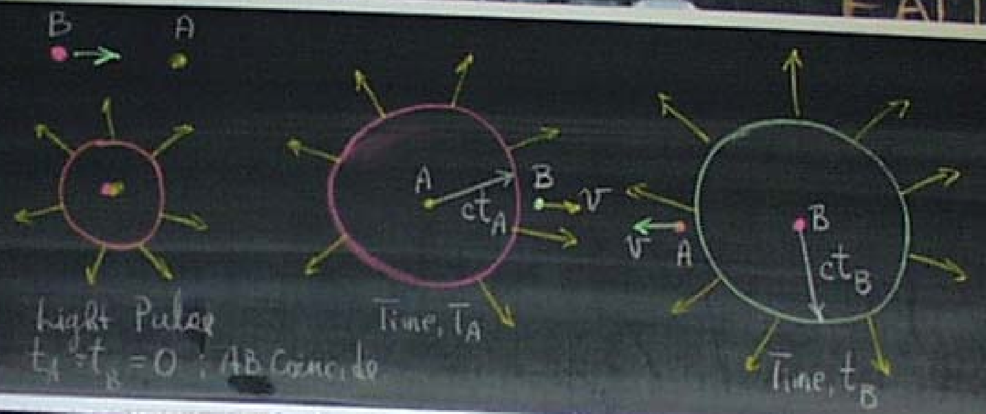
FALL CONCERT

## Paradox: Light Spheres

M-M Expt: velocity of earth through ether = 0.
Light vel rel. to earth = light vel rel. to ether!!
How is it possible for A, B moving with $v$ rel to each other??




①

A: $x^2+y^2+z^2 = r^2 = c^2t^2$
B: $x'^2+y'^2+z'^2 = r'^2 = c^2t'^2$

Galilean Trans: $t = t'$
$r' = r - vt$

A → B: Equations are Inconsistent with Gal. Trans!!

## Special Relativity: Einstein (1905)

Postulates:
1. All inertial frames are equivalent to all the laws of physics
2. The speed of light in empty space always has the same value c.

## Relativity and Measurement

Event: Anything that happens.
4-Coordinates.

System-S: $x, y, z, t$
-S': $x', y', z', t'$ } Space-Time

Events recorded by observers in S or S'.

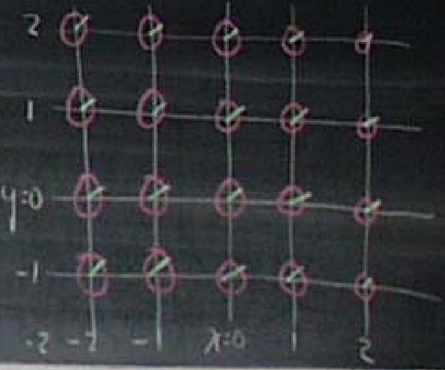
## Space Coord:

Space filled with 3D measuring rods: $x, y, z$
Close packed array.

## Time Coord:

Every intersection point has a clock. All synchronized to $t=0$.

②




Lecture 33 Blackboard #1

## Special Relativity · Einstein (1905)

Postulates:

1. All inertial frames are equivalent to all the laws of physics.
2. The speed of light in empty space always has the same value c.

## Relativity and Measurement

Event: Anything that happens.
4-coordinates.

$$\left.\begin{array}{l}\text{System-S: } x, y, z, t \\ \phantom{\text{System}}\text{-S}': x', y', z', t'\end{array}\right\}\text{Space-Time}$$

Events recorded by observers in S & S'.

## Space Coord:

Space filled with 3D measuring rods: x, y, z
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## Time Coord:

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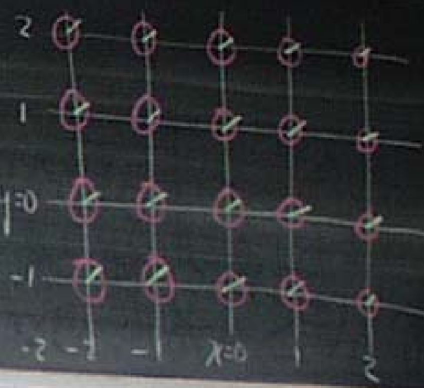


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## Relativity of Simultaneity

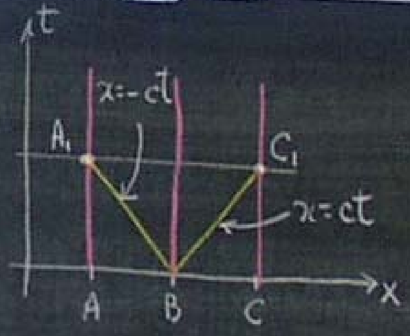
. Is simultaneity absolute?
Are two events which are simult in one frame also simult. in a frame moving rel. to the first?

