Example: Flat Car / Conveyor Belt

Constant Speed = $\vec{v}_0$

Force $\vec{F}$ = ?

$$\vec{F} = \frac{d\vec{P}}{dt} = \frac{d}{dt}(m\vec{v}) = \cancel{m\frac{d\vec{v}}{dt}} + \vec{v}\frac{dm}{dt}$$

$$F = v_0 \frac{dM}{dt} = 2.2 \times 75 = 165\ \text{N}$$

$\frac{dM}{dt} = 75$ kg/s

$v_0 = 2.2$ m/s

Power: $P = \frac{dW}{dt} = FV_0 = V_0^2 \frac{dM}{dt} = 2\frac{d}{dt}\left(\frac{1}{2}MV_0^2\right)$

P = 2× Rate at which KE of Flatcar is increasing !!! Why?

Impulse

(1)

Collisions: Short-Time ⇒ Impulsive forces

Impulse: Large change in motion.

Other forces: g, etc small changes / Ignore.

$\frac{d\vec{P}}{dt} = \vec{F}$ Newton's 2nd Law.

$\therefore d\vec{p} = \vec{F}\,dt$ Change in $\vec{P} \sim \vec{F} \times dt$

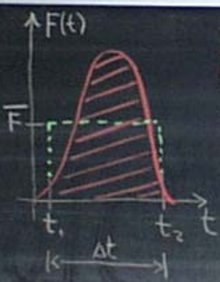
(2)

$$\Delta\vec{p} = \vec{P}_f - \vec{P}_i = \int_{P_i}^{P_f} d\vec{p} = \int_{t_1}^{t_2} \vec{F}\,dt$$

⇒ Area under curve F(t) from $t_1 \rightarrow t_2$

$$\vec{I} = \int_{t_1}^{t_2} \vec{F}(t)\,dt \equiv \text{Impulse.}$$

$$\Delta\vec{p} = \vec{P}_f - \vec{P}_i = \vec{I} \equiv \text{Change in } \vec{P}$$



Average $\vec{F}$

Let $\Delta t = t_2 - t_1$

$$\bar{\vec{F}} = \frac{1}{\Delta t}\int_{t_1}^{t_2} \vec{F}(t)\,dt$$

$$\bar{\vec{F}}\Delta t = \vec{P}_f - \vec{P}_i = \vec{I}$$

Example

$$\vec{I} = \Delta\vec{p} = \int_{t_1}^{t_2} F(t)\,dt$$

$$= \frac{1}{2}\times 4\times 2 + 4\times 1 + \frac{1}{2}\times 4\times 2$$

$\vec{I}$ = 12 N·s [In direction of $\vec{F}$]

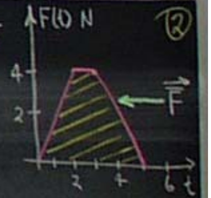
$\vec{I} = \vec{P}_f - \vec{P}_i$ = 12 N.s

$\vec{P}_i = 0$, $\vec{v}_i = 0$

$m v_f$ = 12 N.s

$v_f = \frac{12}{2} = 6$ m/s

$$\bar{\vec{F}} = \frac{1}{\Delta t}\int_{t_1}^{t_2} F(t)\,dt = \frac{12\ \text{Ns}}{5\ \text{s}} = 2.4\ \text{N}$$



m = 2 kg

$v_i = 0$ at $t = 0$

$K_{INT} = 0$ CM Motion Only.

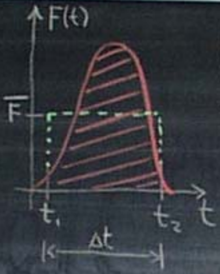
Lecture 19, Blackboard #1

$$\overrightarrow{\Delta p} = \vec{p}_f - \vec{p}_i = \int_{p_i}^{p_f} d\vec{p} = \int_{t_1}^{t_2} \vec{F}\,dt$$

⇒ Area under curve $F(t)$ from $t_1 \to t_2$

$$\vec{I} = \int_{t_1}^{t_2} \vec{F}(t)\,dt \equiv \text{Impulse.}$$

$$\overrightarrow{\Delta p} = \vec{p}_f - \vec{p}_i = \vec{I} \equiv \text{Change in } \vec{p}$$



