Friction

Surfaces in contact exert two forces on each other:

i) Normal; force $\perp$ to surfaces.

ii) Parallel; friction

- Force of friction always opposes relative motion or potential relative motion of the surfaces.

Frictional Forces

- Frictional forces play an important role in the motion of real objects
- Arise from adhesion between atoms in the two surfaces.
- Microscopic level description is very complicated.
- Macroscopic level description is purely empirical (L. da. Vinci)
    - proportional to normal force between surfaces
    - independent of area of contact
    - independent of speed

Kinetic Friction

- Surfaces in relative motion

$$f_k = \mu_k N$$

μ_k = coefficient of kinetic friction $(0 < \mu_k < 1)$
N = contact (normal) force

- $\rightarrow$ proportional to N
- f_k parallel to the surface of contact
- opposite to the direction of motion
- law is approximate and empirical.
- μ_k depends on the nature of the materials.

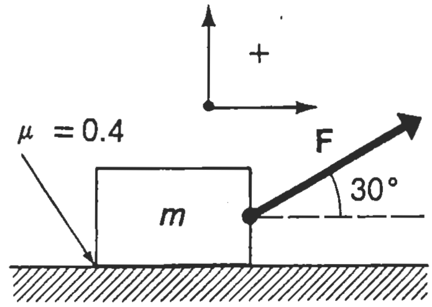
μ_k independent of v over wide range

The friction force on each interacting body is <u>opposite</u> in direction to the <u>motion</u> of <u>that body</u> relative to the other.

Example

$m = 100\,kg$

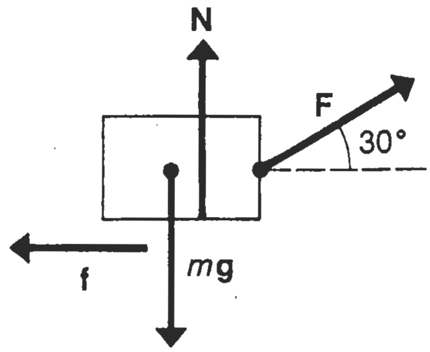
$\mu_k = 0.40$



Crate is moved forward at constant speed.

$\vec{a} \equiv 0.$

What is F ?



Vertical force components:

$$N + F\sin 30^\circ - mg = 0 \qquad (1)$$

Horizontal force components

$$F\cos 30^\circ - f_k = 0 \qquad (2)$$

$$f_k = \mu_k N \qquad (3)$$

$$F\cos 30^\circ - \mu_k\left[mg - F\sin 30^\circ\right] = 0$$

$$F = \frac{\mu_k mg}{\cos 30^\circ + \mu_k \sin 30^\circ}$$

$$F = \frac{0.40 \times 100 \times 9.81}{0.866 + 0.40 \times 0.50} = 368\,N$$

$\theta = 0^\circ \quad F = 392\,N$

$\theta = 45^\circ \quad F = 396\,N$

$\theta = 90^\circ \quad F = 981\,N$

Static Friction

- Frictional forces also act between surfaces that are at rest (no relative motion)
- Objects at rest require a non-zero force to start them moving.

$$f_s \leq \mu_s N$$

μ_s = coefficient of static friction
N = contact (normal) force

- Force of static friction can have any magnitude between zero (when there is no other force parallel to the surface) up to a maximum value of $\mu_s N$. Equality sign holds when motion is about to start.

  - proportional to normal force
  - independent of area
  - empirical law
  - opposite to lateral push that tries to move the body.
  - Usually $\mu_s > \mu_k$, so that once block starts moving it will take less force to keep it from accelerating.
  - μ_s depends on nature and condition of surfaces

Example: Block on Surface.