Frame-S: Three stations at rest in S at x = A, B, C. Equally spaced.
Send light pulse from B at t = 0.

Light Signal:

$$x = x_B \pm ct$$

travels along ±x-axes
Signals arrive at A and C at $A_1$ and $C_1$
hence $A_1C_1 \rightarrow$ simultaneous time T in Frame-S.




Frame-S

③

## Frame-S'

S' moving with velocity v along +x.
A, B, C at rest in S'
World lines of A, B, C are inclined in S.

$$x = x_A + vt, \text{ etc.}$$

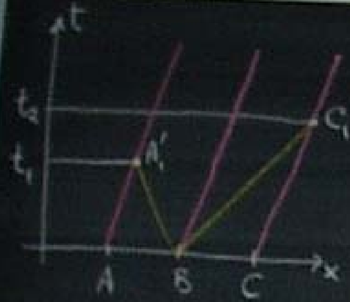
Light signal from B, still the same

$$x = x_B \pm ct$$

Signal arrives at A and C, at $A_1$, $C_1$

Lecture 33 Blackboard #2

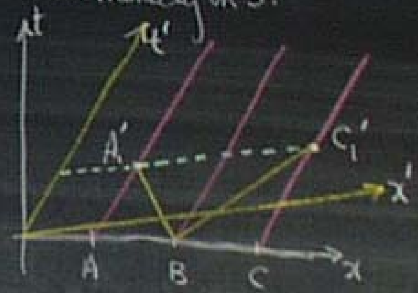
④




$A_1$ occurs at $t_1$ } $t_1 \neq t_2$
$C_1$ occurs at $t_2$ }
Signal reaches A before C
A is moving toward signal
C is running away from it.
$A, C_1$ not simultaneous.

In S' B midway between A and C.
∴ $A_1'$ and $C_1'$ as seen in S' must be simultaneous.
Conclusion: Simultaneity depends on choice of reference frame!!

Simultaneity in S'.




t'-axis → line $x'=0$, worldline of origin in S'
S'-origin: $x = vt$
x'-axis: Parallel to $A_1'C_1'$. Simultaneity in S'.
Use $x=0$, $t=0$ and $x'=0$, $t'=0$

⑤

## Space-Time Coordinates

System S: $x_p, t_p$ 2D
S': $x_p', t_p'$ 2D


## Lorentz-Fitzgerald Transformations

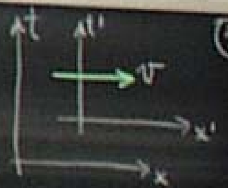
Equations relating $(x,y,z,t)$ in S to $(x'y'z't')$ in S'.

Requirements:
1. Must be linear eqts: single-valued.
2. For $v \ll c \Longrightarrow$ Galilean Transformation
3. Speed of light must have same value in all frames.

$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma(t - vx/c^2)$$
$$\beta = v/c$$
$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\Longrightarrow$$

$$x = \gamma(x' + vt')$$
$$y = y'$$
$$z = z'$$
$$t = \gamma(t' + vx'/c^2)$$



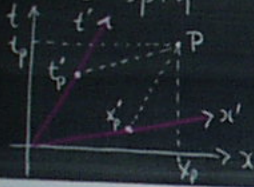

Assume $x=0$
$x'=0$
$t=0$
$t'=0$

Lecture 33 Blackboard #3

## Space-Time Coordinates

System S: $x_p, t_p$ 2D

S': $x'_p, t'_p$ 2D




## Lorentz-Fitzgerald Transformations

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$\Longrightarrow$

$$x = \gamma(x' + vt)$$
$$y = y'$$
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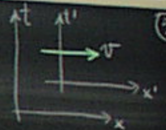
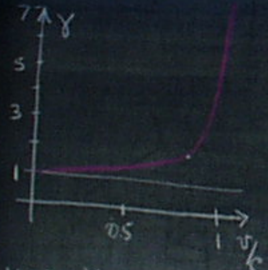
$$t = \gamma(t' + vx'/c^2)$$




Assume $x = 0$, $x' = 0$, $t = 0$, $t' = 0$

⑤




$v = c$; $\gamma = \infty$

$v = 0$; $\gamma = 1$

Light is special. Spreads out as a growing sphere.