Average $\vec{F}$

Let $\Delta t = t_2 - t_1$

$$\bar{\vec{F}} = \frac{1}{\Delta t}\int_{t_1}^{t_2} \vec{F}(t)\,dt$$

$$\bar{\vec{F}}\,\Delta t = \vec{p}_f - \vec{p}_i = \vec{I}$$

Example

$$\vec{I} = \overrightarrow{\Delta p} = \int_{t_1}^{t_2} \vec{F}(t)\,dt$$

$$= \tfrac{1}{2}\times 4\times 2 + 4\times 1 + \tfrac{1}{2}\times 4\times 2$$

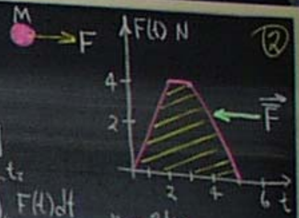
$\vec{I} = 12\ \text{N}\cdot\text{s}$ [In direction of $\vec{F}$]

$\vec{I} = \vec{p}_f - \vec{p}_i = 12\ \text{N.s}$

$\vec{p}_i = 0\quad \vec{v}_i = 0\qquad m v_f = 12\ \text{N.s}$

$v_f = \frac{12}{2} = 6\ \text{m/s}$

$$\bar{\vec{F}} = \frac{1}{\Delta t}\int_{t_1}^{t_2} F(t)\,dt = \frac{12\ \text{Ns}}{5\ \text{s}} = 2.4\ \text{N}$$



$m = 2\,\text{kg}$

$v_i = 0$ at $t = 0$

②

Collisions

I. Elastic

- Interactive forces all conservative
- Momentum is conserved: $\boxed{\vec{p}_i = \vec{p}_f}$
- Total KE is the same before and after. Conserved! $\boxed{K_i = K_f}$

II. Inelastic Collisions:

- Involves non-conservative forces.
- Momentum is conserved:
  If $\sum F_{ext} = 0$ $\quad \boxed{P_i = P_f}$
- KE is not conserved: $K_f < K_i$
- Complicated!!
- Perfectly Inelastic: Special Case
  $K_{INT} \equiv 0$ CM Motion Only.

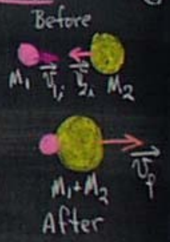
Perfectly Inelastic: Sticking Collision – 1D

1- Dimension

$\vec{p}_i = \vec{p}_f$ Conserved $\sum \vec{F}_{ext} \equiv 0.$

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f$$

$$\boxed{\vec{v}_f = \frac{m_1\vec{v}_{1i} + m_2\vec{v}_{2i}}{m_1 + m_2}}$$



③

Lecture 19, Blackboard #2

Example: Body-2 initially at rest.

$v_{2i}=0 \qquad v_f = \frac{m_1}{m_1+m_2} v_{1i}$

$K_i = \frac{1}{2} m_1 v_{1i}^2$

$K_f = \frac{1}{2}(m_1+m_2)\frac{m_1^2}{(m_1+m_2)^2} v_{1i}^2 = \frac{1}{2}(m_1+m_2) V_{cm}^2$

$\frac{K_f}{K_i} = \frac{m_1}{m_1+m_2} < 1$ Final KE is less than initial KE!!

KE is LOST!

Example

$\vec{P}_i = \vec{P}_f$

"Sticking"

$5(2)-3(2) = (5+3)\vec{v}_f$

$v_f = \frac{10-6}{8} = +0.5$ m/s

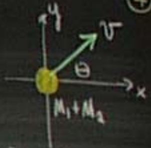
$K_i = \frac{1}{2} m_A v_{Ai}^2 + \frac{1}{2} m_B v_{Bi}^2 = \frac{1}{2}\times5\times2^2 + \frac{1}{2}\times3\times2^2 = 16$ J

$K_f = \frac{1}{2}(m_A+m_B) v_f^2 = \frac{1}{2}(5+3)\left(\frac{1}{2}\right)^2 = 1$ J

Sticking Collisions: 2D

④

Before



- Linear Momentum Conserved: $\vec{P}_i = \vec{P}_f$
- $\Sigma F_{ext} = 0$
- Two equations: One for each component.

