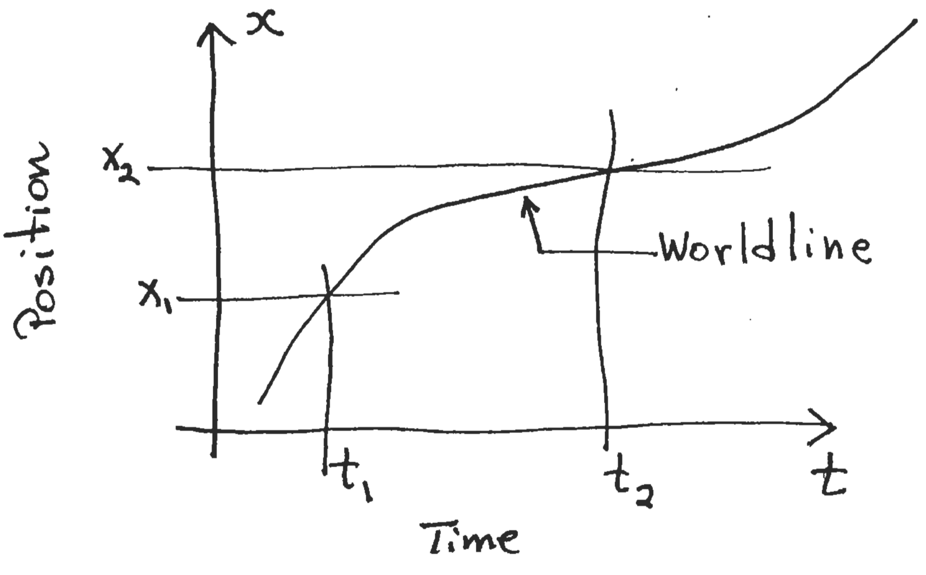
Kinematics

- Describes the geometry of motion of a particle
- Uses mathematics to describe the motion in terms of position, velocity and acceleration
- Introduction to dynamics $\Longrightarrow$ study of why things move.
- Next few lectures will involve a study of translational motion.
- Simplify the physics : use concept of an ideal particle:
  - no size
  - no internal structure
  - mass
  - position as a function of time gives a complete description

- world line $\rightarrow$ position vs time



Speed

Consider a particle moving along a worldline

- straight
- curved

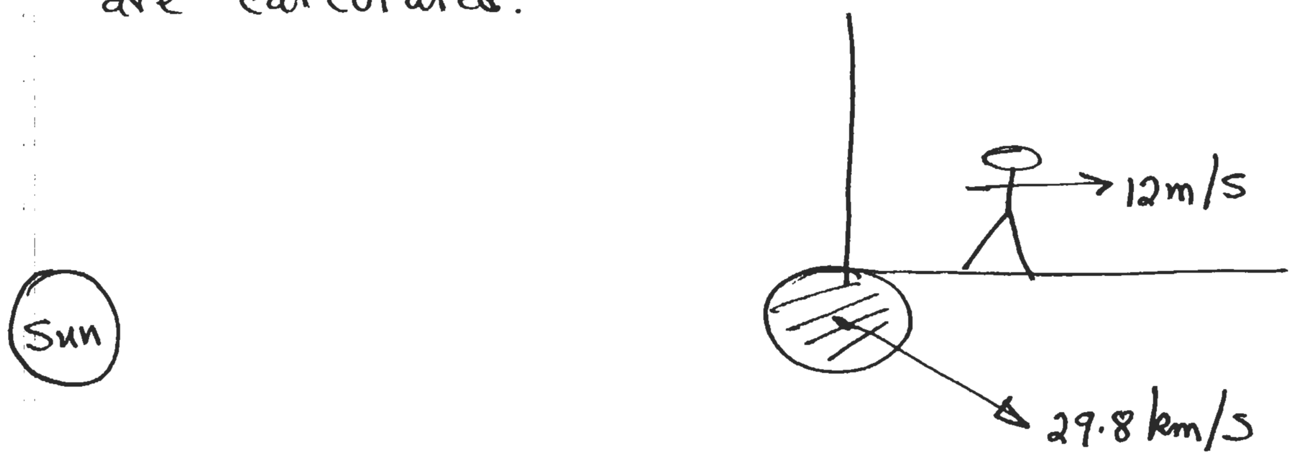
Average Speed = $\dfrac{\text{distance travelled}}{\text{time taken}}$ $\left[\dfrac{L}{T}\right]$

$s = \dfrac{d}{t} > 0$ always (m/s) (ft/s)

