HW Solutions # 5 - 8.01 MIT - Prof. Kowalski

Friction, circular dynamics, and Work-Kinetic Energy.

1) **5.80**

If the block were to remain at rest relative to the truck, the friction force would need to cause an acceleration of 2.20 m/s^2, ;However, the **maximum** acceleration possible to to **static** friction is (0.19)(9.80)=1.86 m/s^2, so the block will accelerate relative to the truck.
Acceleration a with respect to ground is (from $\sum F_x = +\mu_s mg = ma_{box}$):

$$a = \mu_k g = (0.15)(9.80) = 1.47 m/s^2. \tag{1}$$

The acceleration a' with respect to truck is:

$$a' = a_{box} - a_{truck} = 1.47 - 2.20 = -0.73 \tag{2}$$

The box will move 1.80 m relative to truck with a' acceleration.

$$t = \sqrt{\frac{2\Delta x}{a_{box} - a_{truck}}} = \sqrt{\frac{2(-1.80)}{-0.73}} = 2.22\text{s} \tag{3}$$

In this time, the truck moves the distance D:

$$D = \frac{1}{2}a_{truck}t^2 = \frac{1}{2}(2.20)(2.22)^2 = 5.43\text{m}. \tag{4}$$

2) **5.86**

Please refer to figure (1).

Let's denote the **common** magnitude of acceleration as a. If block **B** is to remain at rest on **A** , the sum of forces acting on **B** should produce the acceleration a. Setting up the $\sum \overrightarrow{\mathbf{F}_B} = m\overrightarrow{\mathbf{a}}$:

B:

$$\sum F_{By}: \quad N_{AB} - m_B g = 0 \Rightarrow N_{AB} = m_B g \tag{5}$$

$$\sum F_{Bx}: \quad f_s = m_B a \leq \mu_s N_{AB} \tag{6}$$

NOTE: Here is a case in which friction **causes** positive acceleration.

A:

$$\sum F_{Ay}: \quad N_A - N_{AB} - m_A g = 0 \Rightarrow N_A = (m_A + m_B)g \tag{7}$$

$$\sum F_{Ax}: \; T - \mu_k N_A - f_s = m_A a \Rightarrow T = \mu_k g(m_A + m_B) + (m_A + m_B)a \tag{8}$$

$$\therefore \quad T = (m_A + m_B)(a + \mu_k g) \tag{9}$$

NOTE: The friction force between block **A** and **B** is LESS THAN or EQUAL to $\mu_s N_{AB}$. We don't know it yet and must assign it an unknown variable f_s.

C:

$$\sum F_{Cy} = T - m_C g = m_C a_{Cy} = -m_C a \tag{10}$$

Replacing T from above into equation (10) and solving for a:

$$a = g\frac{m_C - \mu_k(m_A + m_B)}{m_A + m_B + m_C} \tag{11}$$

To simplify (6) in terms of a, we combine (5) and (6):

$$m_B a \leq \mu_s m_B g \tag{12}$$

$$\therefore \quad a \leq \mu_s g \tag{13}$$

Using (11):

$$g\frac{m_C - \mu_k(m_A + m_B)}{m_A + m_B + m_C} \leq \mu_s g \tag{14}$$

Solving the inequality for m_C gives:

$$m_C \leq \frac{(m_A + m_B)(\mu_s + \mu_k)}{1 - \mu_s}. \tag{15}$$

3) **5.90**

Since the lower block has less coefficient of friction it will accelerate more rapidly downward in the absence of the string. However the role of string is to make them have the same acceleration by having the tension T.
S:Smaller
L:Larger
Writing equations in the coordinate system which x is **parallel** to plane and pointing **up** and y perpendicular to plane and pointing away from it.

Smaller mass:

$$\sum F_{Sy} = 0 \Rightarrow N_S - m_S g \cos\theta = 0 \tag{16}$$

$$\sum F_{Sx} = m_S a \Rightarrow -m_S g \sin\theta + N_S \mu_{Sk} + T = m_S a \tag{17}$$

Larger mass:

$$\sum F_{Ly} = 0 \Rightarrow N_L - m_L g \cos\theta = 0 \tag{18}$$

$$\sum F_{Lx} = m_L a \Rightarrow -m_L g \sin\theta + N_L \mu_{Lk} - T = m_L a \tag{19}$$

a)Adding these two equations and solving for a:

$$a = \frac{(m_S + m_L)g[\cos\theta(\mu_{Sk} + \mu_{Lk}) - \sin\theta]}{m_S + m_L} \tag{20}$$

$$a = g[\cos\theta(\mu_{Sk} + \mu_{Lk}) - \sin\theta] \tag{21}$$

substitution gives

$$a = -2.21 m/s^2. \tag{22}$$

b)Substituting the above acceleration in either of (17) or (19) gives T=2.27 N.

c)The upper block will slide down with more acceleration until they collide! You could solve this section with the above formalism and you will get **negative** T which means that the string must be a stiff rod ,which supports compressive forces, in order to prevent the collision.

