Dynamics of Uniform Circular Motion

Speed = v, mass = m

Radius = R

$a_c = \frac{v^2}{R} = \omega^2 R$

$F_c = m\frac{v^2}{R} = m\omega^2 R$

If mass m moves in a circle with a speed v, the force F_c must be provided by some agent

Motion in a Vertical Circle

- Circular motion
- Non-uniform; speed increases/decreases
- v changes, energy/next week
- ∴ have $a_{\parallel}$ and $a_{\perp}$

$F_{\parallel} = mg\sin\theta$

$F_{\perp} = T - mg\cos\theta$

$a_{\parallel} = \frac{F_{\parallel}}{m} = g\sin\theta$

$a_{\perp} = \frac{F_{\perp}}{m} = T - mg\cos\theta = \frac{mv^2}{R}$

①

$T = m\left(\frac{v^2}{R} + g\cos\theta\right)$

Lowest Point $\theta = 0°$

$F_{\parallel} = 0$; $a_{\parallel} = 0$

Acceleration is only radial

$T = m\left(\frac{v^2}{R} + g\right)$

Highest Point: $\theta = \pi$

$T = m\left(\frac{v^2}{R} - g\right)$

Let $v = v_c$ when $T = 0$

$0 = m\left(\frac{v_c^2}{R} - g\right)$

$v_c = \sqrt{Rg}$

lowest allowed speed!

$\theta = 90°$

$a_{\parallel} = g$

$a_{\perp} = v^2/R$

Example: Loop-the-Loop

$m = 75\,kg$

$R = 10\,m$

$v = 15\,m/s$ const.

②

Lecture 12, Blackboard #1

$T = m\left(\frac{v^2}{R} + g\cos\theta\right)$

Lowest Point: $\theta = 0^\circ$

$F_{\parallel} = 0$; $a_{\parallel} = 0$

Acceleration is only radial

$T = m\left(\frac{v^2}{R} + g\right)$

Highest Point: $\theta = \pi$

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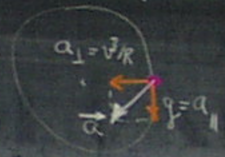
Lowest allowed speed!

$\theta = 90^\circ$

$a_{\parallel} = g$

$a_{\perp} = v^2/R$



Example: Loop-the-Loop

$m = 75\,kg$

$R = 10\,m$

$v = 15\,m/s$ const.

(2)

Bottom

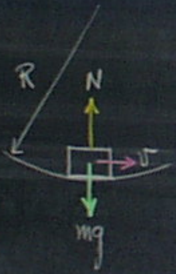
$N - mg = \frac{mv^2}{R}$

$N = m\left(g + \frac{v^2}{R}\right)$

$= mg\left(1 + \frac{v^2}{gR}\right)$

$= mg\left(1 + \frac{1}{9.81} \times \frac{15^2}{10}\right)$

$= mg\,(3.29)$



Top

$N + mg = \frac{mv^2}{R}$

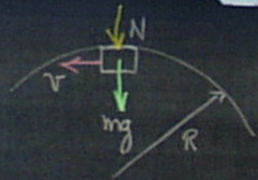
$N = mg\left(\frac{v^2}{Rg} - 1\right)$

$= mg\,(2.29 - 1)$

$= 1.29\,mg$

Suppose $\frac{v^2}{Rg} = 1 \Rightarrow v = \sqrt{Rg} = \sqrt{10 \times 9.81}$

$= 9.9\,m/s$



For $v = \sqrt{Rg}$

$N = 0$ Weight-less!

If $v < \sqrt{Rg}$??

What happens?

(3)

Lecture 12, Blackboard #2

Work and Energy

Forces → acceleration
↳ velocity and position

Force × Displacement = Work
Mass × Velocity^2 = Kinetic Energy
Work + KE = Work-Energy Principle
⇒ Analagous to Newton's 2nd Law.

Work-Energy Principle → Cons of Energy.
Force × ΔTime = Impulse → Cons of Momentum.
Force × Position = Torque → Cons. of Ang. Mom

Cons Laws ⟷ Symmetry
Mom ⟷ Inv. Transf in Space
Energy ⟷ Inv. Transf in Time
Ang. Mom ⟷ Inv. to Rotations

Work - 1D

- Constant Force

$W = F_x \, \Delta x$

- Δx: displacement
- F_x: Force on particle
- W: Work done

$[W] = N \cdot m = \underline{J}oules.$

If:
W>0 Force + Disp. same dir.
W<0 Force + Disp. Opposite
W<0 Work done by particle!!