Speeds

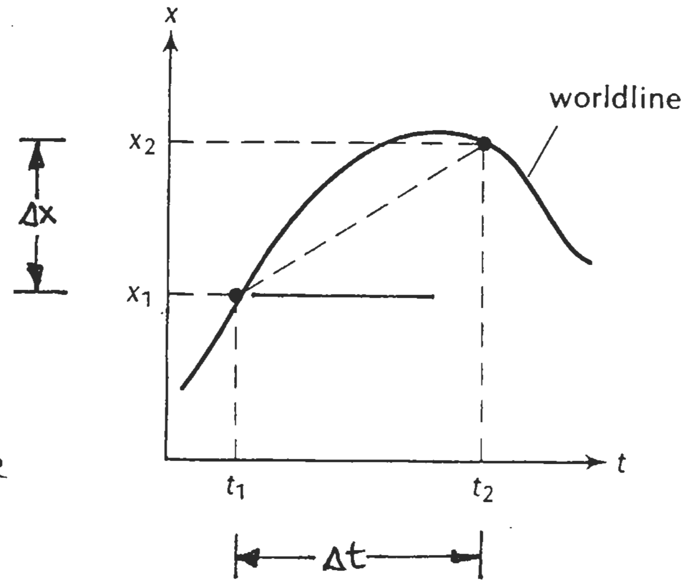
Light	3×10^{8} m/s
Sound	330
Man	12
Glacier	10^{-6}
Continental Drift	10^{-9}

Motion and speed are <u>relative</u>
Depend on the <u>frame of reference</u> where they are calculated.



Average Velocity

- 1 Dimension look at motion of a particle moving along a straight line.



- if position of particle ch[illegible]th time, it is moving $\rightarrow$ velocity.
- We define an <u>average velocity</u> for the particle

$$\bar{v} = \frac{\text{change in position}}{\text{change in time}} \leftarrow \text{displacement}$$

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \begin{array}{l} \leftarrow \text{change in position} \\ \leftarrow \text{change in time} \end{array}$$

$\pm$ sign of velocity indicates direction of motion

$\bar{v} =$ slope of straight line connecting points $(x_1 t_1)$ and $(x_2 t_2)$.

$\bar{v} > 0$ motion to right along x-axis

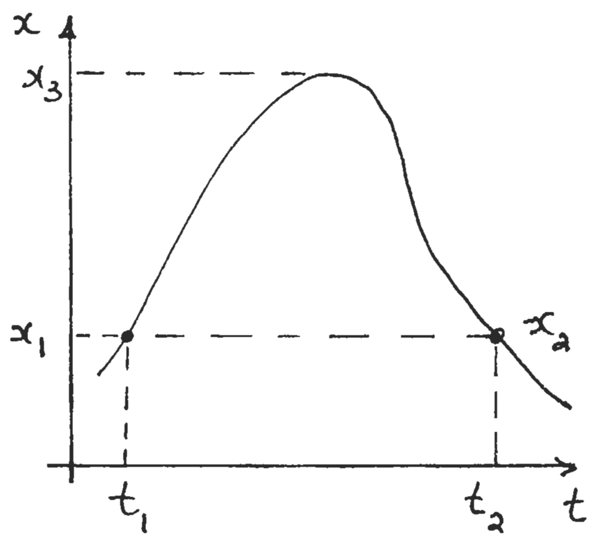
$\bar{v} < 0$ motion to left along x-axis average speed = $\dfrac{\text{distance travelled}}{\text{time}}$

