Paradox: Light Spheres

- M-M experiment gave a zero value for the velocity of the earth through the ether — the light transmitting medium.
- The velocity of light relative to the earth must be the same as the velocity relative to the medium.
- How can the velocity be the same for two observers, A, B moving relative to each other.

- A light signal is sent out when observers coincide at $t_A = t_B = 0$
- A with his rulers and clocks measures a spherically outgoing light wave centered on A

- B with his rulers and clocks also sees a light wave moving out in a sphere centered on B.
- If we have absolute space and absolute time how can both experiments be correct?

Dilema!!!

$$\left.\begin{array}{l} A: x^2 + y^2 + z^2 = r^2 = c^2 t^2 \\ B: x'^2 + y'^2 + z'^2 = r'^2 = c^2 t'^2 \end{array}\right\} \quad \begin{array}{l} \text{Galilean Trans} \\ t = t' \\ r' = r - vt \end{array}$$

A – B: Inconsistent with Galilean Trans.!!

R2-2

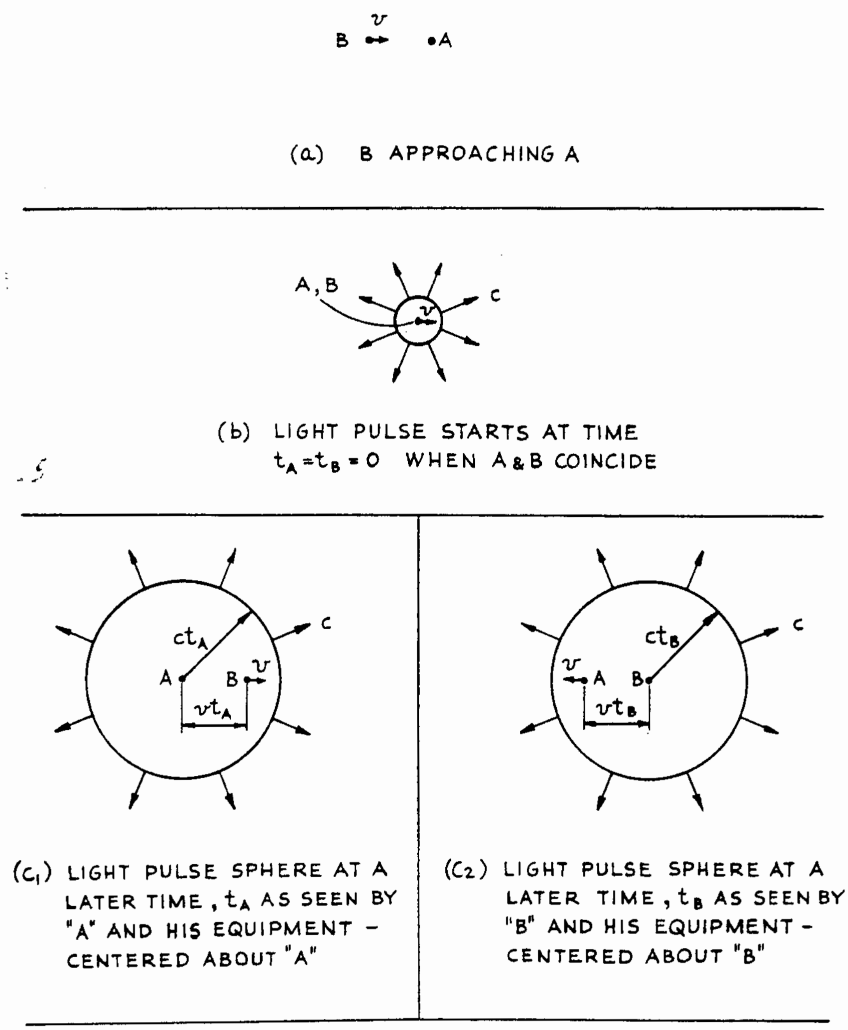


Figure 2. The paradox of the light spheres.

Special Relativity : Einstein (1905)

Postulates :

1: All inertial frames are equivalent to all the laws of physics.

2: The speed of light in empty space always has the same value $\underline{c}$.

Relativity and Measurement

Event: Anything that happens. Defined by four coordinates:

system $-S: (x, y, z, t)$
$-S': (x', y', z', t')$ } Space-Time Coord.

- Given event can be recorded by any number of observers in any inertial frame.
- Practical problems for events separated in space.

Space Coordinates:
- Coordinate frame is filled with a close-packed array of 3-D measuring rods, one set parallel to each axis. Rods provide precise location of an event.

