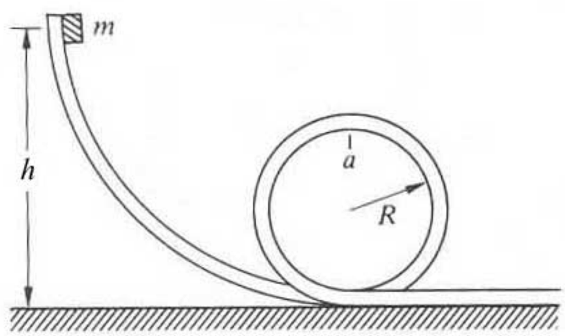
Conservation of Energy
Challenge Problems

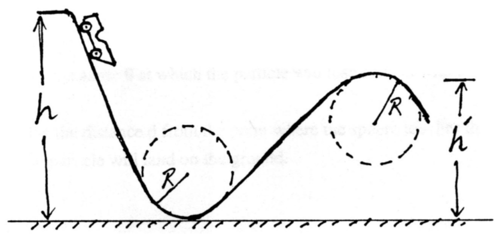
Problem 1

An object of mass m is released from rest at a height h above the surface of a table. The object slides along the inside of the loop-the-loop track consisting of a ramp and a circular loop of radius R shown in the figure. Assume that the track is frictionless. When the object is at the top of the track it pushes against the track with a force equal to three times it's weight. What height was the object dropped from?



Problem 2

Consider a roller coaster in which cars start from rest at a height h_0, and roll down into a valley whose shape is circular with radius R, and then up a mountain whose top is also circular with radius R, as shown in the figure. Assume the contact between the car and the roller coaster is frictionless. The gravitational constant is g. (Note: the following comment was not in the original problem description.) Assume that the wheels of the car run inside a track which follows the path shown in the figure below, so the car is constrained to follow the track.



a) Find an expression the speed of the cars at the bottom of the valley.

b) If the net force on the passengers is equal to $8\ mg$ at the bottom of the valley, find an expression for the radius R of the arc of a circle that fits the bottom of the valley.

c) The top of the next mountain is an arc of a circle of the same radius R. If the normal force between the car and the track is zero at the top of the mountain, what is the height h_{top} of the mountain? (Note that this is a change from the original question, which asked for the value of h_{top} for which the car loses contact with the road at the top of the mountain. The problem has been changed because we realized that the car would have to be attached to the track to prevent it from flying off before it reached the top of the mountain. If the normal force is zero at the top, it would have to be negative before it reached the top.)

Problem 3

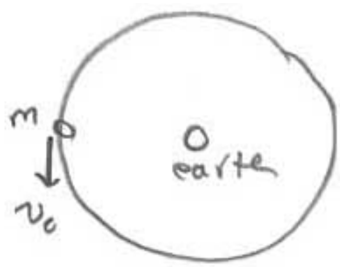
Find the escape speed of a rocket from the moon. Ignore the rotational motion of the moon. The mass of the moon is $m = 7.36 \times 10^{22}\, kg$. The radius of the moon is $R = 1.74 \times 10^{6}\, m$. The universal gravitation constant $G = 6.7 \times 10^{-11}\, N \cdot m^2 \cdot kg^{-2}$.

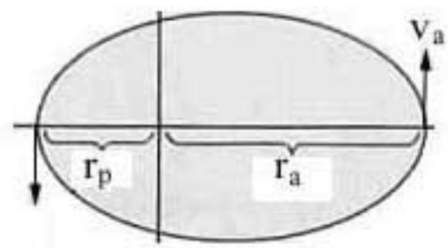
Problem 4

A satellite of mass $m = 5.0\times10^{2}\,kg$ is initially in a circular orbit about the earth with a radius $r_0 = 4.1\times10^{7}\,m$ and velocity $v_0 = 3.1\times10^{3}\,m/s$ around the earth. Assume the earth has mass $m_e = 6.0\times10^{24}\,kg$ and radius $r_e = 6.4\times10^{6}\,m$. Since $m_e >> m$, you may find it convenient to ignore certain terms. Justify any terms you choose to ignore.

a) What is the magnitude of the gravitational force acting on the satellite? What is the centripetal acceleration of the satellite?

b) What are the kinetic and potential energies of the satellite earth system? State any assumptions that you make. Specify your reference point for zero potential energy. What is the total energy?





