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8.012 Physics I: Classical Mechanics
Fall 2008

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.012 Fall 2008

Final Exam
Monday, December 15, 2008

NAME: ____

Instructions:
1. Do all **SEVEN (7)** problems. You have **2.5 hours**.
2. **Show all work**. Be sure to **CIRCLE YOUR FINAL ANSWER**.
3. Read the questions carefully
4. All work and solutions must be done in the answer booklets provided
5. NO books, notes, calculators or computers are permitted. **A sheet of useful equations is provided on the last page**.

Your Scores

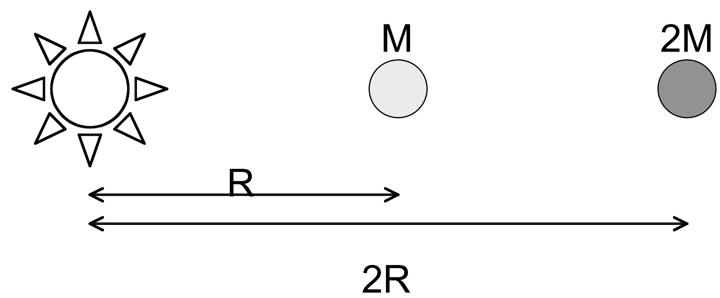
Problem	Maximum	Score	Grader
1	10		
2	15		
3	15		
4	15		
5	15		
6	15		
7	15		
Total	100		

[NO TEST MATERIAL ON THIS PAGE]

Problem 1: Multiple Choice & Short Answer Questions [10 pts]

For each of the following questions enter the correct multiple choice option or write/draw out a short answer in your answer booklet. You do not need to show any work beyond your answer.

(a) [2 pts] Two planets of mass M and 2M are in circular orbits around a star at radii R and 2R, respectively (assume the star's mass is >> M). Which planet has the greater orbital velocity and which planet has the greater orbital angular momentum?



(b) [2 pts] What is Chasles' theorem?

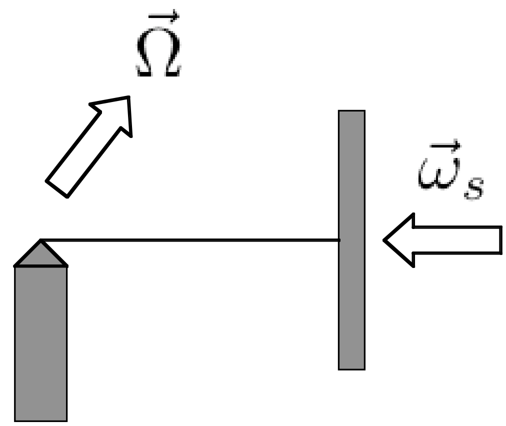
(1)	(2)	(3)	(4)
Every force has an equal and opposite pair	Gravitational orbits form ellipses	Motion can be separated in translation of center of mass and rotation about center of mass	Inertial mass equals gravitational mass

(c) [2 pts] A stationary ice skater is spinning about her center of mass (along a principal axis) on a frictionless surface. She pulls in her arms and spins up faster. Which of the following is conserved in this motion (write down all that apply)?

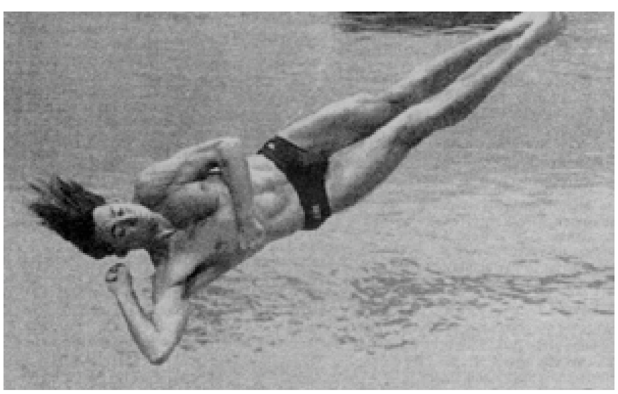
Energy Momentum Angular momentum

(d) [2 pts] What are the dimensions of the gravitational constant G?