average velocity = $\dfrac{\text{change in position}}{\text{time}}$

<u>Example</u>

$$\bar{S}_{12} = \frac{2(x_3 - x_1)}{t_2 - t_1} \neq 0$$

$$\bar{v}_{12} = \frac{x_2 - x_1}{t_2 - t_1} = 0$$

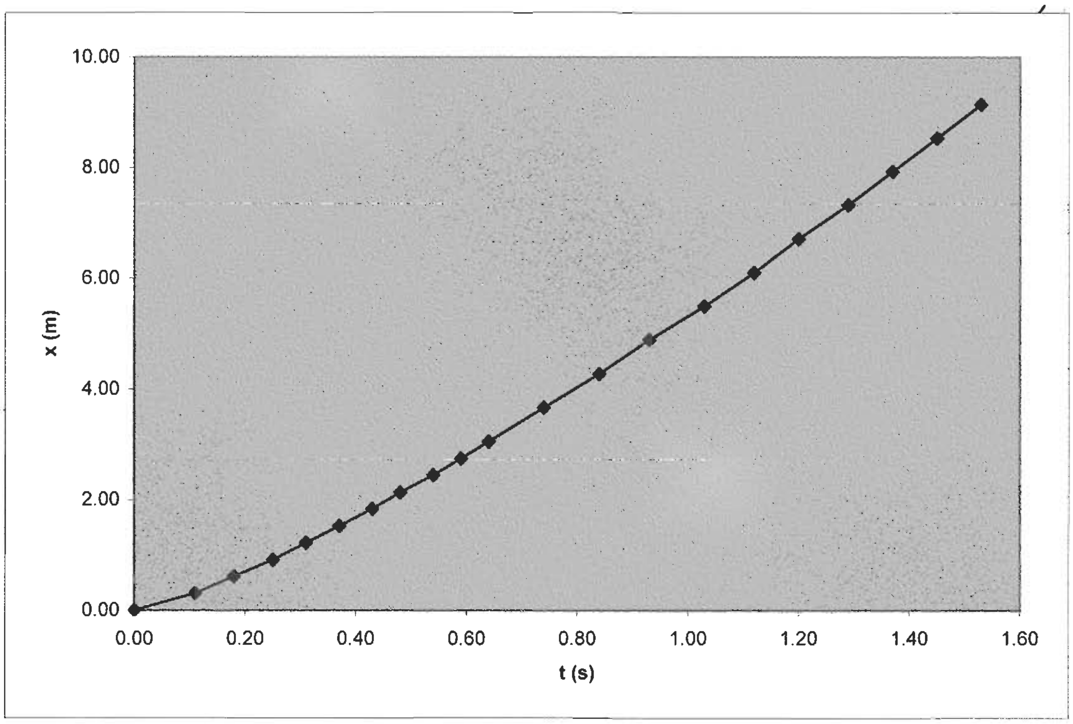


Example.

-Data for a runner

Positions and times of a runner for the initial portion of a race

x (m)	t (s)
0.00	0.00
0.31	0.11
0.61	0.18
0.91	0.25
1.22	0.31
1.52	0.37
1.83	0.43
2.13	0.48
2.44	0.54
2.74	0.59
3.05	0.64
3.66	0.74
4.27	0.84
4.88	0.93
5.49	1.03
6.10	1.12
6.71	1.20
7.32	1.29
7.93	1.37
8.53	1.45
9.14	1.53



-Find $\bar{v}$ for the first 1.53 seconds of race.

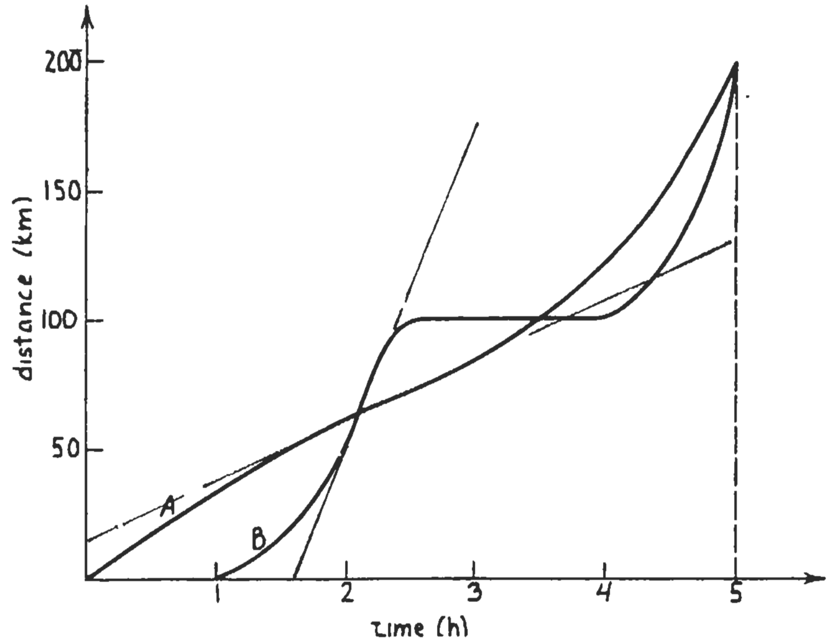
$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{9.14 - 0}{1.53 - 0} = 5.97 \text{ m/s}.$$

-Find $\bar{v}$ for the time interval $t_1 = 0.54$s and $t_2 = 0.93$s

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{4.88 - 2.44}{0.93 - 0.54} = 6.3 \text{ m/s}$$

Example

Two automobiles make a 5h trip over a total distance of 200km.



