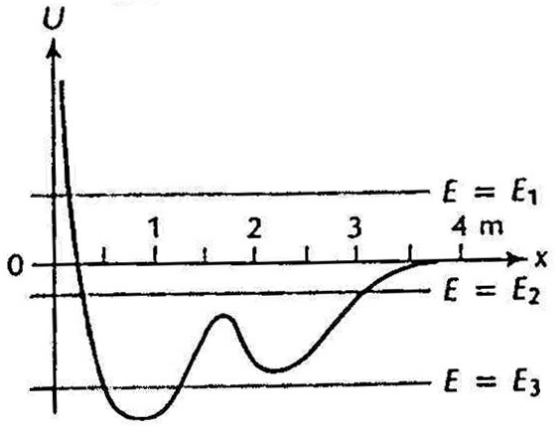
Potential Energy Diagrams
Challenge Problems

Problem 1



A particle moves along the x-axis under the influence of a conservative force with a potential energy $U(x)$. A plot of $U(x)$ vs. x is shown in the figure above. The figure shops several alternative energy levels for the particle: $E = E_1$, $E = E_2$, and $E = E_3$. Assume that the particle is initially at $x = x_0$. For each of the three alternative energy levels describe the motion qualitatively, answering the following questions.

a) Roughly, where are the turning points (right and left)?

b) Where is the speed of the particle maximum? Where is the speed minimum?

c) Is the orbit bound or unbound?

Problem 2: The force of interaction between a particle of mass m_1 and a second particle of mass m_2 separated by a distance r is given by an attractive gravitational force and a repulsive force that is proportional to r^{-3}, with a proportionality constant C,

$$\vec{\mathbf{F}}(r) = \left(-\frac{Gm_1m_2}{r^2} + C\frac{1}{r^3} \right) \hat{\mathbf{r}} .$$

a) Choose your zero point for potential energy at infinity. If the masses start off an infinite distance apart and are then moved until they are a distance r apart, what is the potential energy difference $U(r) - U(\infty) = -\int_{\infty}^{r} \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}}$?

b) What is the distance r_0 between the two masses when they are in stable equilibrium? What is the value of the potential energy $U(r_0)$ at stable equilibrium?

Problem 3

A particle of mass m moves in one dimension. Its potential energy is given by

$$U(x) = -U_0 e^{-x^2/a^2},$$

where U_0 and a are constants.

a) Draw an energy diagram showing the potential energy $U(x)$, the kinetic energy $K(x)$, and the total energy $E < 0$ for a motion which is bound between turning points $\pm a$.

b) Find the force on the particle, $F(x)$, as a function of position x.

c) Find the speed at the origin $x = 0$ such that the when the particle reaches the positions $x = \pm a$, it will reverse its motion.

Problem 4

The force on a particle is given by

$$\vec{\mathbf{F}}(x) = F_0(e^{-2(x-x_0)/x_0} - e^{-x/x_0})\hat{\mathbf{i}}$$

where F_0 and x_0 are positive and $\hat{\mathbf{i}}$ is a unit vector in the positive x-direction.

a) For what value of x is the force zero?

b) What is $U(x) - U(x_0)$, the potential energy, when the particles are a distance x apart?

c) Sketch $U(x)$ with the choice that $U(x_0) = (F_0 x_0 / 2)(1 - 2e^{-1})$

8.01SC Physics I: Classical Mechanics