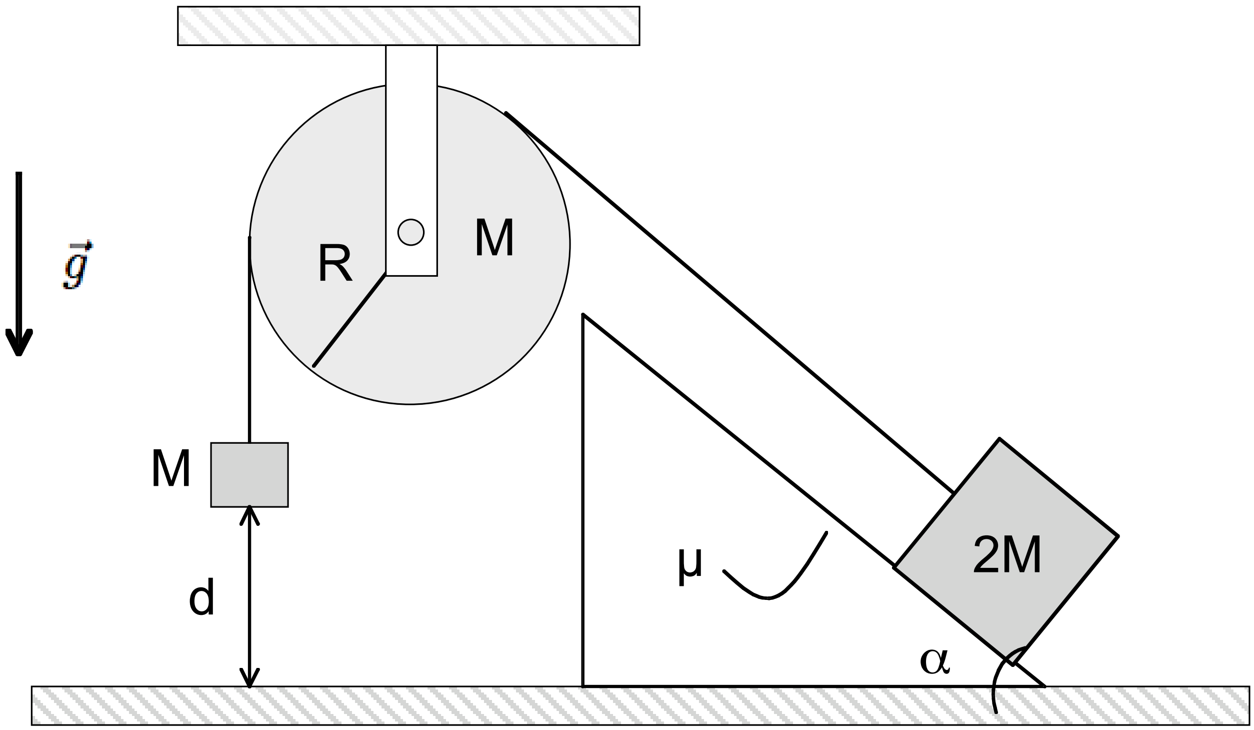
(e) [2 pts] A gyroscope whose spin angular velocity vector points toward the left is observed to precess such that its precession angular velocity vector points at an angle as shown. In which direction does the gravity vector point?



(f) [BONUS 2 pts] A diver is the middle of a dive as shown below. Based on clues in the photo, indicate in your answer booklet the direction that his total spin vector points, and determine whether the diver is doing a front flip or a back flip.