Average velocity for the trip:

$$\bar{v}_A = \frac{200\text{km}}{5\text{h}} = 40\text{ km/h} \qquad \text{Car - A}$$

$$\bar{v}_B = \frac{200\text{km}}{4\text{h}} = 50\text{km/h} \qquad \text{Car - B}$$

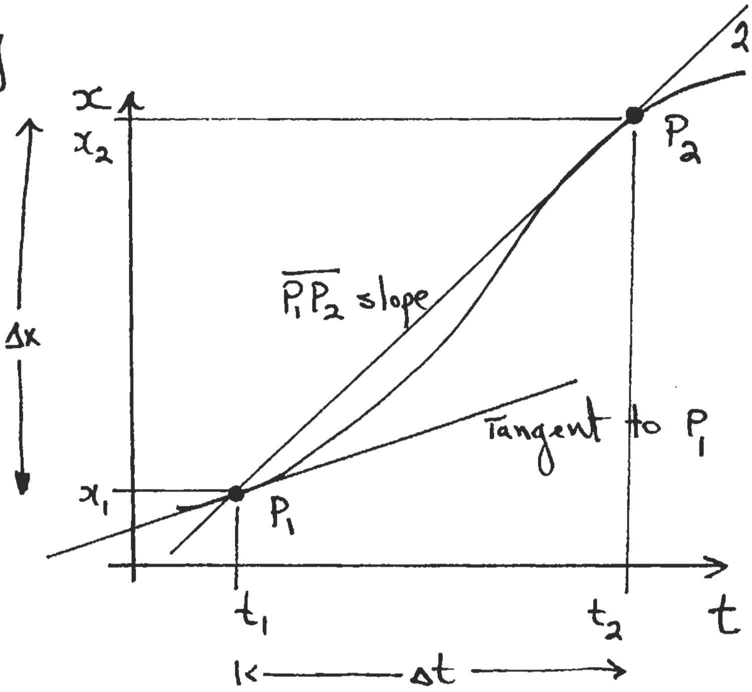
Average velocity at $t = 2h$:

$\bar{v}_A \sim 25$ km/h

$\bar{v}_B \sim 100$ km/h

B starts 1h later than A
A+B pass each other at $t = 2h$ and $t = 3.5h$
At $t = 2.5h$, B stops for 1.5h. A speeds up.
B starts up and speeds quickly to catch A at 200km. Both arrive at $t = 5h$.

Instantaneous Velocity



Slope of the line $\overline{P_1P_2}$ represents the average velocity $\bar{v}$ between t_1 and t_2.

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

Q: what is the velocity, exactly at P_1?

Pick a new point closer to P_1 to get a better measure.

$$\left.\begin{array}{l}\text{Reduce } \Delta t \\ \text{Reduce } \Delta x\end{array}\right\} \qquad \bar{v} = \frac{x_i - x_1}{t_i - t_1} = \frac{\Delta x_i}{\Delta t_i}$$

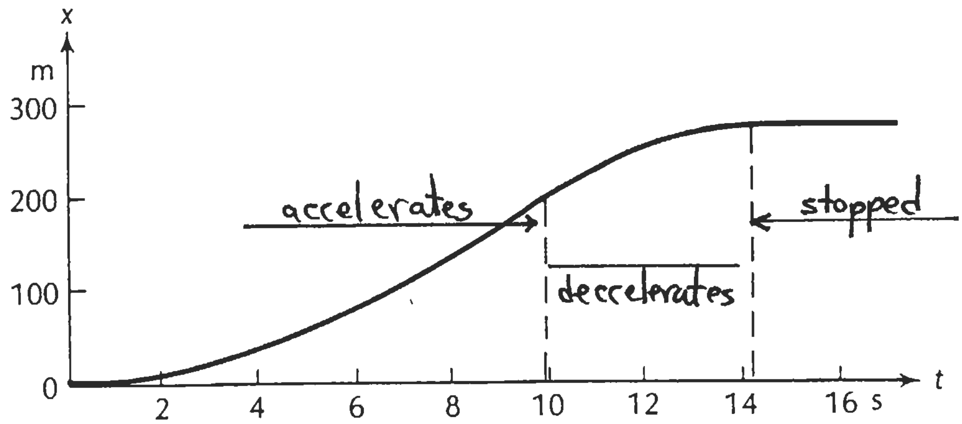
Instantaneous velocity $\equiv$ velocity defined as the limit as we let $\Delta t \to 0$.