⑤

$P_{xi} = P_{xf}$ and $P_{yi} = P_{yf}$

$m_1 v_1 = (m_1+m_2) v \cos\theta$ x-axis ①

$m_2 v_2 = (m_1+m_2) v \sin\theta$ y-axis ②

②/① $\tan\theta = \frac{m_2}{m_1}\frac{v_2}{v_1}$

From ② $v = \frac{m_2}{m_1+m_2}\frac{v_2}{\sin\theta}$

$m_1 = 70$ kg $\quad v_1 = 2$ m/s

$m_2 = 50$ kg $\quad v_2 = 3$ m/s

$\tan\theta = \frac{50}{70}\times\frac{3}{2} \Rightarrow \theta = 47°$

$v = \frac{50}{50+70}\times\frac{3}{\sin 47°} = 1.71$ m/s $= V_{cm}$

$K_i = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

$= \frac{1}{2}\times70\times2^2 + \frac{1}{2}\times50\times3^2 = 365$ J

$K_f = \frac{1}{2}(m_1+m_2) v^2 = \frac{1}{2}(50+70)\times1.71^2 = 175$ J

$\Delta K = K_f - K_i = 175 - 365 = -190$ J

$\longrightarrow$ Lost to Friction/Heat

$K_{INT} \equiv 0 \qquad K_f = \frac{1}{2}(m_1+m_2) v_{cm}^2$

Use cons. of $\vec{P}$ and rel. velocities to solve problems!!

Lecture 19, Blackboard #3

⑤

$P_{xi} = P_{xf}$ and $P_{yi} = P_{yf}$

$m_1 v_1 = (m_1 + m_2) v \cos\theta$ x-axis ①

$m_2 v_2 = (m_1 + m_2) v \sin\theta$ y-axis ②

②/① $\tan\theta = \dfrac{m_2}{m_1} \dfrac{v_2}{v_1}$

From ② $v = \dfrac{m_2}{m_1 + m_2} \dfrac{v_2}{\sin\theta}$

$m_1 = 70\,kg$ $v_1 = 2\,m/s$

$m_2 = 50\,kg$ $v_2 = 3\,m/s$

$\tan\theta = \dfrac{50}{70} \times \dfrac{3}{2} \Rightarrow \theta = 47^\circ$

$v = \dfrac{50}{50+70} \times \dfrac{3}{\sin 47^\circ} = 1.71\,m/s = v_{CM}.$

$K_i = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

$= \frac{1}{2} \times 70 \times 2^2 + \frac{1}{2} \times 50 \times 3^2 = 365\,J$

$K_f = \frac{1}{2}(m_1 + m_2) v^2 = \frac{1}{2}(50+70) \times 1.71^2 = 175\,J$

$\Delta K = K_f - K_i = 175 - 365 = -190\,J$

$\longrightarrow$ Lost to Friction/Heat

$K_{INT} \equiv 0$ $K_f = \frac{1}{2}(m_1 + m_2) v_{CM}^2$

⑥

Elastic Collisions : 1D

- Total E is conserved
- Total Linear Momentum is Conserved.

Cons of $\vec{P}$: $m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{v}_1{}' + m_2 \vec{v}_2{}'$ ①

Cons of E: $\frac{1}{2} m_1 \vec{v}_1^{\,2} + \frac{1}{2} m_2 \vec{v}_2^{\,2} = \frac{1}{2} m_1 \vec{v}_1{}'^2 + \frac{1}{2} m_2 \vec{v}_2{}'^2$ ②

Given $(m_1, m_2, \vec{v}_1, \vec{v}_2) \Longrightarrow (v_1'$ and $v_2')$

From ①: $m_1(\vec{v}_1 - \vec{v}_1{}') = m_2(\vec{v}_2{}' - \vec{v}_2)$ ③

From ② $m_1(v_1^2 - v_1'^2) = m_2(v_2'^2 - v_2^2)$ ④

④/③ $\vec{v}_1 + \vec{v}_1{}' = \vec{v}_2{}' + \vec{v}_2$

OR $\boxed{-(\vec{v}_2 - \vec{v}_1) = +(v_2' - v_1')}$ ⑤

$\vec{v}_1 - \vec{v}_2 = -(v_1' - v_2')$

Relative velocities after collision of the two objects is the negative of the relative velocities before the collision.