Problem 2: Atwood Machine [15 pts]

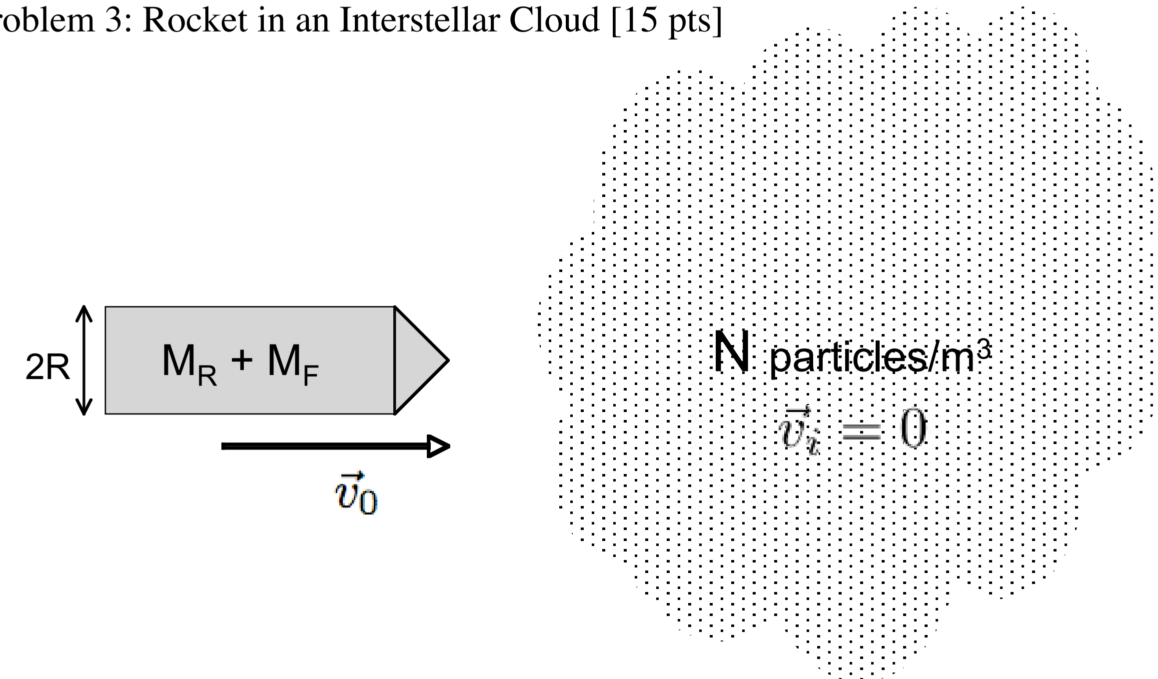


An Atwood machine consists of a fixed pulley wheel of radius R and uniform mass M (a disk), around which an effectively massless string passes connecting two blocks of mass M and 2M. The lighter block is initially positioned a distance d above the ground. The heavier block sits on an inclined plane with opening angle α. There is a coefficient of friction μ between the surfaces of this block and the inclined plane. Constant gravitational force acts downwards, and assume that the string never slips.

(a) [5 pts] Determine two conditions on the angle α which allow the lighter block to move up or move down.

(b) [10 pts] Assuming that the lighter block moves down, determine its acceleration.

Problem 3: Rocket in an Interstellar Cloud [15 pts]



A cylindrical rocket of diameter 2R, mass M_R and containing fuel of mass M_F is coasting through empty space at velocity v_0. At some point the rocket enters a uniform cloud of interstellar particles with number density N (e.g., particles/m^3), with each particle having mass m ($<< M_R$) and initially at rest. To compensate for the dissipative force of the particles colliding with the rocket, the rocket engines emit fuel at a rate $dm/dt = \gamma$ at a constant velocity u with respect to the rocket. Ignore gravitational effects between the rocket and cloud particles.