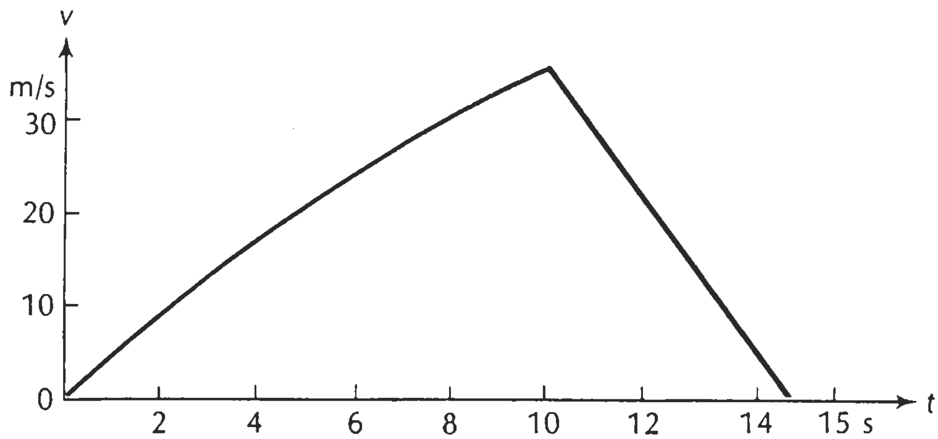
It is equal to the slope of the tangent to the curve at the point.

$$v(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

calculus

- velocity is the general time derivative of the position function.





Example: Racing car.

- Position vs. time.
- Resulting velocity vs. time.

Note: Neither $\bar{v}(t)$ nor $v(t)$ depend on the choice of a coordinate system (if there is no relative motion) since they involve only differences in position.

$\longleftrightarrow$ in variant to choice of origins/systems

Example

Ideal particle moving in a straight line with position given by:

$$x = 2.1\,t^2 + 2.80 \quad (m) \qquad t \rightarrow \text{sec.}$$

Q: What is average velocity between $t_1 = 3.0s$ and $t_2 = 5.0s$?

$t_1 = 3.0s \qquad x_1 = 2.1\,(3.0)^2 + 2.80 = 21.7m$

$t_2 = 5.0s \qquad x_2 = 2.1\,(5.0)^2 + 2.80 = 55.3m$

$$\text{Average } \bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{55.3 - 21.7}{5.0 - 3.0} = 16.8 \text{ m/s}$$

Instantaneous v?

① $\qquad x = 2.1t^2 + 2.80$

$\qquad x + \Delta x = 2.1(t + \Delta t)^2 + 2.80$

② $\qquad x + \Delta x = 2.1t^2 + 4.2t(\Delta t) + 2.1(\Delta t)^2 + 2.80$

②-① $\qquad \Delta x = 4.2t\,(\Delta t) + 2.1(\Delta t)^2$

$$\bar{v} = \frac{\Delta x}{\Delta t} = 4.2t + 2.1(\Delta t)$$

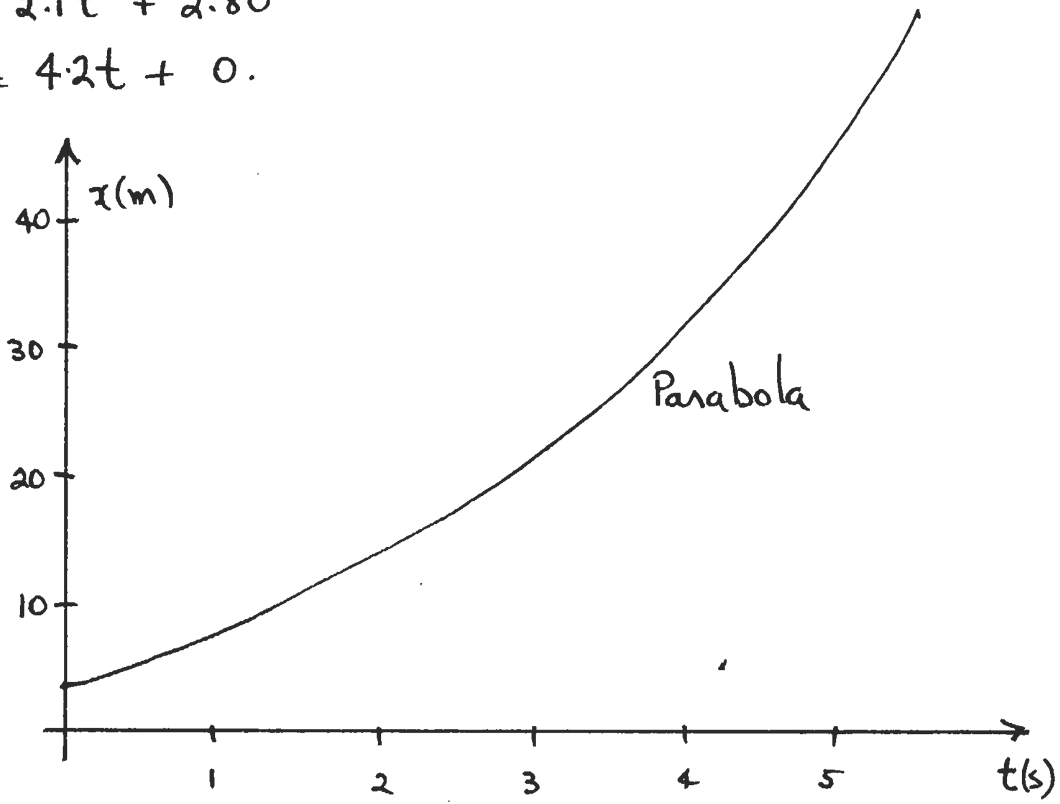
$$\text{Instantaneous } v(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = 4.2t$$