Time Coordinates:
- Every point at the intersection of each measuring rod contains a clock.
- Clocks put in place and simultaneously synchronized at $t=0$. Use light signals!
- All clocks are the same and run at the same rate.

Space-Time Coordinates

- When an event occurs the spacetime coordinates of an event are assigned by reading of the nearest measuring rod and the nearest clock.

- For two events the time difference is computed as the difference in the times recorded nearest each event. The separation in space is derived from the differences of coordinates on the rods near each event.

- Procedure avoids using travel times of signals to an observer.

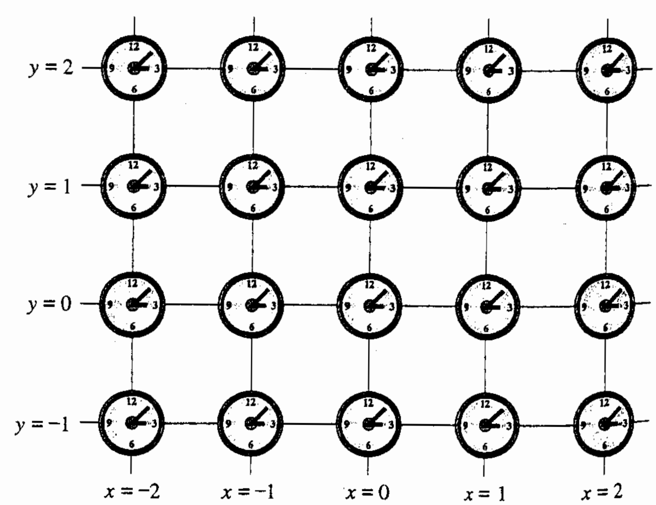


FIGURE 40–7 Clocks attached to lattice points that represent space coordinates separated by fixed distances. The location and the clock reading define the space–time coordinates of an event.

Relativity of Simultaneity

- Is simultaneity absolute? If two events are simultaneous in one reference frame are they also simultaneous in a frame moving relative to the first?

Frame - S:

- Consider three stations A, B and C equally spaced along x-axis of an inertial frame S. They are at rest in S.
- xt-coordinate system shows evolution of A, B and C as time passes. These are the world lines of the points.
- World line is a graph of the position of a point as a function of time. Gives complete history of the point.
- World lines of A, B and C are vertical lines corresponding to x = constant.

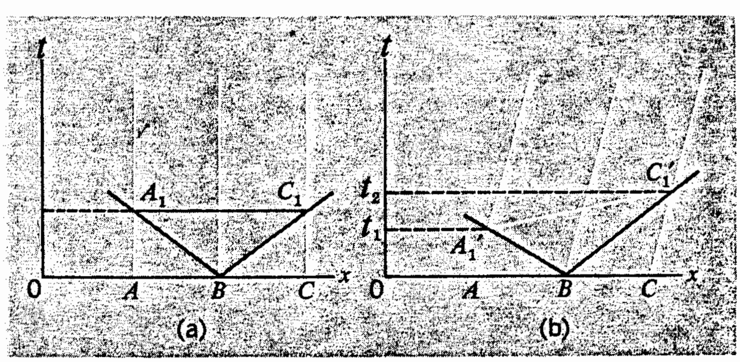


Fig. 3–2 (a) Space-time diagram showing experiment to define simultaneity at stations A and C (at rest in this reference frame) by light signals emitted from a station B midway between them. (b) Equivalent experiment for the case in which A, B, and C all have a velocity with respect to the reference frame.

- Send a light or radio signal from B at $t=0$. It travels forward and backward with speed c along $\pm x$.
- World lines of signal are two sloping lines $x = x_B \pm ct$.

- Signals reach A and C at the points A_1, C_1
- Simultaneity is represented by the line A_1C_1 which has the exact same time t.

Frame - S':

- Frame S' is moving with velocity v along $+x$-axis.
- A, B and C at rest in S'.
- World-lines of A, B, and C as seen in S are inclined: $x = x_A + vt$
- Signal from B at $t=0$ is still described by the lines $x = x_B \pm ct$.
- Signal arrives at A and C, at A_1 and C_1
- Not simultaneously in frame - S.
- $\left.\begin{array}{l} A_1 \text{ occurs at } t_1 \\ C_1 \text{ occurs at } t_2 \end{array}\right\} t_1 \neq t_2$

- Signal reaches A before C because A is moving toward signal and C is running away from it.

- However: B is midway between A and C as seen in S'.

$\therefore$ A_1' and C_1' as seen in S' must be simultaneous.