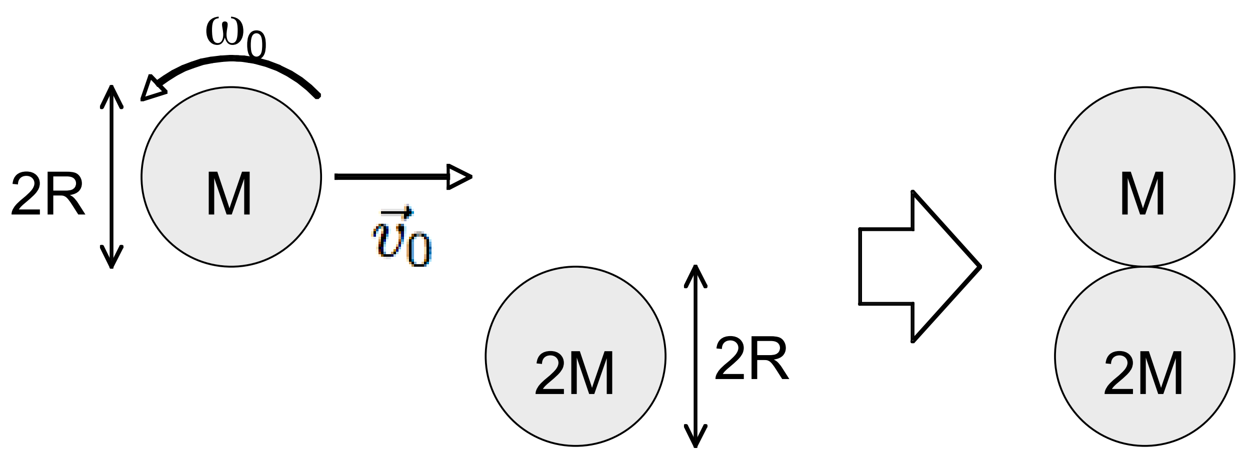
(a) [5 pts] Assuming that the dissipative force from the cloud particles takes the form $F = -Av^2$, where A is a constant, derive the equation of motion of the rocket (F = ma) through the cloud as it is firing its engines.

(b) [5 pts] What must the rocket's thrust be to maintain a constant velocity v_0?

(c) [5 pts] If the rocket suddenly runs out of fuel, what is its velocity as a function of time after this point?

(d) [BONUS 5 pts] Assuming that each cloud particle bounces off the rocket elastically, and collisions happen very frequently (i.e., collisions are continuous), prove that the dissipative force is proportional to v^2, and determine the constant A. Assume that the front nose-cone of the rocket has an opening angle of 90º.

Problem 4: Sticky Disks [15 pts]



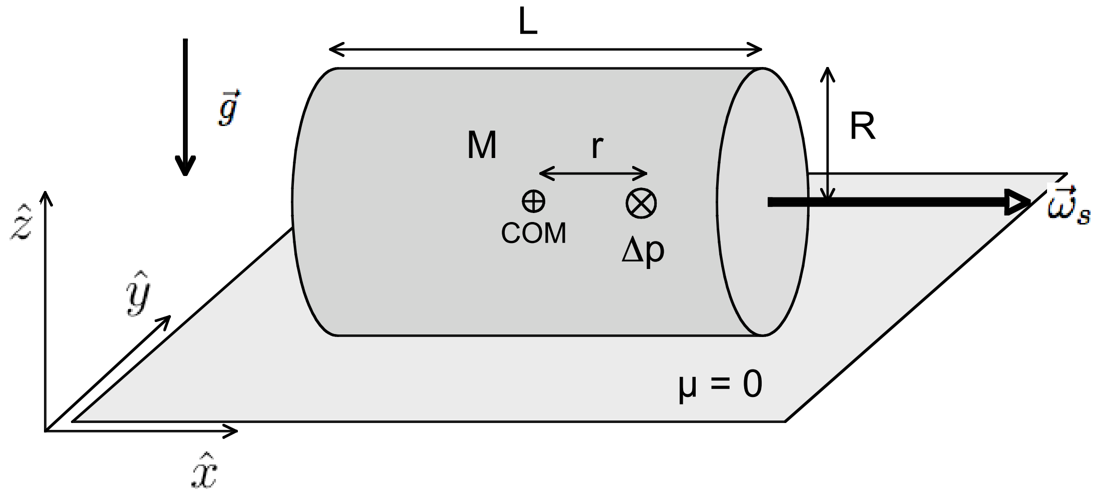
A uniform disk of mass M and diameter 2R moves toward another uniform disk of mass 2M and diameter 2R on the surface of a frictionless table. The first disk has an initial velocity v_0 and spin rate ω_0 as indicated, while the second disk is initially stationary. When the first disk contacts the second (a "glancing" collision), they instantly stick to each other and move as a single object.