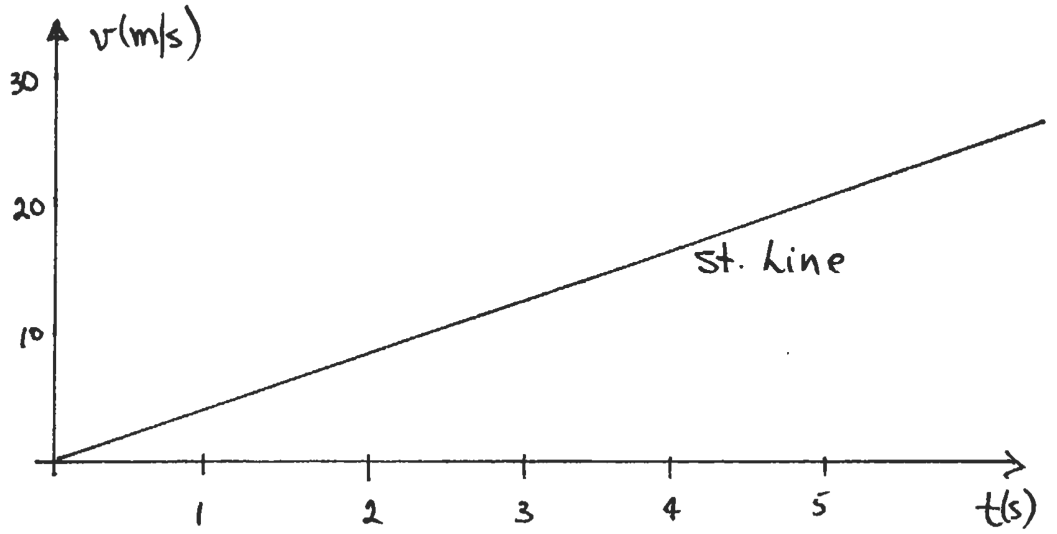
$$\left.\begin{array}{l} v(t=5) = 4.2\times 5 = 21.0\ m/s \\ v(t=3) = 4.2\times 3 = 12.6\ m/s \end{array}\right] 16.8\ m/s\ !!$$

