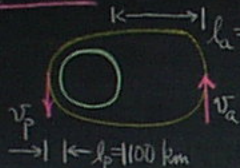
(1)

Earth-Satellite Orbit

$R_e = 6400$ km
$m = 2000$ kg
$m \ll M_e$
$\mu = m$



$$A = (r_p + r_a + 2R_e) = 1.8\times10^7\ \text{m}$$

$$A = \frac{k}{(-E)} = \frac{G\mu M_e}{(-E)} = \frac{GmM_e}{(-E)}$$

$$E = \frac{GmM_e}{A} = -\frac{mgR_e^2}{A} = -4.5\times10^{10}\ \text{J}$$

Energy of Satellite in Orbit

$$E_{Initial} = -\frac{GmM_e}{R_e} = -mgR_e = -12.5\times10^{10}$$

$$\Delta E = E - E_i = 8\times10^{10}\ \text{J}$$

Angular Momentum

$$\epsilon = \frac{r_{max} - r_{min}}{r_{max} + r_{min}} = \frac{3\times10^3}{1.8\times10^4} = \frac{1}{6}$$

$$\epsilon^2 = 1 + \frac{2EL^2}{mk^2}$$

$$L = 1.2\times10^{14}\ \text{kg}\cdot\text{m}^2/\text{s}$$

$$E = \tfrac{1}{2}mv^2 - \frac{k}{r}$$

$$r_p = (1100 + 6400)\ \text{km}$$

$$v_p^2 = \left(E + \frac{k}{r}\right)\frac{2}{m} = 7900\ \text{m/s}$$

Cons of L:

$$\mu r_p v_p = \mu r_a v_a$$

$$v_a = \frac{r_p}{r_a} v_p = 5600\ \text{m/s}$$

(2)

Oscillations

Periodic phenomena: time, pos, etc.

- planets, orbital motion
- pendulum clock
- mass-spring system
- molecular vibrations
- atomic vibrations

Oscillatory Motion: Periodic in Time

Simplest: vibration of a particle about a stable equilibrium position.

- Hooke's Law

Force is prop. to displacement from equilibrium and force is restoring.

Simple Harmonic Motion

- Back and forth motion at a unique frequency.
- Ideal: Mechanical Energy is conserved.
- Real: Friction present, Oscillation decays.

Fourier: Any finite periodic motion can be represented as a sum of SHM's.

Lecture 30 Blackboard #1

②

Oscillations

Periodic phenomena: time, pos, etc.

- planets, orbital motion
- pendulum clock
- mass-spring system
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- Ideal: Mechanical Energy is conserved.
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Fourier: Any finite periodic motion can be represented as a sum of SHM's.

③

Physics Problems

Many examples of small disp. from a condition of stable equilibrium results in SHM about the potential minimum.

Simple Harmonic Motion

$x(t) = A\cos(\omega t + \delta)$

$A, \omega, \delta \equiv$ constants.

$A \equiv$ amplitude of motion (m)

$\omega =$ angular frequency (π/s)

$\delta =$ phase angle.

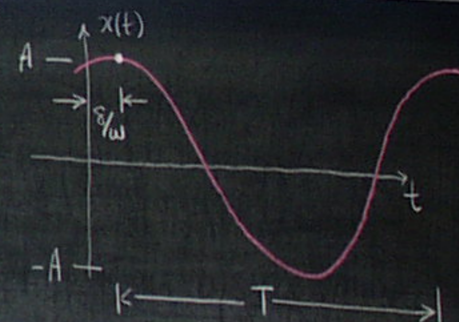
A and δ determined by initial values for $x(t=0) = ?$

$v(t=0) = ?$

$x(t)$ repeats itself when ωt changes by 2π

$$\omega t + \delta + 2\pi = \omega(t+T) + \delta$$

$T = \frac{2\pi}{\omega}$ (s) Period of Motion



Lecture 30 Blackboard #2

$f = \nu = \frac{1}{T} = \frac{\omega}{2\pi}$ (Hz)

$\omega = 2\pi f = \frac{2\pi}{T}$ (s^{-1})

Velocity of Particle:

