Summary Rotations and Rolling

$$\frac{d\vec{P}}{dt} = \Sigma \vec{F}_{ext}$$

$$K = \frac{1}{2} M V_{cm}^2 + \frac{1}{2} I_{cm}\omega^2$$

$$L = L_0 + L_s$$

$$= \vec{R}\times M\vec{V}_{cm} + I_{cm}\omega$$

Always True

$$\frac{d\vec{L}_s}{dt} = \vec{\tau}_{cm}$$

$$\vec{\tau}_s = I_s\alpha$$

①

Example: Rolling down Incline

- Release from rest
- No slipping / static friction
- No energy lost: $W_f = 0$

$$v_c = R\omega$$

$$K = \frac{1}{2} I_c \left(\frac{v_c}{R}\right)^2 + \frac{1}{2} M v_c^2$$

$$K = \frac{1}{2}\left[\frac{I_c}{R^2} + M\right] v_c^2$$

$$\Delta U = MgH$$

$\Delta K = \Delta U$ Cons of E

$$\frac{1}{2}\left[\frac{I_c}{R^2} + M\right] v_c^2 = MgH$$

$$v_c = \sqrt{\frac{2gH}{1 + I_c/MR^2}}$$

Sphere: $I_c = \frac{2}{5} MR^2$

$$v_c = \sqrt{\frac{2gH}{1 + \frac{2}{5}\frac{MR^2}{MR^2}}} = \sqrt{\frac{10}{7} gH}$$

x = distance rolled on plane

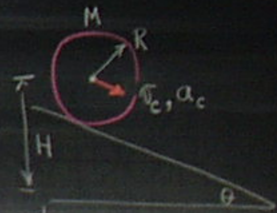
$$H = x\sin\theta$$

$v_c^2 = 2a_c x$ (cons. $\vec{a}$ prob)

$$\therefore a_c = \frac{5}{7} g\sin\theta$$

$$a_c < a_{sliding}$$

- Indep of M
- Indep of R
- All spheres same a_c!



②

Lecture 25 Blackboard #1

Example: Rolling down Incline

- Release from rest
- No slipping / static friction
- No energy lost: $W_f = 0$

$v_c = R\omega$

$K = \frac{1}{2} I_c \left(\frac{v_c}{R}\right)^2 + \frac{1}{2} M v_c^2$

$K = \frac{1}{2}\left[\frac{I_c}{R^2} + M\right] v_c^2$

$\Delta U = MgH$

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$v_c = \sqrt{\frac{2gH}{1 + I_c/MR^2}}$

Sphere: $I_c = \frac{2}{5} MR^2$

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x = distance rolled on plane

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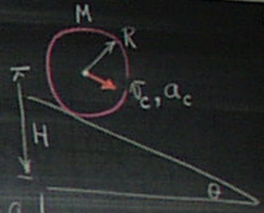
$v_c^2 = 2 a_c x$ (const. $\vec{a}$ prob)

$\therefore a_c = \frac{5}{7} g \sin\theta$

$a_c < a_{sliding}$

- Indep of M
- Indep of R
- All spheres same a_c!



②

Example: Rolling ⟷ Dynamics

Mass = M

Radius = R

$I = \beta MR^2$

$\sum \tau = I\alpha$ (About cm) ①

$\tau_f + \tau_{Mg} + \tau_N = Rf + 0 + 0 = I\alpha$ ②

$\sum F_x = Mg\sin\theta - f = Ma_{cm}$ ③

$v_{cm} = R\omega$ $a_{cm} = R\alpha$ Rolling Motion No Slipping

Sub for f

$Mg\sin\theta - \frac{I\alpha}{R} = Mg\sin\theta - \frac{\beta MR^2}{R}\frac{a_{cm}}{R}$

$Mg\sin\theta - \beta M a_{cm} = M a_{cm}$

$a_{cm} = \frac{g\sin\theta}{1+\beta}$

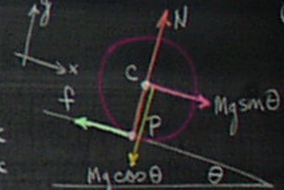
$f \leq \mu_s N$

$\frac{I\alpha}{R} = \frac{\beta MR^2}{R}\frac{1}{R}\frac{g\sin\theta}{1+\beta} \leq \mu Mg\cos\theta$

$\therefore \tan\theta \leq \mu \frac{1+\beta}{\beta}$

Cond on θ for no sliding!