Using calculus

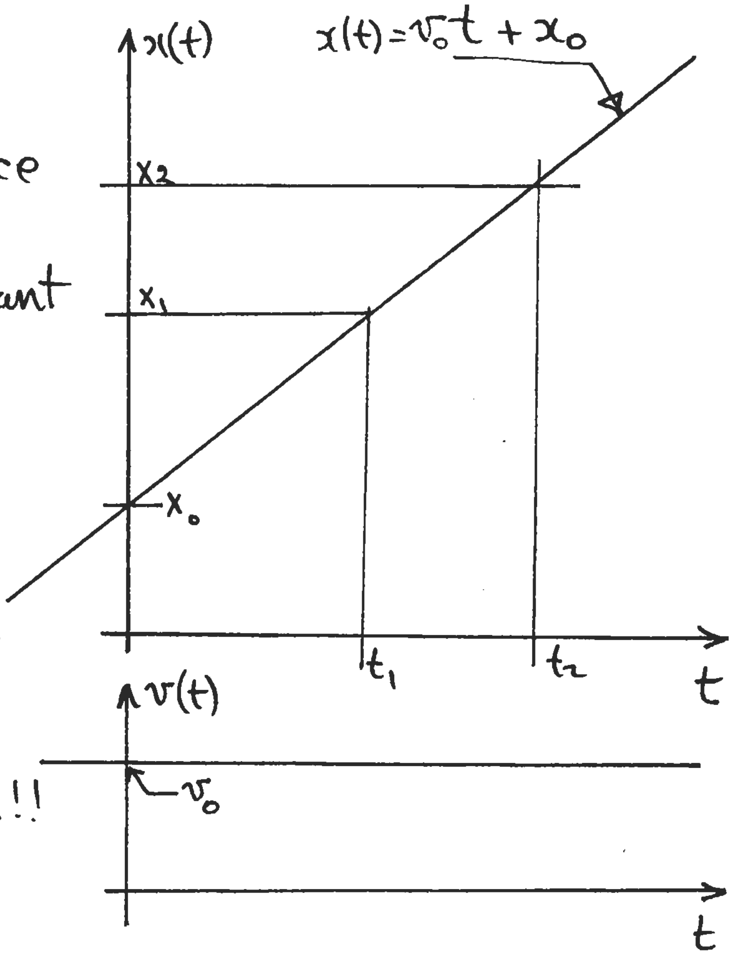
$$x = 2.1t^2 + 2.80$$

$$v(t) = \frac{dx}{dt} = 4.2t + 0.$$





Constant Velocity Motion



- particle moves with a position – time dependence which is a st. line.
- $\therefore$ slope of $x(t) \equiv$ constant

$$\bar{v} = \frac{\Delta x}{\Delta t} = \text{constant.} = v_0$$

Also

$$v(t) = \frac{dx}{dt} = \text{constant} \equiv v_0$$

$$\bar{v} = v$$

average $\equiv$ instantaneous!!

- Motion at constant velocity is called <u>uniform linear motion</u>.

Let $\bar{v} = v(t) = v_0$; a constant.
Suppose at time $t=0$ the position of the particle is at $x = x_0$. Then at any time t its position is at $x(t)$.

$$\therefore \quad \bar{v} = v_0 = \frac{x(t) - x_0}{t - 0}$$

$$\therefore \quad x(t) = x_0 + v_0 t \qquad \text{Eq. of a St. Line.}$$

$\Rightarrow$ General Desc. for 1-Dim. motion at Const. V.

Speed of Rifle Bullet

[15 - 20] min

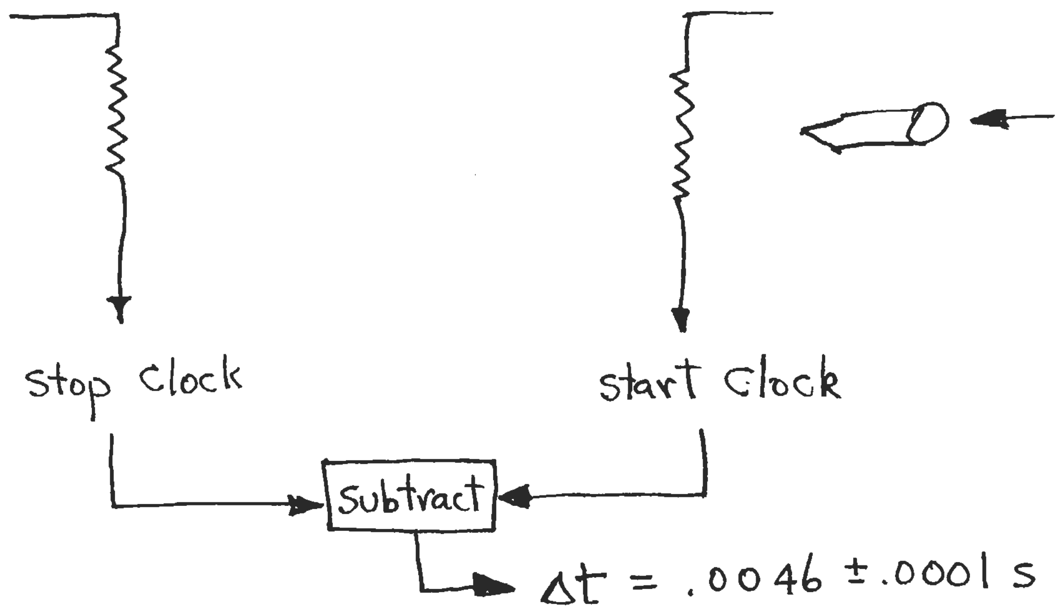
- High speed
- Need to use special techniques

$$s = \frac{d}{t} = \frac{\text{distance}}{\text{time}} = \text{speed}$$

Distance : 1.50 m ± .005 (meter-stick)