$x^2 + y^2 + z^2 = c^2 t^2$ in Frame S

$x'^2 + y'^2 + z'^2 = c^2 t'^2$ in Frame S'

## Lorentz-Fitzgerald (1892)

Equations proposed ad-hoc to provide length shortening seen in M-M expt.

## L-T: Event Pairs

Let $\Delta x = x_2 - x_1$

$\Delta t = t_2 - t_1$

$$\Delta x = \gamma(\Delta x' + v\Delta t') \qquad \Delta x' = \gamma(\Delta x - v\Delta t)$$
$$\Delta y = \Delta y'$$
$$\Delta z = \Delta z'$$
$$\Delta t = \gamma\left(\Delta t' + v\frac{\Delta x}{c^2}\right) \qquad \Delta t' = \gamma\left(\Delta t - v\frac{\Delta x}{c^2}\right)$$

⑥

## Space-Time Invariant

x, t in S   x', t' in S'

$$x' = \gamma(x - vt)$$
$$t' = \gamma(t - vx/c^2)$$

Evaluate:

$$(ct')^2 - (x')^2 = \gamma^2\left[\left(ct - \frac{vx}{c}\right)^2 - (x - vt)^2\right]$$
$$= \gamma^2\left[(c^2 - v^2)t^2 - \left(1 - \frac{v^2}{c^2}\right)x^2\right]$$
$$(ct')^2 - (x')^2 = (ct)^2 - (x)^2 = s^2$$

Invariant!!!

Lecture 33 Blackboard #4

$S^2 \equiv$ Same in any frame!!

Generalize:

$(ct')^2-(x')^2-(y')^2-(z')^2 = (ct)^2-x^2-y^2-z^2$

All observers agree on value of $S^2$.

Assume event pairs on x-axis.

$(c\Delta t)^2-(\Delta x)^2 = (c\Delta t')^2-(\Delta x')^2 = \Delta S^2$

$(\Delta S)^2$ can be $>0$, $<0$ or $=0$!!

## Timelike Events

If $(c\Delta t)^2 > (\Delta x)^2 \longrightarrow (\Delta S)^2 > 0$

Let $\Delta\tau = \frac{\Delta S}{c} = \sqrt{\Delta t^2 - \left(\frac{\Delta x}{c}\right)^2}$

= proper time interval
invariant!

$\Delta\tau = \Delta t$ if $\Delta x = 0$
events in same place

$\Delta t > \Delta\tau$ everywhere else.

## Spacelike Events

If $(c\Delta t)^2 < (\Delta x)^2 \longrightarrow \Delta S^2 < 0$

No proper time interval poss.

Let $\Delta\sigma = \sqrt{-\Delta S^2} = \sqrt{\Delta x^2 - (c\Delta t)^2}$

proper distance.

$\Delta\sigma = \Delta x$ if $\Delta t = 0$.

Simultaneous events.

Timelike: Can find frame where events are in same place.

Spacelike: Frame possible with events simult.

If $(\Delta S)^2 > 0$ Order of events same in S + S'

$(\Delta S)^2 < 0$ Order of events opposite in S + S'.

$(\Delta S)^2 = 0 \Rightarrow (c\Delta t)^2 = (\Delta x)^2$

$\left.\begin{array}{l}\Delta\tau = 0\\ \Delta\sigma = 0\end{array}\right\}$ Light pulse leaving an event arrives just when second event occurs

(7)

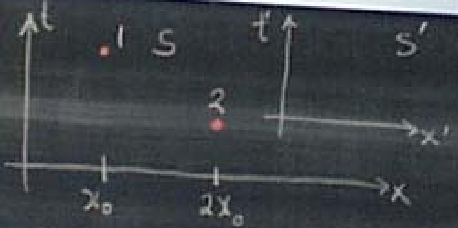
## Example

Frame-S Event 1: $x_1 = x_0$ $t_1 = \frac{x_0}{c}$

Event 2: $x_2 = 2x_0$ $t_2 = \frac{x_0}{2c}$

a. Is there S' where events are simult? What is $v$?

b. What is value of $t'$ in which both events happen in S'?