If object slides: $\omega R \neq v_c$
$\alpha R \neq a_c$



③

Hoop: $\beta = 1$ $a_{Hoop} = \frac{1}{2} g\sin\theta$

Cylinder $\beta = 1/2$ $a_{cyl} = \frac{2}{3} g\sin\theta$

Sphere $\beta = 2/5$ $a_{sphere} = \frac{5}{7} g\sin\theta$

Lecture 25 Blackboard #2

Example: Instantaneous Rotation Axis: P

$\sum \tau_P = I_P \alpha$

$I_P = \beta M R^2 + M R^2 = (1+\beta) M R^2$

$\cancel{\tau_f} + \tau_{Mg} + \cancel{\tau_N} = I_P \alpha$

$Mg \sin\theta = I_P \alpha = (1+\beta) M R^2 \alpha$

$= (1+\beta) M R a_c$

$a_c = \dfrac{g \sin\theta}{1+\beta}$

= Same!!

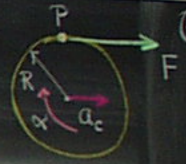
Example: Flat Disk

- Horizontal Frictionless surface
- Disk rotates and translates.

a) Accel of CM: $F = M a_c \quad a_c = \dfrac{F}{M} = \dfrac{5}{2} = 2.5\,m/s^2$

b) Ang Accel $\alpha = \dfrac{\tau_c}{I_c} = \dfrac{FR}{\frac{1}{2}MR^2} = \dfrac{2F}{MR} = \dfrac{2\times5}{2\times0.1} = 50\,rad/s^2$

$R\alpha \neq a_c$!!



④

M = 2 kg
R = 10 cm
F = 5 N

What is $\vec{a}$ of free end of string?

For point P: v_P = vel of cm + v_P due to rot.

$v_P = V_{cm} + R\omega$ (Not rolling!)

$a_P = \dfrac{dv_P}{dt} = a_c + R\alpha$

$= 2.5 + 0.1 \times 50$

$= 7.5\,m/s^2$

Example: Falling Cylinder

$\sum \tau_c = I_c \alpha$

$\cancel{\tau_{Mg}} + \tau_T = I_c \alpha$

① $2TR = \frac{1}{2} M R^2 \alpha$ (Rotation)

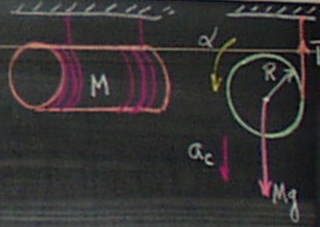
② $Mg - 2T = Ma$ (cm-Motion)

$a = R\alpha$ (String does not slip)

From ① $2T = Ma/2$.

Solve $a = \frac{2}{3} g$

$T = \dfrac{Mg}{6}$



Lecture 25 Blackboard #3

What is a of free end of string?

For point P: v_P = vel of cm + v_{el} due to rot.

$v_p = V_{cm} + R\omega$ (Not rolling!)

$a_p = \frac{dv_p}{dt} = a_c + R\alpha$

$= 2.5 + 0.1 \times 50$

$= 7.5\, m/s^2$

Example: Falling Cylinder

$\sum \tau_c = I_c \alpha$

$\cancel{\tau_{Mg}} + \tau_T = I_c \alpha$

① $2TR = \frac{1}{2} M R^2 \alpha$ (Rotation)

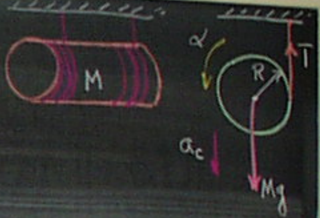
② $Mg - 2T = Ma$ (cm-Motion)

$a = R\alpha$ (String does not slip)

From ① $2T = Ma/2$

Solve $a = \frac{2}{3} g$

$T = \frac{Mg}{6}$



Example: Student + Plank on Ice

M = 50kg   2b = 5m

m = 70kg   v = 3m/s

Student jumps on plank.

Describe motion at t = 1.2s

Initial System: Plank / Student.

Final System: [Plank+Stud] Rigid M.