⑤

Conical Pendulum

$\Sigma F_x = T\sin\theta = \frac{mv^2}{R}$ ①

$\Sigma F_y = T\cos\theta - mg = 0$ ②

$\tan\theta = v^2/Rg$

$R = L\sin\theta$

$v = \frac{2\pi L\sin\theta}{\tau}$

τ = period of rotation

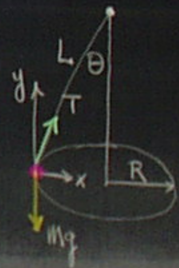
Eq ③

$\cos\theta = \frac{g\tau^2}{4\pi^2 L}$

$\tau = 2\pi\sqrt{\frac{L}{g}\cos\theta}$

$L = 1.0\,m$

$R = 0.57\,m$



④

Lecture 12, Blackboard #3

Work and Energy

Force → acceleration
  ↳ velocity and position

Force × Displacement = Work
Mass × Velocity^2 = Kinetic Energy
Work + KE = Work-Energy Principle
⇒ Analogous to Newton's 2nd Law.

Work-Energy Principle → Cons. of Energy
Force × ΔTime = Impulse → Cons. of Momentum.
Force × Position = Torque → Cons. of Ang. Mom.

Cons. Laws ⟷ Symmetry

Mom ⟷ Inv. Transf. in Space
Energy ⟷ Inv. Transf. in Time
Ang. Mom ⟷ Inv. to Rotations

Work - 1D

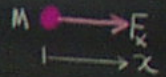
(5)

- Constant Force

$W = F_x \, \Delta x$

- Δx: displacement
- F_x: Force on particle
- W: Work done

$[W] = N \cdot m = $ Joules.

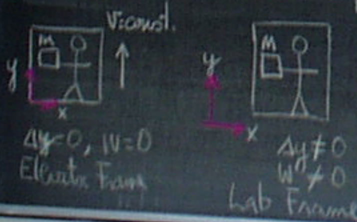


If:
W>0 Force+Disp. same dir.
W<0 Force+Disp. Opposite
W<0 Work done by particle!!

(6)

Several Forces: Superposition
- Add effects due to each force

Work depends on Frame of Ref.



Work - Variable Force

$F_x = F_x(x)$ Function of x.

What is $W(x=a \rightarrow x=b)$

$\Delta W_i = F_i(x_i)\,\Delta x$

$W(a\rightarrow b) = \sum_i \Delta W_i = \sum_i F_x(x_i)\,\Delta x$

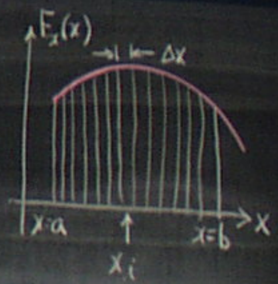
Limiting case: $\Delta x \rightarrow 0$, $i \rightarrow \infty$