Conclusion: Judgement of simultaneity is a function of the reference frame that we use.

Simultaneity in S'

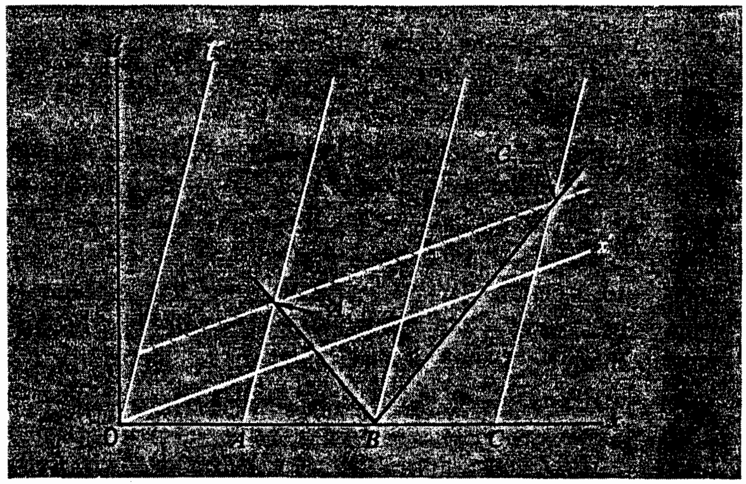


Fig. 3–3 Specification of coordinate axes (x, t) and (x', t') for two reference frames in relative motion.

- Add x', t' axes as seen in S
- A, B, C at rest in S'
- t'-axis is the line $x'=0$, the world line of the origin in S'

  S'-origin : $x = vt$

- x'-axis

  A', C' - line is simultaneous in S'. They therefore have a constant t'.

  $\therefore$ t'-axis has to be parallel to this line.

- Use $x=0, t=0$ and $x'=0, t'=0$ for convenience.

Space-Time Coordinates

System-S : x_P, t_P

System-S$'$: x'_P, t'_P

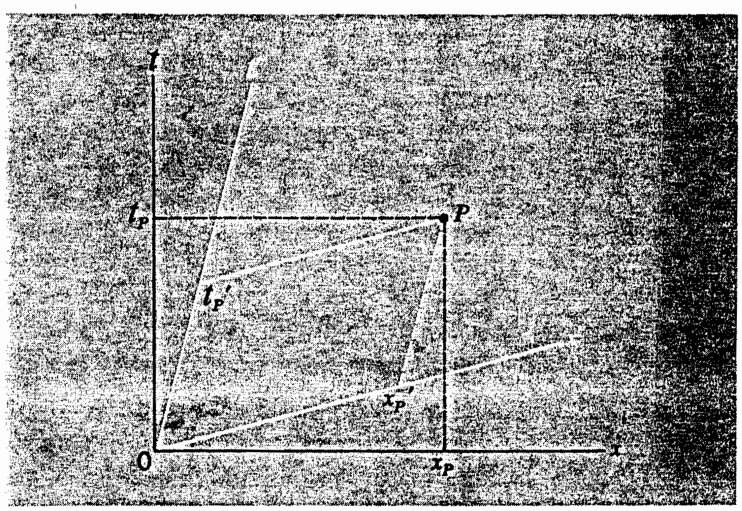


Fig. 3–4 Space-time coordinates of a given point event in two different inertial frames.

Lorentz-Transformations

- Set of equations to relate (x, y, z, t) in S to (x', y', z', t') in S' moving with velocity v relative to S.

Requirements:

1. Transformation must be linear. A single event in S must transform to a single event in S'.

2. For $v \ll c$ the transformation should approach the Galilean transformation.

3. The speed of light must have the value c in every inertial frame.

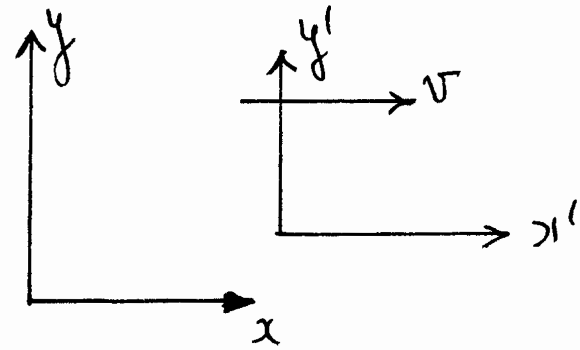
Light is special. A flash of light spreads out as a growing sphere:

$$x^2 + y^2 + z^2 = c^2 t^2 \quad \text{in Frame - S}$$

The same must be true in the primed frame;

$$x'^2 + y'^2 + z'^2 = c^2 t'^2$$

Lorentz transformations meet all these requirements:



