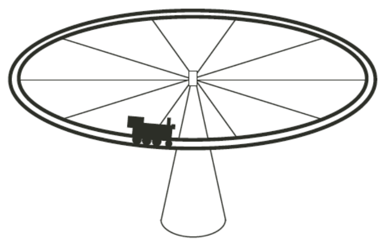
Angular Momentum Problems
Challenge Problems

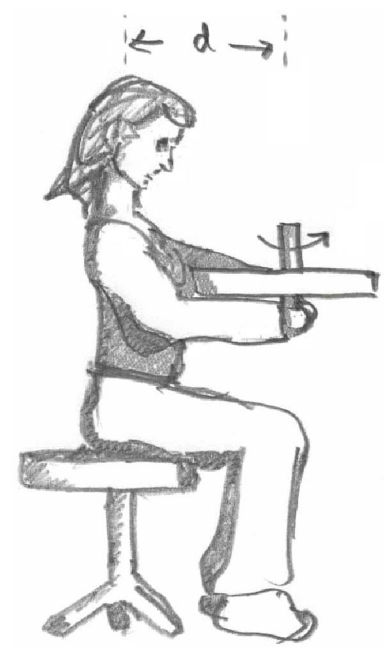
Problem 1:

Toy Locomotive A toy locomotive of mass m_L runs on a horizontal circular track of radius R and total mass m_T. The track forms the rim of an otherwise massless wheel which is free to rotate without friction about a vertical axis. The locomotive is started from rest and accelerated without slipping to a final speed of v relative to the track. What is the locomotive's final speed, v_f, relative to the floor?



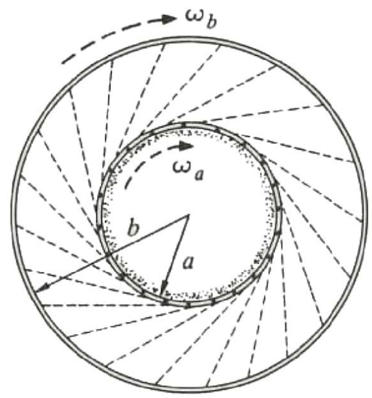
Problem 2: Spinning Chair and Wheel Solution

A person is sitting on a stool that is initially not rotating and is holding a spinning wheel. The moment of inertia of the person and the stool about a vertical axis passing through the center of the stool is $I_{S,p}$. The moment of inertia of the wheel about an axis, perpendicular to the plane of the wheel, passing through the center of mass of the wheel is $I_w = (1/4)I_{S,p}$. The mass of wheel is m_w. Suppose that the person holds the wheel as shown in the sketch such that the distance of an axis passing through the center of mass of the wheel to the axis of rotation of the stool is d and that $m_w d^2 = (1/3)I_w$. Suppose the wheel is spinning initially at an angular speed ω_s. The person then turns the spinning wheel upside down. You may ignore any frictional torque in the bearings of the stool. **What is the angular speed of the person and stool after the spinning wheel is turned upside down?**



Problem 3:

A drum of mass m_A and radius a rotates freely with initial angular velocity $\omega_{A,0}$. A second drum with mass m_B and radius $b > a$ is mounted on the same axle and is at rest, although it is free to rotate. A thin layer of sand with mass m_S is distributed on the inner surface of the smaller drum. At $t = 0$, small perforations in the inner drum are opened. The sand starts to fly out at a constant rate $\lambda\ \mathrm{kg \cdot s^{-1}}$ and sticks to the outer drum. Find the subsequent angular velocities of the two drums ω_A and ω_B. Ignore the transit time of the sand.



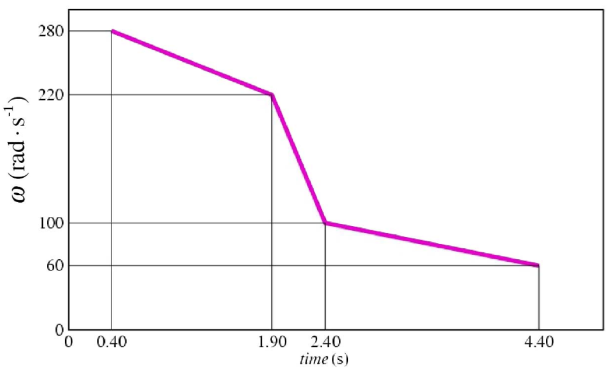
Problem 4: Measuring Moment of Inertia

In the angular momentum experiment, shown to the right, a washer is dropped smooth side down onto the spinning rotor.

The graph below shows the rotor angular velocity $\omega\left(\text{rad}\cdot\text{s}^{-1}\right)$ as a function of time.

