This is 2.D.3.

A frictionless horizontal surface-- the plane of paper is my horizontal surface. I have here the spring, which is the constant k 1, here is a mass m, and here I have a spring, which is the constant k 2. This object is in equilibrium, lying still on the table-- I call this x equals 0, and I'm going to place this in the minus x direction over the distance A. A in the minus x direction-- I call this the plus x direction.

Let us first make the assumption that in this particular configuration the way I have drawn it, both springs are relaxed-- that means that they are not pushed in, and they are not stretched. Therefore, they do not exert any force on the object m-- they are totally relaxed. I'm going to put the object here: that means this spring is pushed in, and so if it's pushed in, it will push outwards to the right. It'll push this back to equilibrium, and the force will be k 1 A in a positive direction. Of course, this spring is longer than it wants to be, so it will pull inwards, so there will also be a force on m in the positive x direction, which is k 2 times A.

The [? restoring ?] force F, which I write down as a factored equation, equals k 1 plus k 2 times x. To remind you that it is in the positive x direction, I will introduce this x roof. This x roof simply represents the unit vector in the plus x direction.

I do not like this equation-- I hate equations like this, because I'm used to seeing here a minus sign, and the only reason why you don't see this minus sign here is that I have moved this object to the minus direction. I'm going to change it-- I'm going to change it in a more general equation, and that doesn't mean this one is wrong, but I'm going to change it in a more general factor equation of minus k 1 plus k 2 times x. It's a one dimensional vector equation: if x is positive, the force is driving back to equilibrium. Is it a negative direction if x is negative? As is the case here, the force will be positive-- as you see here, it is driving it back to equilibrium in the positive x direction.

This represents the equation of a simple harmonic oscillator. The period of the simple harmonic oscillator-- T, this is the period, it has nothing to do with tension, and it equals 2 pi times the square root of the mass of the object divided by k 1 plus k 2. The springs act together: that's why you see a k 1 and ok 2. The net force on m is larger due to both springs than it will be if there were only one spring. Therefore, on a given mass having a larger force, it's obvious that everything will go faster, and so the period will be shorter. That's exactly what you see: the larger this value is, the smaller will be the periods, and this is very intuitive.

Now, it is not intuitive that this period T is independent of the displacement A. A we call the amplitude-- I take the object, and I displace it to one side, and then I release it, and it's starting to oscillate. The period of oscillation which you have here-- I'll get back to that--- this period is independent off the amplitude, and the amplitude is the maximum displacement from equilibrium. In this particular case, we had A-- that is not so intuitive that this period is independent, but I will returns to this.

Let's first address the issue of what would be the case if the two springs were not relaxed when I started? Suppose I have here again this frictionless table: that's the plane of my paper, and I stretch this one. It's now much longer than in the relaxed position. I also stretch this one, and it's also much longer. This is the spring constant k 2, and here is the mass m.

It would mean, then, that in this position, it is the equilibrium position x equals 0, which is being accelerated, it's sitting still-- is equilibrium. It means then, that in this situation, which is now my initial condition, there is a force: go right, because this spring is longer than it wants to be, so it wants to contract so it'll go poof to the right. This string is also longer than it wants to be, so it also wants the contract, so it will be a force to the left-- I will call that F 1. The two forces obviously must be the same in magnitude-- otherwise, this object could never be in equilibrium, so this is now my starting condition.

If now I move this object-- to the side, for instance, in the negative x direction A, then I want you to convince yourself that everything we've done before remains the same. In other words, that the restoring force to equilibrium will be exactly as it was before, and it makes no difference that now these springs are stretched. It makes no difference, and therefore, also that the object will again be starting to oscillate-- simple harmonic oscillation-- and that the period is precisely as we just stated.

Now, I will derive this period for you, and I will do that through a differential equation. I write down a one dimensional differential equation: F equals ma, which is Newton's First Law. This is a factor, and this a factor. A is the second derivative of x, so I can write down for A m x double dot, whereby x double dot equals d 2 x dt squared. That now equals minus capital K, which is the equivalent spring constant this case-- k 1 plus k 2 times x.

Why don't I put arrows over here? Well, because this is a one dimensional vector equation. If x is positive, this minus sign will tell you that the force is the minus direction. If x is negative, the minus sign will tell you that the force is in the positive direction, so I really don't have to put the arrows over here. I'm going to combine these two, and I get the famous equation that you will see a zillion times before this course is over: mx double dot plus kx equals 0. This is the famous equation of a simple harmonic oscillator.

The solution to the equation, I will give you for now. x, as a function of any moment in time--- x is the position of this object m-- equals some amplitude x 0, which is the maximum displacement from equilibrium, times the cosine of omega t plus alpha, or it could be a sign of omega t plus alpha. Omega we call the angular frequency, which is 2 pi divided by period T-- this has nothing to do with tension, this is the number of seconds for one complete oscillation-- and that is the square root of k over m.

