HW Solutions # *6* - 8.01 MIT - Prof. Kowalski

Potential Energy and Mechanical Energy Conservation.

NOTE: We made a mistake on the solution sheet for HW #5. On part **d** of problem *5* on HW #5 we omitted the work due to friction. If you submitted the answer $\frac{1}{2}mv_0^2\frac{x_2-x_1}{x_1} + \frac{1}{2}mv_1^2$ it should have been graded *correct*, and you can recover any lost credit by resubmitting it to your grader with a note to regrade this problem.

1) *7.42*

`Please refer to the figure 7.30 p.277.`

Let's denote the compression length as $\mathbf{\Delta x}$, the velocity after leaving the spring as $\mathbf{v_0}$ and the maximum height it goes on the incline as $\mathbf{h}$.

Using energy conservation law:

$$\frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2 + mgy_1 + W_{other} = \frac{1}{2}mv_2^2 + \frac{1}{2}kx_2^2 + mgy_2 \quad (1)$$

There is no friction present in this problem and $W_{other} = 0$. I will measure the gravitation potential energy with respect to the height of the dashed horizontal bar indicated in the figure 7.30.

a) Comparing the energy initially (when spring compressed):

$$v_1 = 0 \quad x_1 = \Delta x \quad y_1 = 0 \quad (2)$$

with the point it leaves the spring:

$$v_2 = v_0 \quad x_2 = 0 \quad y_2 = 0 \quad (3)$$

$$0 + \frac{1}{2}k\Delta x^2 + 0 = \frac{1}{2}mv_0^2 + 0 + 0 \quad (4)$$

$$\therefore \quad v_0 = \sqrt{\frac{k}{m}}\Delta x \tag{5}$$

b) For this part we should compare the energy of particle at its highest point on the incline:

$$v_3 = 0 \quad x_3 = 0 \quad y_3 = h \tag{6}$$

with the point that it has left the spring:

$$v_2 = v_0 \quad x_2 = 0 \quad y_2 = 0 \tag{7}$$

(We could equally well elect to use the variables subscripted as ***1*** *as the initial energy point)*

$$\frac{1}{2}mv_0^2 + 0 + 0 = 0 + 0 + mgh \tag{8}$$

Using (5):

$$\therefore \quad h = \frac{k(\Delta x)^2}{2mg} \tag{9}$$

The distance **L** it travels up the incline is just:

$$L = \frac{h}{\sin\theta} \tag{10}$$

$$\therefore \quad L = \frac{k(\Delta x)^2}{2mg\sin\theta} \tag{11}$$

Plugging in the numbers given in the problem you'll get:

$$v_0 = 3.11 \text{ m/s} \tag{12}$$

$$L = 0.821 \text{ m} \tag{13}$$

2) *7.58*

Let's denote the 0.500 kg rat as **L** and 0.200 kg one as **S**.
No non-conservative force is present in this problem and we can safely use:

$$K_{S1} + U_{S1} + K_{L1} + U_{L1} = K_{S2} + U_{S2} + K_{L2} + U_{L2} \tag{14}$$

$$\begin{aligned} &\frac{1}{2}m_S v_{S1}^2 + m_S g y_{S1} + \frac{1}{2}m_L v_{L1}^2 + m_L g y_{L1} \\ &= \frac{1}{2}m_S v_{S2}^2 + m_S g y_{S2} + \frac{1}{2}m_L v_{L2}^2 + m_L g y_{L2} \end{aligned} \tag{15}$$

Point 1 refers to the initial condition when it's released from rest. Pint 2 refers to the vertical situation. I will measure the gravitation potential energy with respect to the height of the initial horizontal bar.

$$v_{S1} = v_{L1} = 0 \qquad y_{S1} = y_{L1} = 0 \tag{16}$$

If the animals are equidistant from the center, they have the same speed **V**.

$$v_{S2} = v_{L2} = V \qquad y_{S2} = +\frac{l_0}{2} \qquad y_{L2} = -\frac{l_0}{2} \tag{17}$$

Where l_0 is the length of the wooden rod.

$$\therefore \quad 0 + 0 + 0 + 0 = \frac{1}{2}(m_S + m_L)V^2 + (m_S - m_L)g\frac{l_0}{2} \tag{18}$$

$$V = \sqrt{\frac{m_L - m_S}{(m_L + m_S)(g l_0)}} \tag{19}$$

Plugging in the given numbers you get:

$$V = 1.83 \ \ \text{m/s} \tag{20}$$

3) *7.72*

Following the hint (to express k in terms of m & d), when the fish is lowered slowly, it stops in equilibrium::

$$\sum F_y = kd - mg = 0 \Rightarrow k = \frac{mg}{d} \tag{21}$$

Let's denote the maximum distance it reaches when it falls as d_{max}. Again here no nonconservative force is present and we have:

$$\frac{1}{2}mv_1^2 + \frac{1}{2}ky_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + \frac{1}{2}ky_2^2 + mgy_2 \tag{22}$$

NOTE: With a spring is easiest to pick y=0 at its unstretched length - otherwise the energy is $\frac{1}{2}k(y-y_0)^2$. We also take y>0 to be up.

