Energy Curves:

Given $U(x)$
Want $x(t)$
Assume conservative forces only
Then $E = K + U \equiv \text{constant}$
$= \frac{1}{2}mv^2 + U(x)$

$$E = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + U(x)$$

$$v_x = \frac{dx}{dt} = \sqrt{\frac{2}{m}\left[E - U(x)\right]}$$

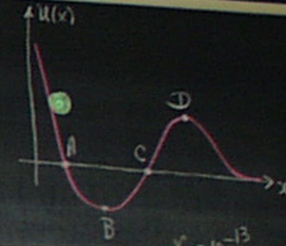
$$\int_{x'=x_0}^{x} \frac{dx'}{\sqrt{\frac{2}{m}\left[E - U(x')\right]}} = \int_{t=0}^{t} dt'$$

where $x' = x_0$ at $t' = 0$

Not easy to solve
Computers, numerically!

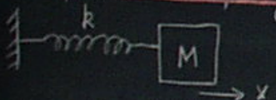
Example: Alpha Particle + Au Nucleus

1. x_B, x_D: $\frac{\partial U}{\partial x} = 0$ $\quad F(x) = 0$
2. $\frac{\partial U}{\partial x} \to 0$ as $x \to \infty$ $\quad F(x) \to 0$
3. $F(x) > 0$ $\quad 0 < x < x_B$ Repulsive Force.
4. $F(x) < 0$ $\quad x_B < x < x_D$ Attractive Force



①

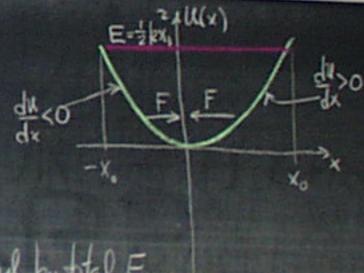
Example Mass on Spring

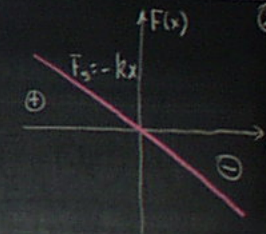


$E = K + U = \frac{1}{2}kx_0^2$ Initial Total E
$= \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kx_0^2$

i) $x > 0$ $\quad \frac{\partial U}{\partial x} = kx > 0$ $\quad F < 0$ Attractive
ii) $x < 0$ $\quad \frac{\partial U}{\partial x} = -kx < 0$ $\quad F > 0$ Attractive
iii) $x = 0$ $\quad \frac{\partial U}{\partial x} = 0$ $\quad F = 0$ Neutral

Harmonic Oscillator!
Motion bounded
Moves between turning points determined by total E.





②

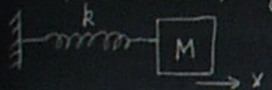
Unbounded $x_4 < x < \infty$

$\rightarrow x$ $\quad \frac{\partial^2 U}{\partial x^2} = 0$ Neutral

Lecture 16, Blackboard #1

2

Example Mass on Spring



$$E = K + U = \tfrac{1}{2} k x_0^2 \quad \text{Initial Total } E$$

$$= \tfrac{1}{2} m v^2 + \tfrac{1}{2} k x^2 = \tfrac{1}{2} k x_0^2$$

i) $x > 0$ $\quad \frac{\partial U}{\partial x} = kx > 0$ $\quad F < 0$ Attractive

ii) $x < 0$ $\quad \frac{\partial U}{\partial x} = -kx < 0$ $\quad F > 0$ Attractive

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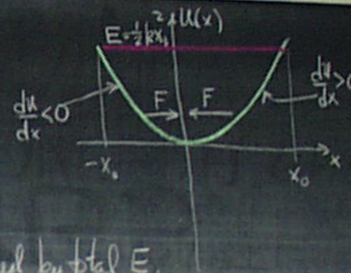
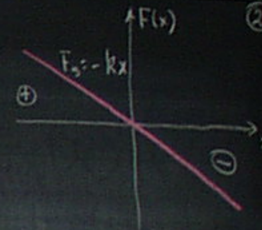




Harmonic Oscillator!

Motion bounded

Moves between turning points determined by total E.