$v_p = (5/4)\,v_0$

As a result of a satellite maneuver, the satellite trajectory is changed to an elliptical orbit. This is accomplished by firing a rocket for a short time interval and increasing the tangential speed of the satellite to $(5/4)v_0$. You may assume that during the firing, the satellite does not noticeably change the distance from the center of the earth.

c) What are the kinetic and potential energies of the earth-satellite at the point of closest approach? Specify your reference point for zero potential energy. What is the total energy?

d) How much energy was necessary to change the orbit of the satellite?

e) What speed would the satellite need to acquire so that it can escape to infinity?

Problem 5

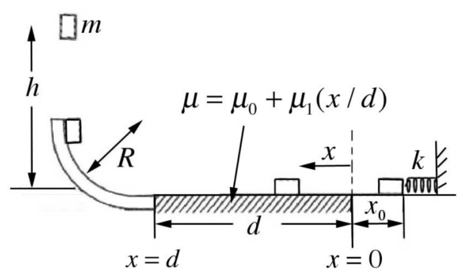
A ball of negligible radius is tied to a string of radius R. A man whirls the string and stone in a vertical circle. Assume that any non-conservative forces have negligible effect. Show that if the string is to remain taut at the top of the circle, the speed at the bottom of the circle must be at least $\sqrt{5gR}$.

Problem 6:

An object of mass m is released from an initial state of rest from a spring of constant k that has been compressed a distance x_0. After leaving the spring (at the position $x = 0$ when the spring is unstretched) the object travels a distance d along a horizontal track that has a coefficient of friction that varies with position as

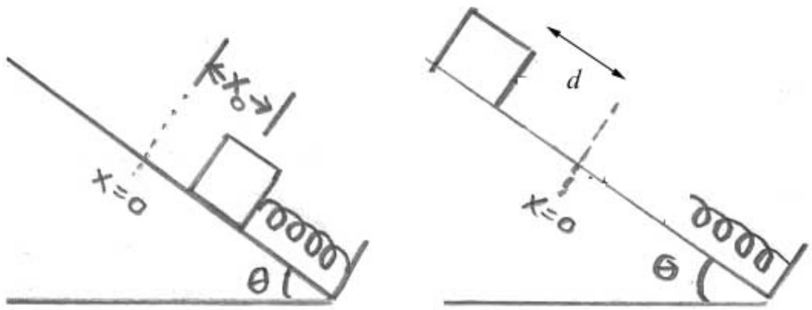
$$\mu = \mu_0 + \mu_1(x / d).$$

Following the horizontal track, the object enters a quarter turn of a frictionless loop whose radius is R. Finally, after exiting the quarter turn of the loop the object travels vertically upward to a maximum height, h, (as measured from the horizontal surface). Let g be the gravitational constant. Find the maximum height, h, that the object attains. Express all answers in terms of m, k, x_0, g, μ_0, μ_1, d and R; not all variables may be needed.



Problem 7

A object of mass m is pushed against a spring at the bottom of a plane that is inclined at an angle θ with respect to the horizontal and held in place with a catch. The spring compresses a distance x_0 and has spring constant k. The catch is released and the object slides up the inclined plane. At x = 0 the object detaches from the spring and continues to slide up the inclined plane.



Assume that the incline plane has a coefficient of kinetic friction μ_k. How far up the inclined plane does the object move from the point where the object detaches from the spring?

Problem 8

Two children are playing a game, which they try to hit a small box using a spring -loaded marble gun, which is fixed rigidly to a table and projects a marble of mass m horizontally from the edge of the table. The edge of the table is a height h above the top of the box. The spring has a spring constant k and the edge of the box is some unknown horizontal distance l away from the table. The first child compresses the spring a distance x and finds that the marble falls short of its target by a horizontal distance y. How far should the second child compress the spring in order to land in the box? Let g denote the gravitational acceleration. Express your answer in terms of k, m, x, g, h, and y as needed but do not use the unknown distance l.