$\vec{P}_i = \vec{P}_f$ [No friction, no ext. forces)

$mv = (m+M)V$

$V_{cm} = \frac{mv}{m+M} = \frac{70 \times 3}{70+50} = 1.75\, m/s$

cm: $(m+M)d = Mb$

$d = \frac{Mb}{m+M} = \frac{50 \times 2.5}{70+50} = 1.04\, m$

$\sum \vec{\tau}_{ext} = 0 \quad \therefore \vec{L}_i = \vec{L}_f$

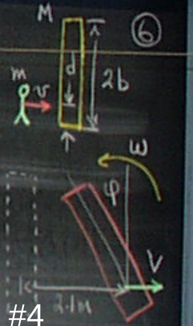
$mvd = I_{cm}\omega = \left[md^2 + M\left(\frac{1}{3}b^2 + (b-d)^2\right)\right]\omega$

$\omega = \frac{mvd}{I} = 0.762\, \frac{rad}{s} = 46.3°/s$

$\Delta t = 1.2s$

$\varphi = \omega \Delta t = 52.9°$

$V\Delta t = 2.10\, m$



Lecture 25 Blackboard #4

Gyroscope Motion

(7)

Support at origin 'O'

Precessing ??

$N = Mg$ (no vertical motion)

$\tau_0 = \vec{R} \times \vec{F} = mgl \sin\gamma$

Direction $\perp$ to $\vec{\omega}_s$ and $\vec{Mg}$

Normal to plane of $\vec{\omega}_s$ and $\vec{\omega}_p = \vec{\Omega}$

Assume $\vec{\omega}_p \ll \vec{\omega}_s$

$L_s = I\omega_s$

$$\frac{d\vec{L}_s}{dt} = \vec{\tau}_0$$

$$d\vec{L}_s = \vec{\tau}_0\, dt = (Mgl \sin\gamma)\, dt$$

Angle $d\theta$ through which spin axis swings in time dt:

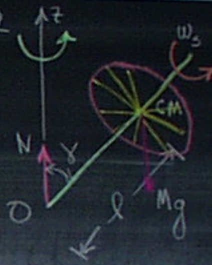
$$d\theta = \frac{dL_s}{L_s \sin\gamma} = \frac{Mgl \sin\gamma\, dt}{L_s \sin\gamma} \qquad \omega_p = \Omega$$

$$\Omega = \omega_p = \frac{d\theta}{dt} = \frac{Mgl}{L_s} \quad : \quad \underline{\omega_p \text{ indep of } \gamma}$$

$$\omega_p = \frac{Mgl}{L_s}\,\frac{\sin\gamma}{\sin\gamma}$$

$$\omega_p L_s \sin\gamma = Mgl \sin\gamma$$

$$\boxed{\vec{\omega}_p \times \vec{L}_s = \vec{\tau}_0}$$



Gyroscope Dynamics

(8)

Torque applied for short time Δt:

$\Sigma[(\vec{F}) + (-\vec{F})] \equiv 0$ CM stays constant.

$\Delta\vec{p} = m\Delta\vec{v} = \vec{F}\Delta t$ Impulse

$\Delta\vec{v} \perp \vec{v}_0$

Axis of rotation tilts:

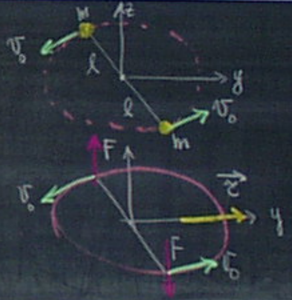
$$\Delta\varphi \approx \frac{\Delta v}{v_0} = \frac{F\Delta t}{m v_0}$$

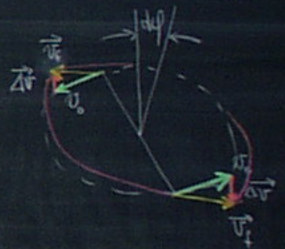
Torque: $\tau = 2Fl$

$L_s = 2 m v_0 l$

$$\therefore \Delta\varphi = \frac{F\Delta t}{m v_0} = \frac{2lF\Delta t}{2lm v_0} = \frac{\tau\Delta t}{L_s}$$

$$\Omega = \frac{\Delta\varphi}{\Delta t} = \frac{\tau}{L_s} \quad \text{Precession!!!}$$





Lecture 25 Blackboard #5