$v(t) = \frac{dx}{dt} = -\omega A \sin(\omega t + \delta)$

$a(t) = \frac{dv}{dt} = -\omega^2 A \cos(\omega t + \delta)$

$= -\omega^2 x(t)$

$v_{max} = \pm \omega A$

$a_{max} = \pm \omega^2 A$

$x(t) = A\cos(\omega t + \delta)$

Example: $t = 0$ $x = x_0$, $v = v_0$

$x_0 = A\cos\delta$

$v_0 = -\omega A \sin\delta$

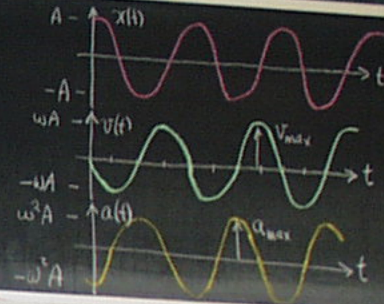
$\therefore \tan\delta = -v_0/\omega x_0$

$x_0^2 + \left(\frac{v_0}{\omega}\right)^2 = A^2(\cos^2\delta + \sin^2\delta) = A^2$

$A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2}$

If $x(t)$ is motion of a mass m: $F = ma(t) = -m\omega^2 x$

$\Rightarrow$ Linear Restoring Force.



④

Properties of SHM

$x(t) = A\cos(\omega t + \delta)$

1. Disp, vel, and acc all vary sinusoidally with time but are not in phase.
2. Accel prop to disp. but in opposite direction.
3. Frequency and period of the motion are indep. of amplitude.

Example: Mass-Spring System

Spring: k

No friction

$F(x) = -kx$ stretch & compress

$m\frac{d^2x}{dt^2} = -kx$ $(\vec{F} = m\vec{a})$

$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$ d. eq. of motion

Let $\frac{k}{m} = \omega^2$

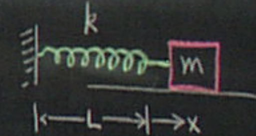
$\frac{d^2x}{dt^2} + \omega^2 x = 0$ Solve DE!

Let $x = A\cos(\omega t + \delta)$

$v = \frac{dx}{dt} = -\omega A \sin(\omega t + \delta)$

$a = -\omega^2 A\cos(\omega t + \delta)$

$= -\omega^2 x(t)$



⑤

Lecture 30 Blackboard #3

Properties of SHM

$x(t) = A\cos(\omega t + \delta)$

1. Disp, vel, and acc all vary sinusoidally with time but are not in phase.
2. Accel prop to disp. but in opposite direction.
3. Frequency and period of the motion are indep. of amplitude.

Example: Mass-Spring System

Spring: k

No friction

$F(x) = -kx$ stretch or compress

$m\dfrac{d^2x}{dt^2} = -kx \quad (\vec{F} = m\vec{a})$

$\dfrac{d^2x}{dt^2} + \dfrac{k}{m}x = 0$ d. eq. of motion

Let $\dfrac{k}{m} = \omega^2$

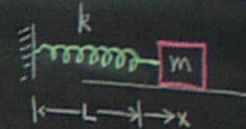
$\dfrac{d^2x}{dt^2} + \omega^2 x = 0$ Solve DE!

Let $x = A\cos(\omega t + \delta)$

$v = \dfrac{dx}{dt} = -\omega A\sin(\omega t + \delta)$

$a = -\omega^2 A\cos(\omega t + \delta)$

$= -\omega^2 x(t)$



⑤

$\dfrac{d^2x}{dt^2} + \omega^2 x = 0$

$-\omega^2 x + \omega^2 x = 0$ (OK)

$\omega = \sqrt{k/m}$

$T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{m}{k}}$

$f = \dfrac{1}{T} = \dfrac{1}{2\pi}\sqrt{k/m}$

Equivalent SHM Solutions

$x(t) = A\sin(\omega t + \delta)$

$x(t) = A\cos\omega t + B\sin\omega t$

2'nd Order Dif. Eq.

Must have 2 Constants.

Constants fixed by 2 initial conditions!!