a) No motion

$f_1 < \mu_s N$

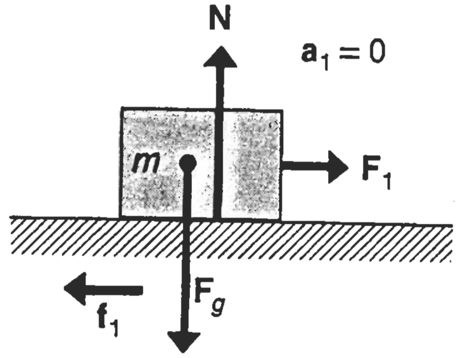
b) Motion impends

$f_2 = \mu_s N$

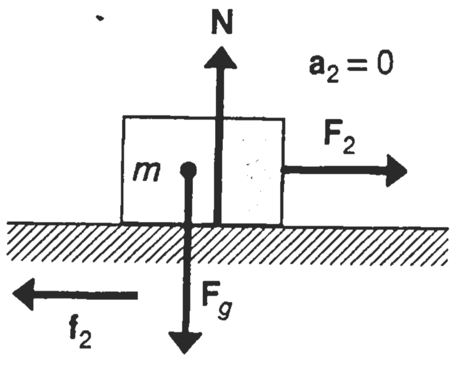
c) Motion exists

$f_3 = \mu_k N$

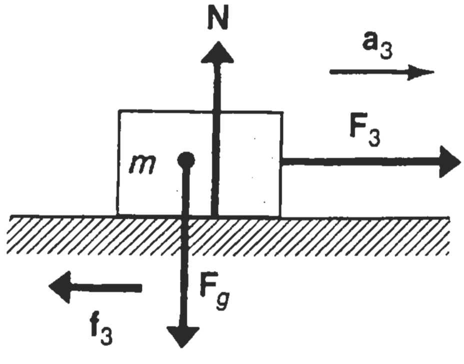
Fig. (a, b) For $f_s < f_{s,max}$, the frictional force exactly balances the applied force; then, there is no acceleration. (c) When a force sufficient to cause motion is applied, the frictional force is equal to $\mu_k N$ and the acceleration is $(F - \mu_k N)/m$.



(a) $\mathbf{f}_1 = -\mathbf{F}_1$

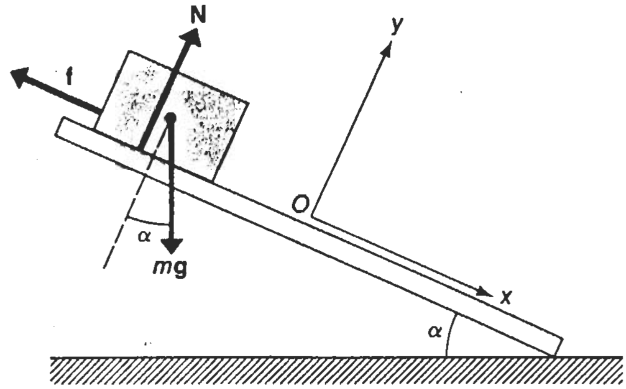


(b) $\mathbf{f}_2 = -\mathbf{F}_2$



(c) $f_3 = \mu_k N$

Example : Block-on-Plane



Block starts to slip at $\alpha = 23°$, what is coefficient of static friction, μ_s?

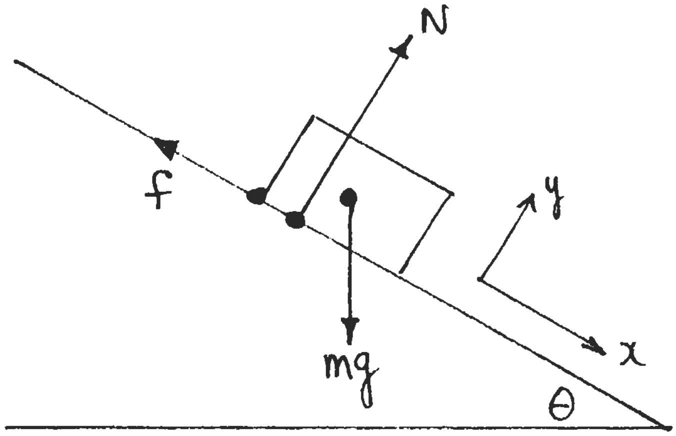
$$\left.\begin{array}{ll} \text{(y-axis)} & N - mg\cos\alpha = 0 \\ \text{(x-axis)} & mg\sin\alpha - f = 0 \end{array}\right\} \vec{a} = 0$$

$$\frac{f}{N} = \frac{mg\sin\alpha}{mg\cos\alpha} = \tan\alpha$$

$$\left(\frac{f}{N}\right)_{Max} = \mu_s = \tan 23° = 0.424$$

Maximum angle is called the angle of repose.
It is independent of the mass of the block.