3

Example: Potential Well

$$v_x = \sqrt{\frac{2}{m}\left[E - U(x)\right]}$$

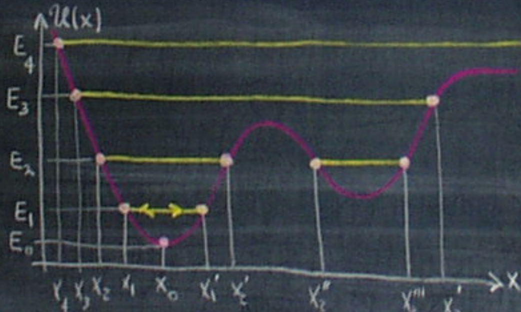
i) $E = E_0$ Fixed pos. $K = 0$

ii) $E = E_1$ Oscillates $x_1 < x < x_1'$

iii) $E = E_2$ 4 turning points

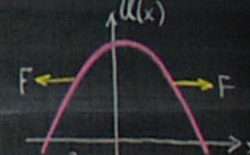
iv $E = E_4$ 1-turning point

Unbounded $x_4 < x < \infty$

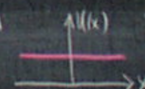


Equilibrium/Stability

$\frac{\partial^2 U}{\partial x^2} > 0$ Pot. Minimum Stable!



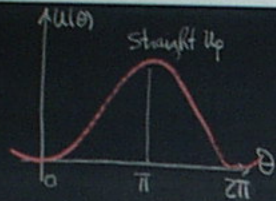
$\frac{\partial^2 U}{\partial x^2} < 0$ Pot. Max Unstable



$\frac{\partial^2 U}{\partial x^2} = 0$ Neutral.

Lecture 16, Blackboard #2

Pendulum



$U(\theta) = mgL(1-\cos\theta)$

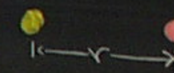
$U(0) = 0 \qquad U(\pi) = 2mgL$

$\theta = 0 \quad \frac{\partial^2 U}{\partial\theta^2} > 0$ Stable

$\theta = \pi \quad \frac{\partial^2 U}{\partial\theta^2} < 0$ Unstable

Example Diatomic Molecule

Leonard-Jones Potential (6,12)



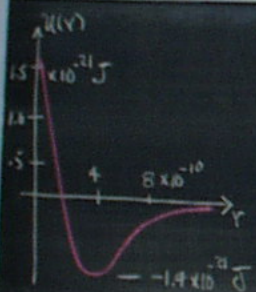
$U(r) = U_0\left[\left(\frac{a}{r}\right)^{12} - \left(\frac{a}{r}\right)^{6}\right]$

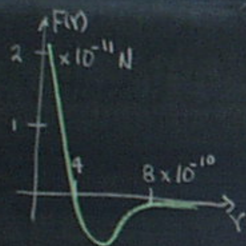
$F(r) = -\frac{dU}{dr} = \frac{6U_0}{a}\left[2\left(\frac{a}{r}\right)^{13} - \left(\frac{a}{r}\right)^{7}\right]$

$\left.\begin{array}{l} U_0 = 5.6\times10^{-21}\ J \\ a = 3.5\times10^{-10}\ m \end{array}\right\} O_2$

At equil: $F(r) = 0 \Rightarrow \left(\frac{a}{r_0}\right)^6 = \frac{1}{2} \Rightarrow r_0 = 3.9\times10^{-10}\ m$

④





Example: Repulsive Square Law Force

$F(x) = \frac{A}{x^2} \qquad A > 0$

$U(x) = \frac{A}{x}$

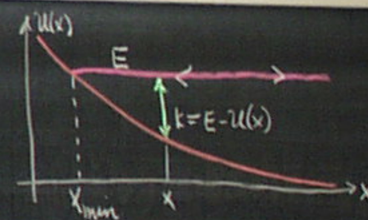
Like two equal charges.

Assume total energy is E

Then minimum x:

$x_{min} = \frac{A}{U} = \frac{A}{E}$

i.e. $U(x_{min}) = E$



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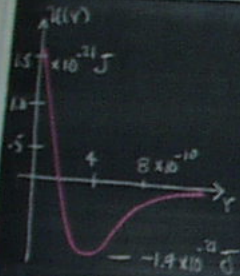
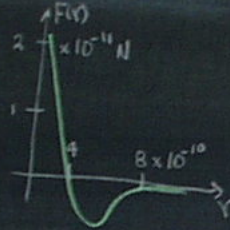
Lecture 16, Blackboard #3





Example: Repulsive Square Law Force

$F(x) = \dfrac{A}{x^2}$ $\quad A>0$

$U(x) = \dfrac{A}{x}$

Like two equal charges.