Special Case: I

$t = 0 \quad x = A$

$v = 0$

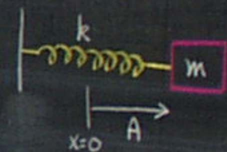
$x = A\cos(\omega t + \delta)$

$t = 0$: $A = A\cos\delta \Rightarrow \delta = 0$

$v = -\omega A\sin(\omega t + \delta)$

$t = 0 \quad v = 0$ OK

Solution: $\boxed{x(t) = A\cos(\omega t)}$



⑥

Lecture 30 Blackboard #4

Special Case II

$x=0$, $v=v_0$ } $t=0$

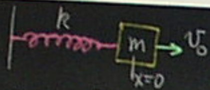
$0 = A\cos(0+\delta)$ ①

$v_0 = -\omega A\sin(0+\delta)$ ②

From ① $\delta = \pm\pi/2$

② $\delta = -\pi/2$

$A = v_0/\omega$



$\therefore x(t) = \frac{v_0}{\omega}\cos(\omega t - \frac{\pi}{2})$

$= \frac{v_0}{\omega}\sin\omega t$

$v(t) = v_0\cos\omega t$

$a(t) = -\omega v_0\sin\omega t$

Example: Mass + 2 Springs.

$-F_x - F_x = ma_x$

$-kx - kx = ma_x$

$-2kx = m\frac{d^2x}{dt^2}$

$\frac{d^2x}{dt^2} + \frac{2k}{m}x = 0$

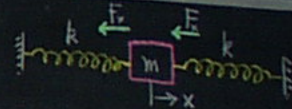
$x(t) = A\cos(\omega t+\delta)$

$\omega^2 = 2k/m$

$\omega = \sqrt{2k/m}$

$\nu = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$ Hz

$T = 2\pi\sqrt{\frac{m}{2k}}$ s



⑦

Energy in SHM

$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2A^2\sin^2(\omega t+\delta)$

$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t+\delta)$

let $\omega^2 = k/m$

$E = K+U = \frac{1}{2}kA^2[\sin^2(\ ) + \cos^2(\ )]$

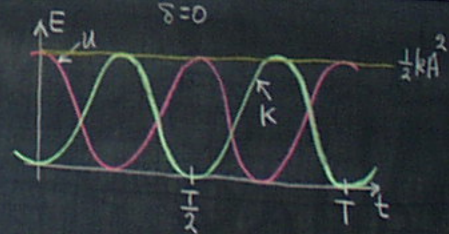
$\boxed{E = \frac{1}{2}kA^2} \equiv$ constant.

$E = K+U$

$= \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$

$v = \pm\sqrt{\frac{k}{m}(A^2-x^2)}$

$\delta = 0$



⑧

Lecture 30 Blackboard #5

Energy in SHM

⑧

$K = \frac{1}{2} m v^2 = \frac{1}{2} m \omega^2 A^2 \sin^2(\omega t + \delta)$

$U = \frac{1}{2} k x^2 = \frac{1}{2} k A^2 \cos^2(\omega t + \delta)$

Let $\omega^2 = k/m$

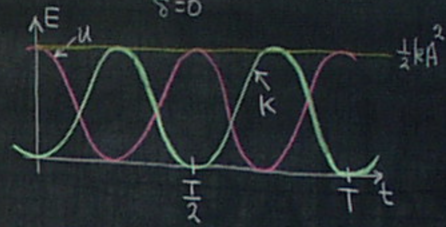
$E = K + U = \frac{1}{2} k A^2 \left[\sin^2(\ ) + \cos^2(\ )\right]$

$\boxed{E = \frac{1}{2} k A^2} \equiv$ constant.

$E = K + U$

$= \frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \frac{1}{2} k A^2$

$v = \pm \sqrt{\frac{k}{m}\left(A^2 - x^2\right)}$



Mass + Spring + Gravity

⑨

$m \frac{d^2 x}{dt^2} = -k(d+y) + mg$

At equil: $y = 0$, net force $= 0$.

$0 = mg - kd$

$\therefore kd = mg$

$m \frac{d^2 y}{dt^2} = -ky$

$\frac{d^2 y}{dt^2} + \frac{k}{m} y = 0$

$\omega^2 = k/m$

$y(t) = A \cos(\omega t + \delta)$