$(\Delta S)^2 = (c\Delta t)^2 - (\Delta x)^2$

$= \frac{x_0^2}{4} - x_0^2 < 0$

Spacelike Events!!

$\Delta t = t_2 - t_1 = \frac{x_0}{c}\left(\frac{1}{2} - 1\right) = -\frac{x_0}{2c}$

$\Delta x = x_2 - x_1 = 2x_0 - x_0 = x_0$

Frame-S'

$\Delta t' = \gamma\left(\Delta t - v\frac{\Delta x}{c^2}\right) \equiv 0$

$\therefore v = \frac{\Delta t}{\Delta x}c^2 = \frac{-x_0/2c}{x_0}c^2$

$v = -c/2$

$\gamma = \frac{1}{\sqrt{1-\left(\frac{1}{2}\right)^2}} = \frac{2}{\sqrt{3}}$

$t' = \gamma\left(t - vx/c^2\right)$

$t_1' = \frac{2}{\sqrt{3}}\left(\frac{x_0}{c} + \frac{c}{2}\frac{x_0}{c^2}\right) = \sqrt{3}\frac{x_0}{c}$ Event-1

$t_2' = \frac{2}{\sqrt{3}}\left(\frac{x_0}{2c} + \frac{c}{2}\frac{2x_0}{c^2}\right) = \sqrt{3}\frac{x_0}{c}$ Event-2

Simultaneous.

(8)

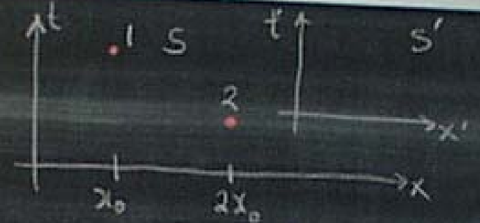
## Example

Frame-S  Event 1: $x_1 = x_0$  $t_1 = \frac{x_0}{c}$

Event 2: $x_2 = 2x_0$  $t_2 = \frac{x_0}{2c}$

a. Is there S′ where events are simult? What is $v$?

b. What is value of $t'$ in which both events happen in S′?




$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$$
$$= \frac{x_0^2}{4} - x_0^2 < 0$$

Spacelike Events!!

$$\Delta t = t_2 - t_1 = \frac{x_0}{c}\left(\frac{1}{2} - 1\right) = \frac{-x_0}{2c}$$
$$\Delta x = x_2 - x_1 = 2x_0 - x_0 = x_0$$

Frame-S′

$$\Delta t' = \gamma\left(\Delta t - v\frac{\Delta x}{c^2}\right) \equiv 0$$
$$\therefore v = \frac{\Delta t}{\Delta x}c^2 = \frac{-x_0/2c}{x_0}c^2$$
$$v = -c/2$$

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{1}{2}\right)^2}} = \frac{2}{\sqrt{3}}$$
$$t' = \gamma\left(t - vx/c^2\right)$$
$$t_1' = \frac{2}{\sqrt{3}}\left(\frac{x_0}{c} + \frac{c}{2}\frac{x_0}{c^2}\right) = \sqrt{3}\frac{x_0}{c} \quad \text{Event-1}$$
$$t_2' = \frac{2}{\sqrt{3}}\left(\frac{x_0}{2c} + \frac{c}{2}\frac{2x_0}{c^2}\right) = \sqrt{3}\frac{x_0}{c} \quad \text{Event-2}$$

Simultaneous

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## Example: Train + Simultaneity

- Railcar in System-S.
- length 2L
- Light Flash at Center $T_A = T_B = \frac{L}{c}$
- System-S′ moving to right velocity $v$
- Car appears to move to left.

$A \to A^*$ } Light reaches

$B \to B^*$ } $B^*$ before $A^*$

Event-1: Pulse at A: $x_1 = -L$, $t_1 = \frac{L}{c} = T$

Event-2: Pulse at B: $x_2 = L$, $t_2 = \frac{L}{c} = T$

$$t_1' = \gamma\left(t_1 - \frac{vx_1}{c^2}\right) = \gamma\left(T + \frac{vL}{c^2}\right)$$
$$= \gamma\left(T + \frac{v}{c}T\right) = T\sqrt{\frac{1+v/c}{1-v/c}}$$
$$t_2' = \gamma\left(t_2 - \frac{vx_2}{c^2}\right) = T\sqrt{\frac{1-v/c}{1+v/c}}$$

In S′, pulse at B is earlier than pulse at A!!!

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