Assume the following:

- The rotor and washer have the same moment of inertia I.
- The friction torque $\vec{\tau}_f$ on the rotor is constant during the measurement.



a) Find an expression for the magnitude $\left|\vec{\tau}_f\right|$ in terms of I and numbers you obtain from the graph.

b) What torque does the rotor exert on the washer during the collision (between $t = 1.90\text{ s}$ and $t = 2.40\text{ s}$)? Express your answer in terms of I and numbers you obtain from the graph.

c) How many radians does the rotor rotate during the collision (between $t = 1.90\text{ s}$ and $t = 2.40\text{ s}$)? Give a numerical answer.

d) How many radians does the washer rotate during the collision (between $t = 1.90\text{ s}$ and $t = 2.40\text{ s}$)? Give a numerical answer.

Note: express all of your answers in terms of I and numbers you obtain from the graph. Be sure to give an analytic expression prior to substituting the numbers from the graph.

Problem 5

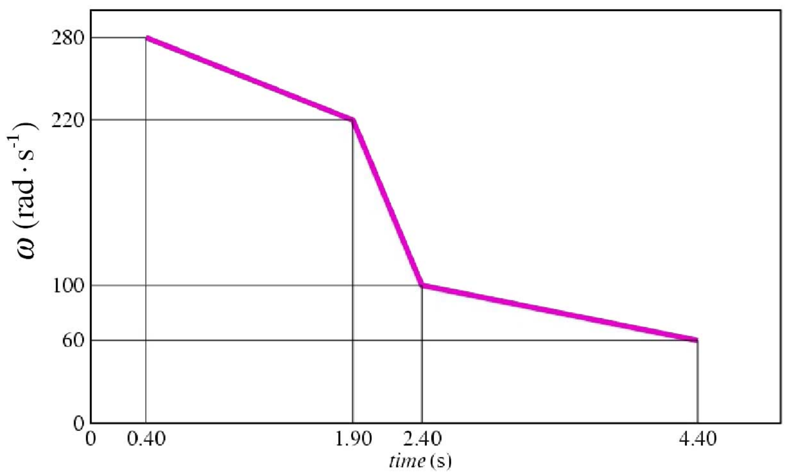
In the angular momentum experiment, shown to the right, a washer is dropped smooth side down onto the spinning rotor.

The graph below shows the rotor angular velocity $\omega\,(\text{rad}\cdot\text{s}^{-1})$ as a function of time.

Assume the following:

- The rotor and washer have the same moment of inertia I.
- The friction torque $\vec{\tau}_f$ on the rotor is constant during the measurement.

Note: express all of your answers in terms of I and numbers you obtain from the graph. Be sure to give an analytic expression prior to substituting the numbers from the graph.



a. Find an expression for the magnitude $|\vec{\tau}_f|$ in terms of I and numbers you obtain from the graph.

b. How much mechanical energy is lost to bearing friction during the collision (between $t = 1.90\text{ s}$ and $t = 2.40\text{ s}$)?

c. How much mechanical energy is lost to friction *between the rotor and the washer* during the collision (between $t = 1.90\text{ s}$ and $t = 2.40\text{ s}$)?

Problem 6: Work Done by Frictional Torque

A steel washer is mounted on the shaft of a small motor. The moment of inertia of the motor and washer is I_0. The washer is set into motion. When it reaches an initial angular velocity ω_0, at $t = 0$, the power to the motor is shut off, and the washer slows down during a time interval $\Delta t_1 = t_a$ until it reaches an angular velocity of ω_a at time t_a. At that instant, a second steel washer with a moment of inertia I_w is dropped on top of the first washer. Assume that the second washer is only in contact with the first washer. The collision takes place over a time $\Delta t_{col} = t_b - t_a$ after which the two washers and rotor rotate with angular speed ω_b. Assume the frictional torque on the axle is independent of speed, and remains the same when the second washer is dropped.

a) What angle does the rotor rotate through during the collision?

b) What is the work done by the friction torque τ_f from the bearings during the collision?

Problem 7: Experiment Angular Momentum: ***Angular Collision***

A steel washer is mounted on the shaft of a small motor. The moment of inertia of the motor and washer is I_0. The washer is set into motion. When it reaches an initial angular velocity ω_0, at $t = 0$, the power to the motor is shut off, and the washer slows down until it reaches an angular velocity of ω_a at time t_a. At that instant, a second steel washer with a moment of inertia I_{w} is dropped on top of the first washer. Assume that the second washer is only in contact with the first washer. The collision takes place over a time $\Delta t_{\mathrm{col}} = t_b - t_a$. Assume the frictional torque on the axle is independent of speed, and remains the same when the second washer is dropped. The two washers continue to slow down during the time interval $\Delta t_2 = t_f - t_b$ until they stop at time $t = t_f$. Your answers should be in terms of all or any of I_0, I_{w}, ω_0, ω_a, Δt_1, Δt_{col}, and Δt_2 (these would be measured or observed values).

a) What is the angular deceleration α_1 while the washer and motor are slowing down during the interval $\Delta t_1 = t_a$?

b) What is the angular impulse during the collision?

c) What is the angular velocity ω_b of the two washers immediately after the collision is finished (when the washers rotate together)?

d) What is the angular deceleration α_2 after the collision?