Assume total energy is E

Then minimum x:

$x_{min} = \dfrac{A}{U} = \dfrac{A}{E}$

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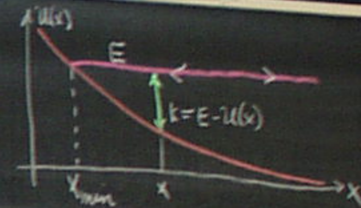

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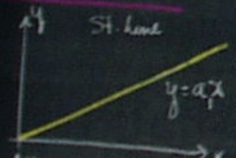
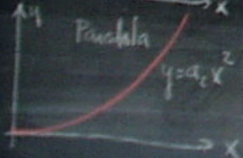
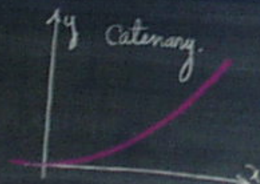
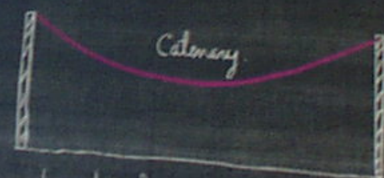
Least Time:







$$y = \frac{1}{2a}\left(e^{ax} + e^{-ax}\right)$$
$$= \frac{1}{a}\cosh ax$$



- Lowest c-of-g.
- Smallest PE
- Nature shapes curve to minimize PE

⑥

Lecture 16, Blackboard #4

Power : Time Rate of Doing Work

$\bar{P} = \frac{\Delta W}{\Delta t}$ Average Power

$P(t) = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$ Inst. Power.

$[P] = J/s =$ Watts.

$1\,HP = 550\ ft \cdot lb/s = 745.7\,W$

Power ⟷ Force

$dW = \vec{F} \cdot d\vec{r}$

$P = \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$ $[\vec{F} = \text{const.}]$

Power Depends on Reference Frame.
Since velocity depends on frame.

Energy ⟷ Power

⑦

$E = \int_{t_1}^{t_2} P(t)\,dt = Pt$ for constant P.

$1\,kWh = 3.6 \times 10^6\,J = 1000\,W$ for 1-hour.

$1\,kcalorie - 4.19 \times 10^3\,J$ (fuels)

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⑧

Example: Car up Incline

· Moves at constant $\vec{v}$

$F - f + mg\sin\theta = 0$ $[a = 0]$

$F = 700 + 1400 \times 9.8 \times \sin 10^\circ$

$= 3100\,N$

$P = \vec{F} \cdot \vec{v}$ $[\vec{F} \parallel \vec{v}]$

$= Fv$

$= 3100 \times 22$

$= 6.8 \times 10^4\,W$

$= 91\,HP$

$M = 1400\,kg$
$v = 22\,m/s$ (80 km/h)
$f = 700\,N$ friction

Example: Car up Incline

· Moves at constant $\vec{v}$

$F - f + mg\sin\theta = 0 \quad [a=0]$

$F = 700 + 1400 \times 9.8 \times \sin 10^\circ$

$= 3100\,N$

$P = \vec{F}\cdot\vec{v} \quad [\vec{F} \parallel \vec{v}]$

$= Fv$

$= 3100 \times 22$

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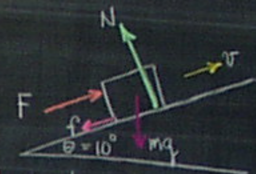
$= 91\,HP$



$M = 1400\,kg$

$v = 22\ m/s \quad (80\,km/h)$

$f = 700\,N$ friction

⑧

Example: Elevator

$T - f - Mg = 0$

$T = f + Mg = 4\times10^3 + 1.8\times10^3 \times 9.81$

$= 2.16 \times 10^4\ N$

$P = \vec{T}\cdot\vec{v} = Tv = 2.16\times10^4 \times 3$

$= 6.48 \times 10^4\ W = 86.9\,HP.$

Suppose $a = 1.0\,m/s^2$

$T - f - Mg = ma$

$T = M(a+g) + f$

$= 2.34 \times 10^4\ N$

$P = Tv$

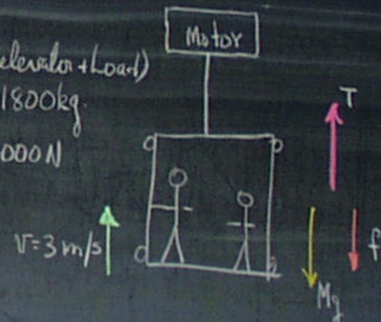
$= 2.34\times10^4\ v$

Power increases as speed increases!

M(elevator + Load)

$= 1800\,kg$

$f = 4000\,N$



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Lecture 16, Blackboard #6