Example: Block-down-Plane



$f = \mu_k N$

(x-Axis) $\quad mg\sin\theta - f = m a_x$

(y-Axis) $\quad N - mg\cos\theta = m a_y = 0$

$$\therefore\ N = mg\cos\theta$$

$$\cancel{mg}(\sin\theta - \mu_k\cos\theta) = \cancel{m}a_x$$

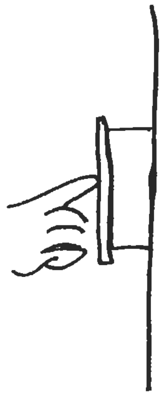
$$a_x = (\sin\theta - \mu_k\cos\theta)\,g$$

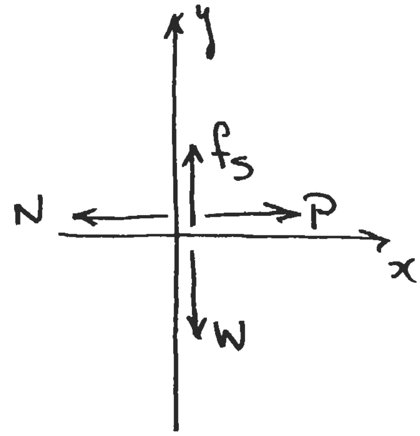
If $\quad a_x = 0$

$$\mu_k = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$\Rightarrow$ angle at which object will move down the plane at constant velocity for a given μ_k. Measure of μ_k.

Example





$$\sum F_x = P - N = 0 \qquad ①$$

$$\sum F_y = f_s - W = 0 \qquad ②$$

From ② $\quad f_s = W$

① $\quad P = N$

$f_s \le \mu_s P$

For no slipping $f_s \ge W$

$\therefore \mu_s P \ge W$

$$P \ge \frac{W}{\mu_s}$$

$P = \dfrac{W}{\mu_s} \equiv$ minimom push needed.

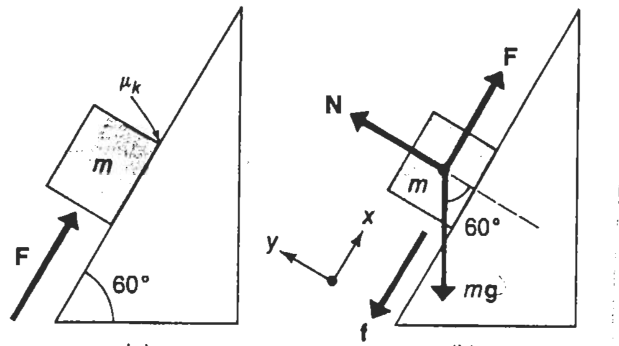
Example: Block-Up-Plane?

$m = 5\ kg$

$F = 20\ N$

$\mu_k = 0.42$

$a = ?$



(a) (b)

Assume upward motion:

Along-x $\quad F - f - mg \sin 60° = ma_x$

Along-y $\quad N - mg \cos 60° = 0 \quad$ (No acceleration)

$$\therefore N = mg \cos 60°$$

$$f = \mu_k N = \mu_k mg \cos 60°$$

$$a_x = \frac{F - mg \sin 60° - \mu_k mg \cos 60°}{m}$$

$$= \frac{F}{m} - g \sin 60° - \mu_k g \cos 60°$$

$$= \frac{20}{5} - 9.81 \times 0.866 - 0.42 \times 9.81 \times \frac{1}{2}$$

$$= -6.55\ m/s^2$$ [Block moves down plane. Need to redo.]

Change direction of frictional force.

$F - mg \sin 60° + f = ma_x$

$N - mg \cos 60° = 0$

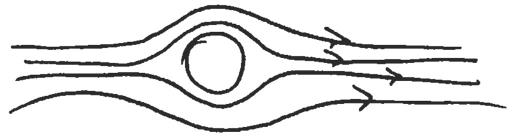
Solve $\quad a = -2.43\ m/s^2 \quad$ [Direction consistent with assump.]