(a) [5 pts] What are the velocity and spin angular velocity of the combined disks after the collision? Indicate both magnitudes and directions.

(b) [5 pts] For what value of ω_0 would the combined disks not rotate?

(c) [5 pts] How much total mechanical energy is lost in this collision?

Problem 5: Cylindrical Top [15 pts]



A cylinder of mass M, length L and radius R is spinning about its long axis with angular velocity $\vec{\omega} = \omega_s \hat{x}$ on a frictionless horizontal surface. The cylinder is given a sharp, horizontal strike with impulse $\Delta p\,\hat{y}$ at a distance r from its center of mass (COM). Assume that constant gravitational acceleration acts downward. NOTE: you do not need to use Euler's equations to solve this problem.

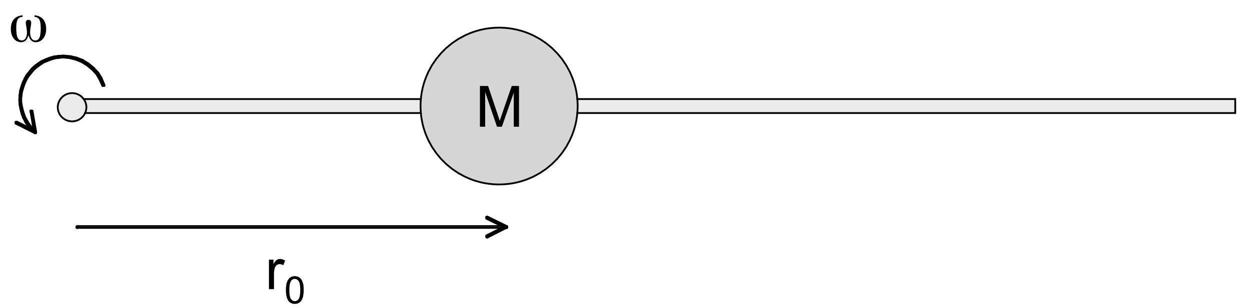
(a) [5 pts] What is the translational velocity of the cylinder after the impulse (magnitude and direction)?

(b) [5 pts] The strike imparts an angular momentum impulse to the cylinder which causes it to lift up at one end. At what angle α will the cylinder be tilted after the impulse and which end of the cylinder lifts up? Assume that the angular momentum impulse is much smaller than the spin angular momentum.

(c) [5 pts] After the cylinder tilts up, it effectively becomes a top. Determine its precessional rate and the direction of precession. Assume that nutational motion is negligible (i.e., α remains effectively constant) and that R << L (i.e., that the cylinder can be approximated as a thin rod for this part).

(d) [5 pts BONUS] For a strong enough impulse, the cylinder will tilt high enough to precess in the opposite direction. What is the minimum tilt angle for this to happen and what is the minimum impulse required? (Note that you cannot assume R << L here. This problem is similar to the "tipping battery" trick pointed out by one of the 8.012 students.)

Problem 6: Bead on a Spinning Rod [15 pts]



A bead of mass M is placed on a frictionless, rigid rod that is spun about at one end at a rate ω. The bead is initially held at a distance r_0 from the end of the wire. For the questions below, treat the bead as a point mass. Ignore gravitational forces.

(a) [5 pts] What force is necessary to hold the bead in place at r_0? Indicate both magnitude and direction.

(b) [5 pts] After the bead is released, what is its position in the inertial frame (in polar coordinates) as a function of time?