Point **1** refers to the point it *starts* falling from rest and pint **2** as the point it **stops** moving after it stretches the spring the amount d_{max}. As a matter of convention I'll pick the y_1 we have released our mass as the **0**.

$$v_1 = 0 \qquad y_1 = 0 \tag{23}$$

$$v_2 = 0 \qquad y_2 = -d_{max} \tag{24}$$

$$0 + 0 + 0 = 0 + \frac{1}{2}kd_{max}^2 + mg(-d_{max}) \tag{25}$$

$$\therefore \quad (\frac{1}{2}kd_{max} - mg)d_{max} = 0 \tag{26}$$

There are two solutions:

The trivial one:

$$d_{max_1} = 0 \tag{27}$$

which results from the fact that the energy of the starting point is equal to itself!

The other solution:

$$\frac{1}{2}kd_{max} - mg = 0 \tag{28}$$

$$d_{max_2} = d_{\mathbf{max}} = \frac{2mg}{k} \tag{29}$$

Replacing k from equation (21) we get:

$$d_{\mathbf{max}} = \frac{2mg}{\frac{mg}{d}} \tag{30}$$

$$\therefore \quad d_{\mathbf{max}} = \mathbf{2}d \tag{31}$$

This is not surprising - the fish oscillates symmetrically about its equilibrium position.

4) ***Bead Slides Around and Up Wire***

`Please refer to the figure in the problem set.`

There is no non-conservative forces, so mechanical energy is conserved. The thing to remember is that for a circular motion acceleration is *radial* and is equal in magnitude to $\frac{v^2}{R}$.

Our first job is to obtain v at different points and look carefully what radial at that point means and use the component of Newton's law $\sum \overrightarrow{\mathbf{F}} = m\overrightarrow{\mathbf{a}}$ in the radial direction. As for convention I choose the horizontal line shown at the bottom of the loop in the figure as the **0** of potential energy.

a)

$$\frac{1}{2}mv_a^2 + mgy_a = E_a = E_b = \frac{1}{2}mv_b^2 + mgy_b \tag{32}$$

$$v_a = 0 \quad y_a = 2R \tag{33}$$

$$y_b = R \tag{34}$$

$$0 + mg(2R) = \frac{1}{2}mv_b^2 + mgR \tag{35}$$

$$\therefore \quad v_b = \sqrt{2gR} \tag{36}$$

Radial at point $\boldsymbol{b}$ is the $\boldsymbol{x}$ *direction*:

$$\sum F_{b_x} = N_b = m\frac{v_b^2}{R} = m\frac{2gR}{R} = 2mg \tag{37}$$

(This is twice the weight)

b)

$$\frac{1}{2}mv_a^2 + mgy_a = E_a = E_c = \frac{1}{2}mv_c^2 + mgy_c \tag{38}$$

$$v_a = 0 \quad y_a = 2R \tag{39}$$

$$y_c = 0 \tag{40}$$

$$0 + mg(2R) = \frac{1}{2}mv_c^2 + 0 \tag{41}$$

$$\therefore \quad v_c = 2\sqrt{gR} \tag{42}$$

Radial at point $\boldsymbol{c}$ means the $\boldsymbol{y}$ *direction*:

$$\sum F_{c_x} = N_c - mg = m\frac{v_c^2}{R} = m\frac{4gR}{R} = 4mg \Rightarrow N_c = 5mg \tag{43}$$

c) At point d which the mass reverse its direction: $v_d = 0$.

$$\frac{1}{2}mv_a^2 + mgy_a = \frac{1}{2}mv_d^2 + mgy_d \tag{44}$$

$$v_a = 0 \qquad y_a = 2R \qquad v_d = 0 \qquad y_d = H \tag{45}$$

$$0 + mg(2R) = 0 + mgH \tag{46}$$

$$\therefore \quad H = 2R \tag{47}$$

This is not surprising- in the absence of dissipation it rises to the same height from which it was released.

You could have chosen any two points you want between which to apply energy conservation since energy is conserved along the entire path. Let's do this part by comparing points d and c:

$$\frac{1}{2}mv_c^2 + mgy_c = \frac{1}{2}mv_d^2 + mgy_d \tag{48}$$

$$v_c = 2\sqrt{gR} \qquad y_c = 0 \qquad v_d = 0 \qquad y_d = H \tag{49}$$

$$\frac{1}{2}m(4gR) + 0 = 0 + mgH \tag{50}$$

which gives again

$$H = 2R. \tag{51}$$