Drag Force and Terminal Speed

- objects moving through a fluid (air, water, etc) give rise to a drag force which retards the motion

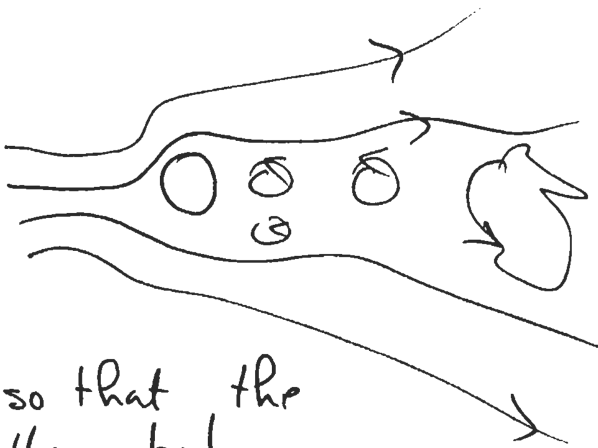
- Complicated problem in detail.
- Two distinct regions of fluid flow around object.

1. Laminar Flow
   - smooth Flow around object
   - small particles in fluids



   - $F_D \sim$ velocity.
   - stokes law.

2. Turbulent Flow



   - baseball (42m/s)
   - parachutist (5m/s)
   - object sinking in water.

   $F_D \sim \text{velocity}^2$

   - speed is high enough so that the flow of air behind falling body is turbulent. Particles in the fluid fluctuate in a random manner, causing disordered whirling and eddying of the fluid.

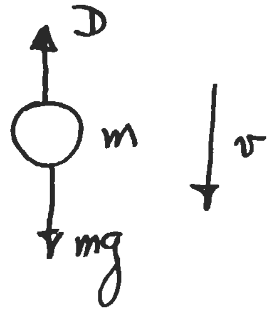
Resistive Force Proportional to Velocity

- Objects falling through a fluid
- Small objects (dust particles) in air.

Resistive Force $\vec{D} = -b\vec{v}$

$\vec{v}$: velocity of object
b: constant which depends on medium and on shape of object. For a sphere $b \sim r$. (kg/s)

Consider dropping a sphere of mass m in a fluid.



Forces which act:

mg = weight (corrected for buoyant force)

$-bv$ = resistive force.

Applying Newton's Second Law

$$\Sigma F_y = m a_y$$

$$mg - bv = m\left(\frac{dv}{dt}\right)$$

$$a = \frac{dv}{dt} = g - \frac{b}{m}v \qquad \text{[Differential Equation]}$$

When object is first started at $t=0$, $v=0$ and resistive force is zero.

Initial acceleration

$$a(t=0) = \frac{dv}{dt} = g \quad \text{[Reasonable]}$$

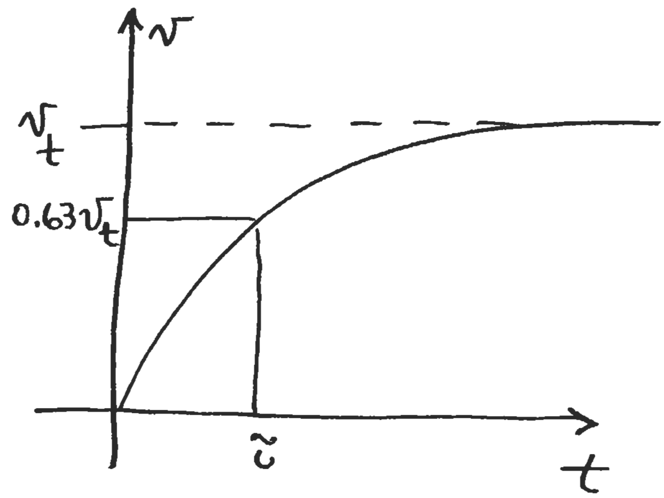
As t increases, v increases, and resistive force increases and acceleration decreases.

When resistive force equals weight the acceleration becomes zero. Body continues to move at its terminal velocity with no acceleration.

Let $a = \frac{dv}{dt} = 0$

$$mg - bv_t = 0$$

$$v_t = \left(\frac{mg}{b}\right)$$



Solve d.e.

$$v = \frac{mg}{b}\left(1 - e^{-bt/m}\right) = v_t\left(1 - e^{-t/\tau}\right)$$

$\tau = m/b$ is the time-constant. i.e. time for object to reach 63% of its terminal velocity.