(c) [5 pts] Now calculate the fictitious forces on the bead in a reference frame that is rotating with the wire. What real force must the rod exert on the bead in both the rotating and inertial frames?

Problem 7: Central Potential [15 pts]

A particle of mass m moves within a region under the influence of a force of the form

$$\vec{F}(r) = -Ar^3\hat{r}$$

The particle is initially at a distance r_0 from the origin of the force, and initially moves with velocity v_0 in a tangential direction.

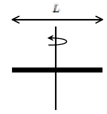
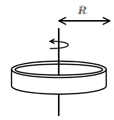
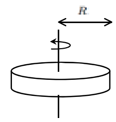
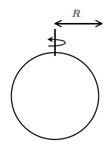
(a) [5 pts] Derive and sketch the effective potential of this system as a function of radius from the origin. Indicate all important inflection points. Can the particle pass through the origin of this reference frame?

(b) [5 pts] Find the velocity v_0 required for the particle to move in a purely circular orbit at a radius r_0 with this force law.

(c) [5 pts] Compute the frequency of small oscillations about this equilibrium radius. How does the period of these oscillations compare to the orbital period?

USEFUL EQUATIONS

Velocity in polar coordinates	$\dot{\vec{r}} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$
Acceleration in polar coordinates	$\ddot{\vec{r}} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}$
Center of mass (COM) of a rigid body	$\vec{R} = \frac{\sum m_i \vec{r}_i}{\sum m_i} = \frac{1}{M}\int \rho \vec{r} dV$
Volume element in cylindrical coordinates	$dV = r dr d\theta dz$
Kinetic energy	$K = \frac{1}{2}M(\vec{v}\cdot\vec{v}) + \frac{1}{2}\vec{\omega}\cdot\mathbf{I}\cdot\vec{\omega}$
Work	$W = \Delta K = \int \vec{F}\cdot d\vec{r} = \int \vec{\tau}\cdot d\vec{\theta}$
Potential Energy (for conservative forces)	$U = -\int \vec{F}_c \cdot d\vec{r}$ where $\vec{\nabla}\times\vec{F}_c = 0$
Angular momentum	$\vec{L} = \vec{r}\times\vec{p} = \mathbf{I}\cdot\vec{w}$
Torque	$\vec{\tau} = \vec{r}\times\vec{F} = \frac{d\vec{L}}{dt}$ Fixed axis rotation: $\tau_z = I_{zz}\dot{\omega}$

COM Moment of inertia for a uniform bar	 $I_{zz} = \frac{1}{12} ML^2$
COM Moment of inertia for a uniform hoop	 $I_{zz} = MR^2$
COM Moment of inertia for a uniform disk	 $I_{zz} = \frac{1}{2} MR^2$
COM Moment of inertia for a uniform sphere	 $I_{zz} = \frac{2}{5} MR^2$
Scalar parallel axis theorem	$I = I_{COM} + MR^2$
Moments of inertia tensor (permute x→y→z)	$I_{xx} = \sum_i m_i(y_i^2 + z_i^2) = \int dV \rho(y^2 + z^2)$
Euler's Equations (permute 1→2→3)	$\tau_1 = I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3$
Time derivative between inertial and rotating frames	$\left(\frac{d\vec{B}}{dt}\right)_{inertial} = \left(\frac{d\vec{B}}{dt}\right)_{rotating} + \vec{\Omega} \times \vec{B}$
Fictitious force in an accelerating frame	$\vec{F}_f = -m\vec{A}$
Fictitious force in a rotating frame (Ω constant)	$\vec{F}_f = -2m\vec{\Omega} \times \vec{v}_{rot} - m\vec{\Omega} \times (\vec{\Omega} \times \vec{r})$
Taylor Expansion of f(x)	$f(x) = f(a) + \frac{1}{1!}\frac{df}{dx}\vert_a(x-a) + \frac{1}{2!}\frac{d^2f}{dx^2}\vert_a(x-a)^2$