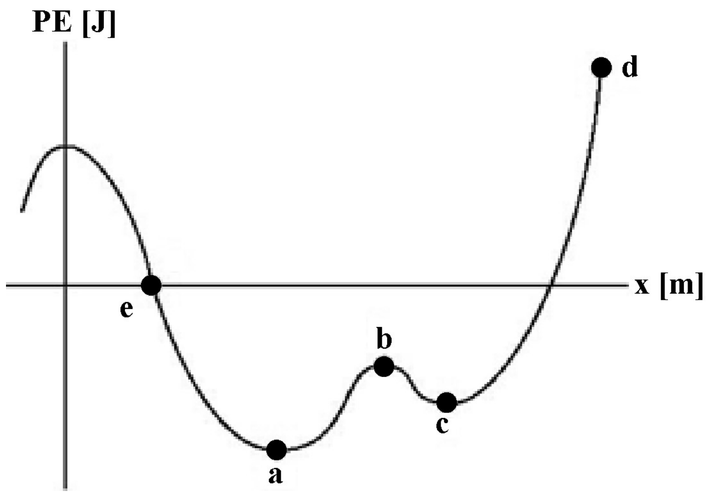
Answer False: The derivative of the potential energy is not continuous at $x = x_0$. Since the force is equal to the negative derivative of the potential energy, the force is therefore not continuous at $x = x_0$. Since the force is proportional to the acceleration, the acceleration is therefore not continuous at $x = x_0$.

True False The force is conservative.

Answer True: Since the potential energy is defined for all values of x, the force is conservative even though there are different forces acting for $x > 0$ and $x < 0$.

Question 5

Consider the following sketch of potential energy for a particle as a function of position. There are no dissipative forces or internal sources of energy.



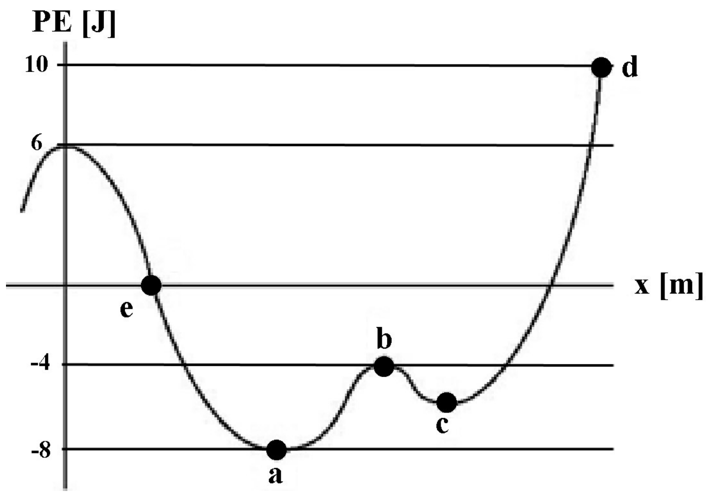
If a particle travels through the entire region of space shown in the diagram, at which point is the particle's velocity a maximum?

1. a
2. b
3. c
4. d
5. e

Answer 1: The if the particle travels through the entire region then the energy must be at least greater or equal to the largest potential energy which occurs at point d. The kinetic energy is equal to the total energy minus the potential energy. So the smallest potential energy has the largest kinetic energy hence when the particle is at the lowest minimum point of the potential energy graph, the kinetic energy is maximum.

Question 6

Consider the following sketch of potential energy for a particle as a function of position. There are no dissipative forces or internal sources of energy.



What is the minimum total mechanical energy that the particle can have if you know that it has traveled over the entire region of X shown?

1. -8
2. 6
3. 10
4. It depends on direction of travel
5. Can't say - Potential Energy uncertain by a constant

Answer 3: The mechanical energy is the sum of kinetic and potential energy. If the mechanical energy were less that 10 J, then when the particle was at the position corresponding to the point *d* on the figure, the PE is equal to 10 J so the kinetic energy would be less than zero, but this is forbidden for a particle using just the principles of classical mechanics. (In quantum mechanics, one would have a small but finite probability of finding the particle in a classical forbidden region.)

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