[Prob: show solution satisfies d.e.]

Falling Bodies in Air.

Drag Force

$$D = \frac{1}{2} C \rho A v^2$$

· Force is proportional to $(\text{velocity})^2$.

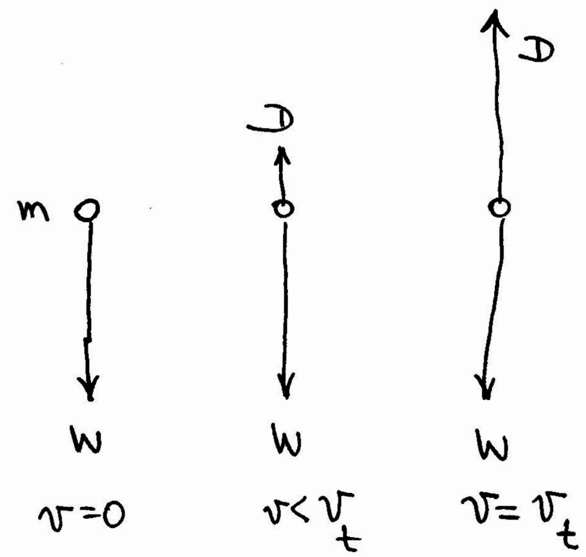
A: effective cross-sectional area of falling body
ρ: density of air
v: speed of fall
C: dimensionless drag coefficient — depends on shape of object (usually $C = 0.5 \rightarrow 1.0$).

- When body is released it has no velocity and drag force is zero.
- Velocity increases and drag force increases up to point where it equals weight of body.
- At this point acceleration is zero and body falls at constant velocity.

- At terminal speed $D = mg$.

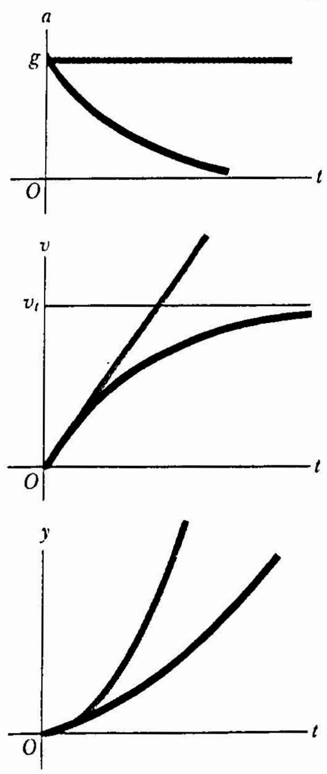
$$\therefore \quad \frac{1}{2} C \rho A v_t^2 = mg$$

$$\therefore \quad v_t = \sqrt{\frac{2mg}{C\rho A}} \quad \text{m/s}.$$



For high terminal speeds:

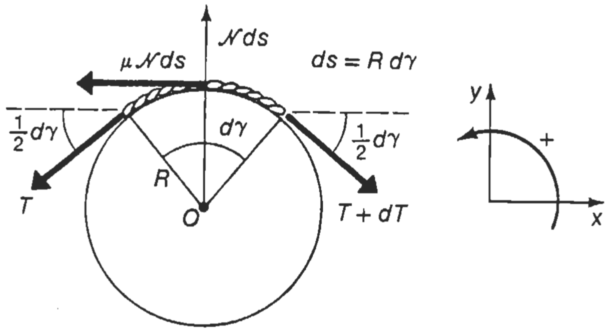
- Reduce effective area
- Reduce drag coefficient → streamlining.



Graphs of acceleration, velocity, and position versus time for a body falling in a viscous fluid, shown as solid color curves. The light color curves show the corresponding relations if there is no viscous friction.

Ropes and Posts

Rope is wrapped around a rough post. We want to relate the forces at the ends of a rope to the length of rope wrapped around the post if the coefficient of static friction is μ.



- Assume no slipping occurs.
- Section of rope makes an angle $d\gamma$ at center.
- Let normal force on the rope be $\mathcal{N}$ per unit length at any point.