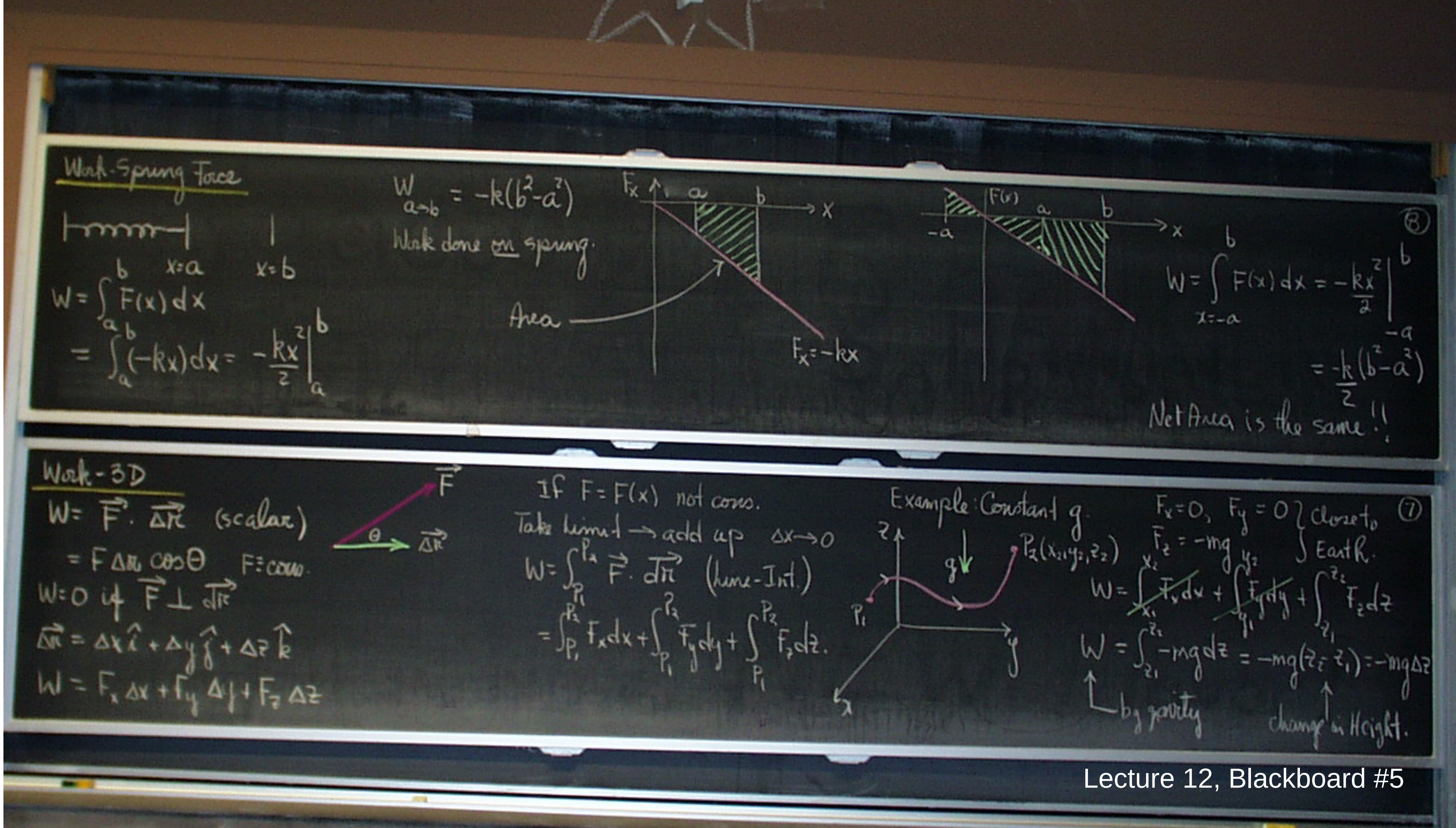
$W = \lim_{\Delta x \rightarrow 0} \sum F_x(x_i)\,\Delta x$

$W = \int_a^b F_x(x)\,dx$: Definite Integral

(integrand)

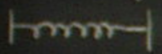
= Area bounded by curve F(x) and the lines x=a and x=b and the x-axis.





Lecture 12, Blackboard #5

Work-Spring Force

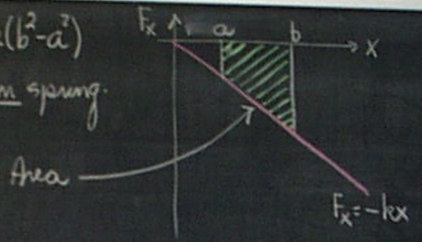


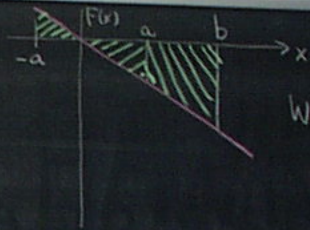
$$W = \int_a^b F(x)\,dx$$

$$= \int_a^b (-kx)\,dx = -\frac{kx^2}{2}\Big|_a^b$$

$$W_{a\to b} = -k(b^2-a^2)$$

Work done on spring.





(8)

$$W = \int_{x=-a}^{b} F(x)\,dx = -\frac{kx^2}{2}\Big|_{-a}^{b}$$

$$= -\frac{k}{2}(b^2-a^2)$$

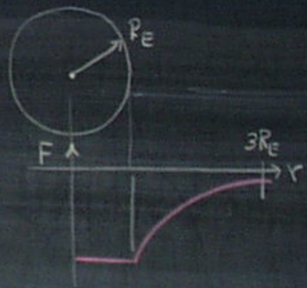
Net Area is the same !!

Example: Work on Astronaut

$$F = -\frac{GM_E m}{r^2}\hat{r}$$

$$W = -\int_{R_E}^{3R_E} \frac{GM_E m}{r^2}\,dr = \frac{GM_E m}{r}\Big|_{R_E}^{3R_E} = GM_E m\left(\frac{1}{3R_E} - \frac{1}{R_E}\right)$$

$$= -\frac{2}{3}\frac{GM_E m}{R_E} = -\frac{2}{3} mgR_E$$



(9)

$m = 80$ kg

$$W = -\frac{2}{3} \times 80 \times 9.81\, R_E$$

$$= -3.34 \times 10^9 \text{ J}$$