Use cons of $\vec{P}$ and rel. velocities to solve problems!!

Lecture 19, Blackboard #4

Special Case: Equal Masses; $m_1 = m_2$

$\vec{P}_i = \vec{P}_f$: $\vec{v}_1 + \vec{v}_2 = \vec{v}_1' + \vec{v}_2'$

Rel v's : $\vec{v}_1 - \vec{v}_2 = \vec{v}_2' - \vec{v}_1'$

Solve $\vec{v}_2' = \vec{v}_1$ and $\vec{v}_1' = \vec{v}_2$

Particles exchange velocities

If $v_2 = 0$ (M_2 at rest) $v_2' = v_1$ $v_1' = 0$

General Solution: Particle-2 at Rest.

Momentum : $m_1 v_1 = m_1 v_1' + m_2 v_2'$ ①

Rel Vel's : $v_1 = -v_1' + v_2'$ ②

Solve: $\boxed{v_2' = v_1 \dfrac{2m_1}{m_1 + m_2}}$ $\boxed{v_1' = v_1 \dfrac{(m_1 - m_2)}{m_1 + m_2}}$

i) $v_2 = 0$ $m_1 = m_2$

$\left.\begin{array}{l} v_2' = v_1 \\ v_1' = 0 \end{array}\right\}$ Same as Before.

⑦

Examples:

$m_2 = 2m_1$; $v_2 = 0$

$v_2' = v_1 \dfrac{2m}{m + 2m} = \dfrac{2}{3} v_1$

$v_1' = v_1 \dfrac{(m - 2m)}{m + 2m} = -\dfrac{1}{3} v_1$

$m_1 = 2m_2$

$v_2' = v_1 \left(\dfrac{4m}{2m + m}\right) = \dfrac{4}{3} v_1$

$v_1' = v_1 \left(\dfrac{2m - m}{2m + m}\right) = \dfrac{1}{3} v_1$

⑧

of heavy object.

and moves with same speed.

Lecture 19, Blackboard #5

Examples:

$m_2 = 2m_1$; $v_2 = 0$

$$v_2' = v_1 \frac{2m}{m+2m} = \frac{2}{3} v_1$$

$$v_1' = v_1 \frac{(m-2m)}{m+2m} = -\frac{1}{3} v_1$$

$m_1 = 2m_2$

$$v_2' = v_1 \left(\frac{4m}{2m+m}\right) = \frac{4}{3} v_1$$

$$v_1' = v_1 \left(\frac{2m-m}{2m+m}\right) = \frac{1}{3} v_1$$

(8)

ii) $v_2 = 0$ $m_1 \gg m_2$

· A heavy object strikes a light object at rest.

$$v_2' \simeq 2v_1$$

$$v_1' \approx v_1$$

· Velocity of incoming particle unchanged.

· Light particle moves with twice velocity of heavy object.

$v_2 = 0$ $m_1 \ll m_2$

A moving light particle strikes a stationary heavy object.

$$v_2' \sim 0$$

$$v_1' \sim -v_1$$

Heavy object stays at rest.

Light particle reverses direction and moves with same speed.

General Solution

$$v_2' = v_1 \left(\frac{2m_1}{m_1+m_2}\right) + v_2 \left(\frac{m_2-m_1}{m_1+m_2}\right)$$

$$v_1' = v_1 \left(\frac{m_1-m_2}{m_1+m_2}\right) + v_2 \left(\frac{2m_2}{m_1+m_2}\right)$$

Forget It!!!

(9)

Lecture 19, Blackboard #6