The frequency, in terms of how many oscillations per second, equals 1 divided by T. That is the number of oscillations per second, which we sometimes often call hertz. Omega is often expressed in radians per second, x 0 is the amplitude-- the maximum displacement from 0-- and alpha is a phase angle, which is entirely dictated by the initial conditions. There's really not that much physics in alpha. I can make alpha 0 if I want that, but I can [UNINTELLIGIBLE] the mass m a T equals 0, and I can do that with 0 speed. If I do that from a location, x equals plus A, then x as a function of t would x 0 times the cosine of omega t.

You can check for yourself that by stating the initial conditions the way I did, I have made alpha 0. The velocity as a function of time is the derivative of this, so that gives me a minus omega-- oh, I was going to make this an A-- omega times A times the sine of omega t. Notice that at t equals 0, the velocity is indeed 0-- that was my initial condition, that t equals 0 at 0 speed. Also notice that when t is just a hair larger than 0, the velocity is negative-- that's clear, because when I release the object from plus A in positive x direction, it wants to go back to equilibrium, and so the velocity is in this direction, so therefore the velocity is negative, and if velocity were in this direction, the velocity would be positive. That's all beautifully taken care of by algebra, and now we get x double dot t, which is a as a function of t that equals minus omega squared times A times cosine omega t, which is also minus omega squared times x of t. Now what you can do is you can substitute this x in here-- oh, where is my differential equation? I have to substitute it the differential equation-- I'm sorry-- I have to substitute this x here, and I have to substitute this x double dot here.

What do I find, then? That this equation-- this differential equation-- is only and only satisfied if omega equals the square root of k over m, which is what I stated earlier verbatim without proof. Here you have it: the square root of k over m, and therefore, that the period of one oscillation equals 2 pi times the square root off m over k. I repeat myself: the frequency of how many oscillations per second is 1 divided by the periods. You'll notice-- which I stated earlier-- that these periods is [UNINTELLIGIBLE] independent off A, which now I proved. That is by no means so intuitive-- I find it always amazing, but it is very characteristic for a simple harmonic oscillation that the periods of oscillations are independent of how far you displace it from its equilibrium point. If you look at this equation in more detail, you see that the larger K is the smaller of the periods-- I discussed that earlier, and that is rather intuitive. The larger the mass m is, the lower the acceleration will be for a given force on the mass. If I make m larger, the acceleration will be slower, therefore the whole motion will be slower. If the motion will be slower, obviously it will take longer to complete one oscillation, so T will be higher, so if m is higher, then also T is higher, and that is intuitive and very pleasing.

In general, the phase angle [UNINTELLIGIBLE] will not be 0, but I have made it zero by choosing my initial conditions very appropriately. You can very often do that.

Now we get to the last part of this problem set-- let me do a time check. Oh, boy, I'm 15 seconds ahead of time-- that's a real treat.

The last part is asking you the following: I now take this same system, k 1, with mass m, k 2, it's now vertical, and there is gravity. Let's first take the situation that there is no gravity: that's the situation here. Here, I have equilibrium. All of a sudden, I turn gravity on-- let this be the floor, and this object will sag because of gravity. It will sag over a distance: s, which stands for it will sag.

The new equilibrium position will now be here. This spring will be longer than we would prefer it to be, and this spring is shorter than it prefers to be. If I take this object m, and I look at the forces now in their new equilibrium, then there is of course mg-- that's clear-- but now this is spring is shorter than it wants to be, so it's going to push up. It will push up with a force F k 1, which equals k 1 times s-- that's the magnitude.

This one is longer than it wants to be, and it wants to contract, so it will also push ups with a force F k 2, and the magnitude of that force in an upwards direction equals k 2 times s. There has to be equilibrium: the sum of all forces in this direction will have to be 0. In other words, mg must be equal to k 1 plus k 2 times s. If you want to, this allows you to calculate what s is-- how far this object is sagging.

Now comes an interesting portion, an interesting part of this problem: suppose now from this new equilibrium position, I move the object either down over a certain distance, or I move it up over a certain distance, and I let it do its thing. The displacement from this new equilibrium position, I call this y, so that we don't have any confusion with my x before. I want you to demonstrate now that you will get F equals ma equals m y double dot equals minus capital K times y, where capital K equals k 1 plus k 2. This is the same simple harmonic oscillation that we had before-- again, we find that omega equals the square root of capital K over m. The period is unchanged, the frequency is unchanged, and both are independent of the amplitude by which I displace it from 0.

Even though I've left you with this a little bit, the bottom line is that even when I put this whole system in gravity, and when there is sag, given the new equilibrium when I displace it away from the new equilibrium, the object will oscillate with exactly the same period and exactly the same frequency as it did before when we had the system on the horizontal frictionless plane.

You may think that's so obvious-- I find it always a little bit surprising. I guess when you come to think about it, maybe it is obvious.