$x = 0$, $x' = 0$ at $t = 0$, $t' = 0$

$$x' = \gamma(x - Vt)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma(t - Vx/c^2)$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

where,

$$\beta = \frac{V}{c}$$

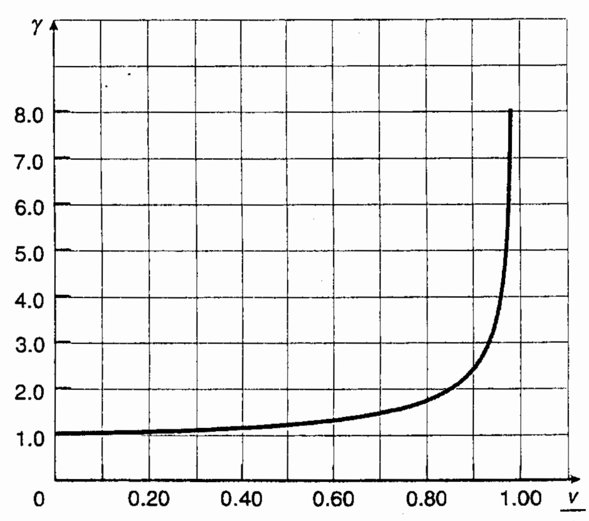


FIGURE 25.14 A graph of γ versus v/c.

Solve equations or by symmetry:

$$x = \gamma(x' + vt')$$

$$y = y'$$

$$z = z'$$

$$t = \gamma(t' + vx'/c^2)$$

Lorentz-Fitzgerald (1892)

- Equations proposed ad-hoc to provide length shortening needed to explain null Michelson-Morley experiment.

Space-Time Invariant

Event: x, t in S
x', t' in S'

$$x' = \gamma(x - vt)$$
$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

Evaluate:
$$(ct')^2 - (x')^2 = \gamma^2\left[(ct - vx/c)^2 - (x - vt)^2\right]$$
$$= \gamma^2\left[(c^2 - v^2)t^2 - (1 - v^2/c^2)x^2\right]$$
$$(ct')^2 - (x')^2 = (ct)^2 - (x)^2 = s^2$$

Invariant

Generalize

$$(ct')^2 - (x')^2 - (y')^2 - (z')^2 = (ct)^2 - x^2 - y^2 - z^2$$

L-T: Event Pairs

$\Delta x = x_2 - x_1$
$\Delta t = t_2 - t_1$

$\Delta x = \gamma(\Delta x' + V\Delta t')$ $\quad\quad$ $\Delta x' = \gamma(\Delta x - V\Delta t)$

$\Delta y = \Delta y'$

$\Delta z = \Delta z'$

$\Delta t = \gamma\left(\Delta t' + \frac{V\Delta x'}{c^2}\right)$ $\quad\quad$ $\Delta t' = \gamma\left(\Delta t - v\frac{\Delta x}{c^2}\right)$

Spacetime Interval – Invariant

- Assume events along x-axis

$$(c\Delta t)^2 - (\Delta x)^2 = (c\Delta t')^2 - (\Delta x')^2 = (\Delta s)^2$$

$\Delta s \equiv$ same in all frames, can show!!

- All observers agree on value.
- $(\Delta s)^2$ can be >0, <0, $=0$.

Timelike Events

If $(c\Delta t)^2 > (\Delta x)^2 \rightarrow (\Delta s)^2 > 0$
$\rightarrow$ time like events

Let $\Delta\tau = \frac{\Delta s}{c} = \sqrt{(\Delta t)^2 - \left(\frac{\Delta x}{c}\right)^2}$ proper time interval invariant.

$\Delta\tau = \Delta t$ if $\Delta x = 0$; events at same place.

$\Delta t > \Delta\tau$ everywhere else. Moving clocks run slower.

Spacelike Events

If $(c\Delta t)^2 < (\Delta x)^2 \longrightarrow (\Delta s)^2 < 0$

$\rightarrow$ space like events
no physically proper time interval possible

Let $\Delta\sigma = \sqrt{-(\Delta s)^2} = \sqrt{(\Delta x)^2 - (c\Delta t)^2}$ proper distance

$\Delta\sigma = \Delta x$ if $\Delta t = 0$; events simultaneous.

Timelike Region: Can find frame in which two events occur at same place.

Spacelike Region: Can find a frame in which two events occur at same time.