Time : 2 Methods

Method I : Direct Timing

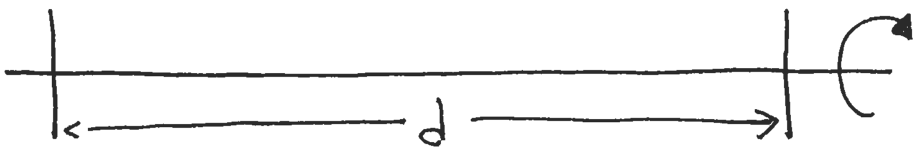


$\Delta t = .0046 \pm .0001$ s

$$s = \frac{1.50}{.0046} = 326 \text{ m/s}$$

Sound = 330 m/s

Method II: Rotating Shaft



- 2 paper disks on a shaft - rotating a distance d apart.
- bullet pierces first disk
- shaft rotates while bullet travels distance d
- bullet pierces second disk.

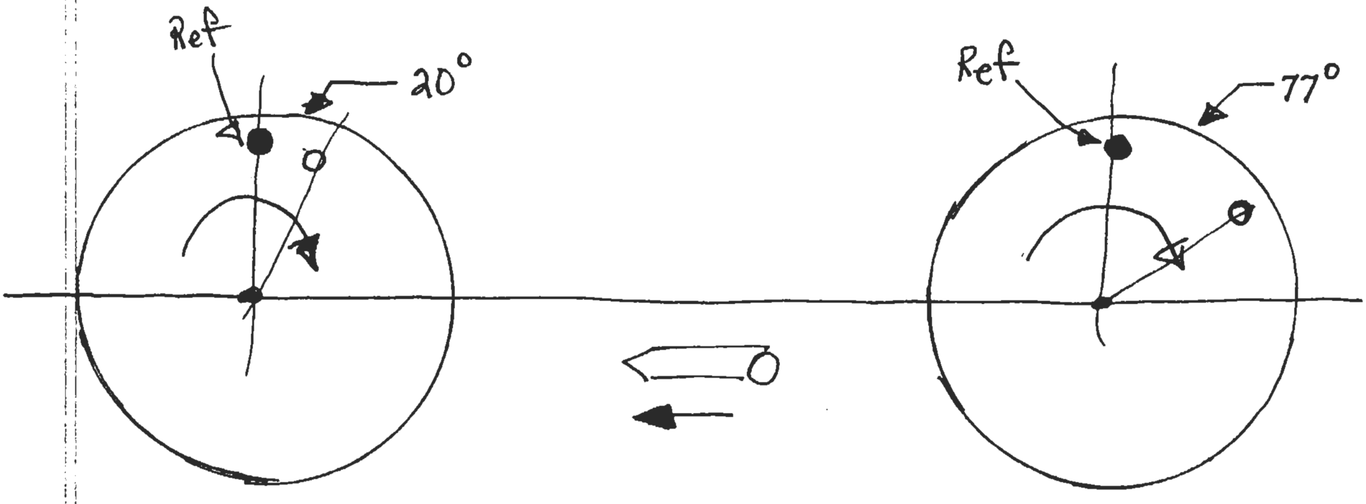
1. Measure time for 1-revolution of the shaft
 $T_R = .0293\,s$ 1-revolution $= .0293\,s$
 $\Longrightarrow 360°$

1800 rpm motor $\rightarrow \frac{1800}{60} = 30$ rev/s
$\hookrightarrow$ 1 rev $= .0333\,s$

(reasonable)

2. Since discs are located arbitrarily on the shaft - need to define st. line by firing bullet with shaft not rotating.

3. Mark direction of shaft rotation
4. Mark bullet reference marks
5. Rotate shaft - fire bullet
6. Measure angular displacement, $\Delta\theta$



Time-of-Flight

$$\Delta t = \frac{\Delta\theta}{360^\circ} \times .0293$$

$$= \frac{77^\circ - 20^\circ}{360^\circ} \times .0293$$

$$= .0046 \text{ s}$$

$$s = \frac{d}{\Delta t} = \frac{1.50 \text{ m}}{.0046 \text{ s}} = 323 \text{ m/s}$$

<u>Uncertainty / Errors</u>

- Time measurement of shaft rotation
  $\Delta t_R = .0001$ < ½%
- Location of Holes (angle measurement)
  $\Delta\theta \sim (5 - 10)\%$
- Length measurement
  $\Delta L \sim .01$ < 1%