For a length of rope ds, the normal force is $\mathcal{N} ds$.

$$\Sigma F_y = 0 \quad \mathcal{N} ds - (T + dT)\sin\left(\frac{d\gamma}{2}\right) - T\sin\left(\frac{d\gamma}{2}\right) = 0 \qquad (1)$$

$$\Sigma F_x = 0 \quad (T + dT)\cos\left(\frac{d\gamma}{2}\right) - T\cos\left(\frac{d\gamma}{2}\right) - \mu \mathcal{N} ds = 0 \qquad (2)$$

For small angles $\cos\left(\frac{d\gamma}{2}\right) \sim 1$

$$\sin \frac{d\gamma}{2} \sim \left(\frac{d\gamma}{2}\right)$$

also $dT \ll T$

$$T\,d\gamma = \mathcal{N}\,ds \qquad \text{From Eq. } ①$$

$$T\left(\frac{d\gamma}{ds}\right) = \mathcal{N} \qquad ③$$

$$dT = \mu\,\mathcal{N}\,ds \qquad \text{From Eq. } ②$$

$$\therefore \frac{dT}{ds} = \mu\,\mathcal{N} \qquad ④$$

Note: Alternate for Eq. ④

Take torques about center at O.

$$\Sigma \vec{\tau}_0 = 0 \qquad \mu\mathcal{N}R\,ds + TR - (T + dT)R = 0$$

$$\frac{dT}{ds} = \mu\,\mathcal{N}$$

④ / ③ $$\frac{1}{T}\frac{dT}{d\gamma} = \mu$$

Integrating from $\gamma' = 0$ to γ and $T' = T_0$ to T

$$\int_{T'=T_0}^{T} \frac{dT'}{T'} = \mu \int_{\gamma'=0}^{\gamma} d\gamma'$$

$$\ln\frac{T}{T_0} = \mu\gamma$$

or $$T = T_0\, e^{\mu\gamma}$$

Angle around post in radians

Coefficient of static friction

Consider a rope of length ℓ wrapped around the post.

$$R\gamma = \ell$$

$$T = T_0 \, e^{\mu \ell / R}$$

Ex: Rope goes once around the post.

i.e. $\gamma = \dfrac{\ell}{R} = 2\pi$ and $\mu = 0.40$

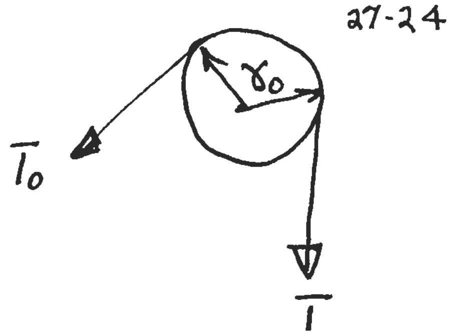
$$T = T_0 \, e^{0.40 \times 2\pi}$$

$$T = 12.3 \, T_0$$ [Tremendous Advantage]

27-24

$$T = T_0 e^{\mu \gamma}$$

$$T_0 = T e^{-\mu \gamma}$$



Let $\gamma_1 = \gamma_0$ $\qquad T_{01} = T e^{-\mu \gamma_1}$

Let $\gamma_2 = \gamma_0 + 2\pi$ $\qquad T_{02} = T e^{-\mu \gamma_2}$ [1-Extra Turn of Rope]

$$r = \frac{T_{01}}{T_{02}} = \frac{e^{-\mu \gamma_1}}{e^{-\mu \gamma_2}} = e^{2\pi \mu}$$

μ	r
0.1	1.87
0.2	3.51
0.3	6.59
0.4	12.34
0.5	23.14

40N
↓
24N
↓) 2π
13N

Initially

$r \sim \left(\frac{3}{4}\pi\right) = 2.36$ $\qquad \left(\frac{2}{3}\pi\right) = 2.09$

$T_0 = 24N$ $\qquad \frac{T}{T_0} = \frac{40}{24} = \left(\frac{5}{3}\right) = 1.667.$

$T = 40N$

μ	T/T_0 $\left(\frac{3}{4}\pi\right)$	$\left(\frac{2}{3}\pi\right)$
0.2	1.60	1.52
0.4	2.57	2.31
0.6	4.12	3.50