If $(\Delta s)^2 > 0$ order of events same in S and S'

$(\Delta s)^2 < 0$ events occur in reverse order
Consider $V = c$, ray of light from event-1 cannot reach ~~event-2~~ point-2 to cause event-2. They are not causally connected.

<u>Lightlike Region:</u>

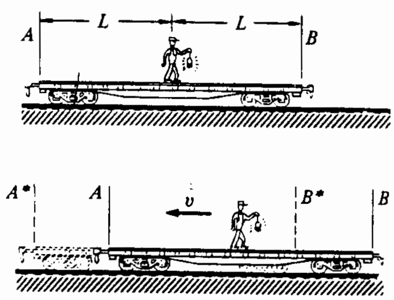
$(\Delta s)^2 = 0 \qquad (c\Delta t)^2 = (\Delta x)^2$

$\Delta\tau = 0 \qquad$ proper time vanishes

$\Delta\sigma = 0 \qquad$ proper distance vanishes.

Light pulse leaving one event arrives at other event just as it occurs.

Example: Simultaneity



- Railcar in System-S
- Length 2L
- Flash of light at center
- Arrival time

$$T_A = T_B = \frac{L}{c}$$

- System-S′ moving to the right, velocity v
- Car appears to move to left.
- A moves to A*
- B moves to B*
- Light reaches B* before A*

Event-1 Pulse arrives at A $\quad x_1 = -L$, $\quad t_1 = \frac{L}{c} = T$

Event-2 Pulse arrives at B $\quad x_2 = L$, $\quad t_2 = \frac{L}{c} = T$

$$t_1' = \gamma\left(t_1 - \frac{v x_1}{c^2}\right) = \gamma\left(T + \frac{vL}{c^2}\right)$$

$$= \gamma\left(T + \frac{v}{c}T\right) = T\sqrt{\frac{1+v/c}{1-v/c}}$$

$$t_2' = \gamma\left(t_2 - \frac{v x_2}{c^2}\right) = T\sqrt{\frac{1-v/c}{1+v/c}}$$

In S′; pulse at B earlier than pulse at A.

Example:

Frame - S

Event1 : $x_1 = x_0$ $t_1 = x_0/c$

Event 2 : $x_2 = 2x_0$ $t_2 = x_0/2c$

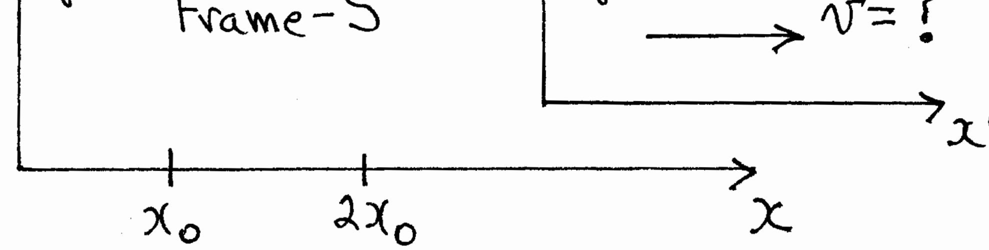
a) Is there a frame, S', in which these events occur at the same time? Find the velocity of this frame S' relative to S.

b) What is the value of t' at which both events occur in S'?

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$$

$$= \frac{x_0^2}{4} - x_0^2$$

$(\Delta s)^2 < 0$!! Spacelike Events



Frame-S:

$$\Delta t = t_2 - t_1 = \frac{x_0}{c}\left[\frac{1}{2} - 1\right] = -\frac{x_0}{2c}$$

$$\Delta x = x_2 - x_1 = 2x_0 - x_0 = x_0$$

Frame-S':

$$\Delta t' = \gamma\left(\Delta t - v\frac{\Delta x}{c^2}\right) = 0 \qquad \text{'simultaneous'}$$

$$\therefore\ v = \frac{\Delta t}{\Delta x}c^2 = \frac{-x_0/2c}{x_0}\,c^2$$

$$v = -c/2$$

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{1}{2}\right)^2}} = \frac{2}{\sqrt{3}}$$

$$t' = \gamma\left(t - \frac{Vx}{c^2}\right)$$

$$= \frac{2}{\sqrt{3}}\left[\frac{x_0}{c} + \frac{c}{2}\frac{x_0}{c^2}\right] = \sqrt{3}\frac{x_0}{c} \qquad \text{Event-1}$$

$$= \frac{2}{\sqrt{3}}\left[\frac{x_0}{2c} + \frac{c}{2}\frac{2x_0}{c^2}\right] = \sqrt{3}\frac{x_0}{c} \qquad \text{Event-2}$$