4) **5.104**

Please refer to figure (2)

Make sure that you have understood `Example 5.22` p.184.

a)

$$\sum F_y = 0 \Rightarrow \cos\beta\ T_U - \cos\beta\ T_L - mg = 0 \tag{23}$$

$$T_L = T_U - \frac{mg}{\cos\beta}. \tag{24}$$

The net force inward is

$$F_{rad} = \sin\beta\ T_U + \sin\beta\ T_L = \sin\beta\ (T_U + T_L) = ma_r. \tag{25}$$

b) Solving

$$F_{rad} = ma_{rad} = m\frac{v^2}{R} = m\frac{4\pi^2 R}{\tau^2} \tag{26}$$

(where τ is the period of oscillations) for period τ:

$$\tau = 2\pi\sqrt{\frac{mR}{F_{rad}}} \tag{27}$$

Using hypotenuse theorem the radius of oscillation is

$$R = \sqrt{(1.25)^2 - 1} = 0.75\text{m} \tag{28}$$

and $\cos\beta = \frac{4}{5}$; $\sin\beta = \frac{3}{5}$.
If you plug in the above numbers into equations we derived you'll get:

$$T_L = 31.0N. \tag{29}$$

$$\tau = 1.334s \tag{30}$$

so the system makes $1/0.02223 \approx 45$ rev/min.

c) When the lower string becomes slack, the system is the same as the conical pendulum considered in `Example 5.22` with $\cos\beta = 0.8$, the period is

$$\tau_{slack} = 2\pi\sqrt{\frac{(1.25)(0.80)}{9.80}} = 2.007\text{s}. \tag{31}$$

which equivalent to 30 rev/min.

d)For oscillation with less revolutions the Tension in lower string is still Zero and the problem is again the conical pendulum problem; the block will drop to a smaller angle.

5) **Work on Sliding Box**

a) Using work-energy theorem

$$W_{tot} = K_2 - K_1 = \Delta K \tag{32}$$

$$K_2 = 0 \tag{33}$$

$$K_1 = \frac{1}{2}mv_0^2 \tag{34}$$

Here gravity and normal forcee is perpendicular to direction of motion and from the definition

$$W = \overrightarrow{\mathbf{F}}.\overrightarrow{\mathbf{s}} \tag{35}$$

$m\overrightarrow{\mathbf{g}}$ and $\overrightarrow{\mathbf{N}}$ don't do any work. So W_{tot} is just W_f which is the work on the box by friction.

$$W_f = W_{tot} = 0 - \frac{1}{2}mv_0^2 = -\frac{1}{2}mv_0^2. \tag{36}$$

b) Using the definition

$$W = \overrightarrow{\mathbf{F}}.\overrightarrow{\mathbf{s}} = Fs\cos\phi \tag{37}$$

because friction is always against the relative motion in this case $\phi = 180^{\circ}$:

$$W_{tot} = W_f = F_f\ (x_1 - x_0)(-1) = -F_f\ (x_1 - x_0). \tag{38}$$

Using the result of part **a** we have

$$-F_f\ (x_1 - x_0) = -\frac{1}{2}mv_0^2 \tag{39}$$

$$\therefore\ F_f = \frac{1}{2}\frac{mv_0^2}{(x_1 - x_0)} \tag{40}$$

c) We have a constant acceleration situation in this problem so

$$v_{ave} = \frac{v_i + v_f}{2} = \frac{v_0 + 0}{2} = \frac{v_0}{2} \tag{41}$$

$$(x_1 - x_0) = v_{ave} t_{stop} \tag{42}$$

$$\therefore\ t_{stop} = \frac{2(x_1 - x_0)}{v_0} \tag{43}$$

d)Using (32)
Here $W_{tot} = W_{person} + W_f$ using (35) and noting that $s = x_2 - x_1$ we have $W_f = -F_f(x_2 - x_1)$. Using the result of part **b** we get

$$W_f = -\frac{1}{2} m v_0^2 \frac{x_2 - x_1}{x_1} \tag{44}$$

Collecting all the information we got we have

$$W_{tot} = W_{person} + W_f = W_{person} - \frac{1}{2} m v_0^2 \frac{x_2 - x_1}{x_1} = +\frac{1}{2} m v_1^2 \tag{45}$$

$$\therefore W_{person} = \frac{1}{2} m v_0^2 \frac{x_2 - x_1}{x_1} + \frac{1}{2} m v_1^2 \tag{46}$$

6) **U-Control Model Airplane**

Please refer to figure (3)

We choose the x axis along the acceleration so we have 0 acceleration along the y axis! If you choose the x axis along the wire you have have T along and F $\perp$. However you should project $\overrightarrow{\mathbf{a}}$ and elimination of two equations becomes difficult.

$$\sum \overrightarrow{\mathbf{F}} = m\overrightarrow{\mathbf{a}} \tag{47}$$

$$\sum F_y = ma_y = 0 \Rightarrow F\cos\theta - T\sin\theta - mg = 0 \tag{48}$$

$$\sum F_x = ma_x = \frac{mv^2}{R} = \frac{mv^2}{L\cos\theta} \Rightarrow F\sin\theta + T\cos\theta = \frac{mv^2}{L\cos\theta} \tag{49}$$

Eliminating F from (9) and (10) we get:

$$T - \frac{mv^2}{L} + mg\sin\theta = 0 \tag{50}$$

The physical condition T$\geq$ 0 will set a restriction on θ for a fixed v.

$$\sin\theta \leq \frac{v^2}{Lg} \tag{51}$$

So

$$\theta_{crit} = \arcsin\frac{v^2}{Lg} \tag{52}$$

Clearly for $v^2 = Lg$ even 90^{o} will be fine. So there exists a finite v_{safe}:

$$v_{safe} = \sqrt{Lg} \tag{53}$$

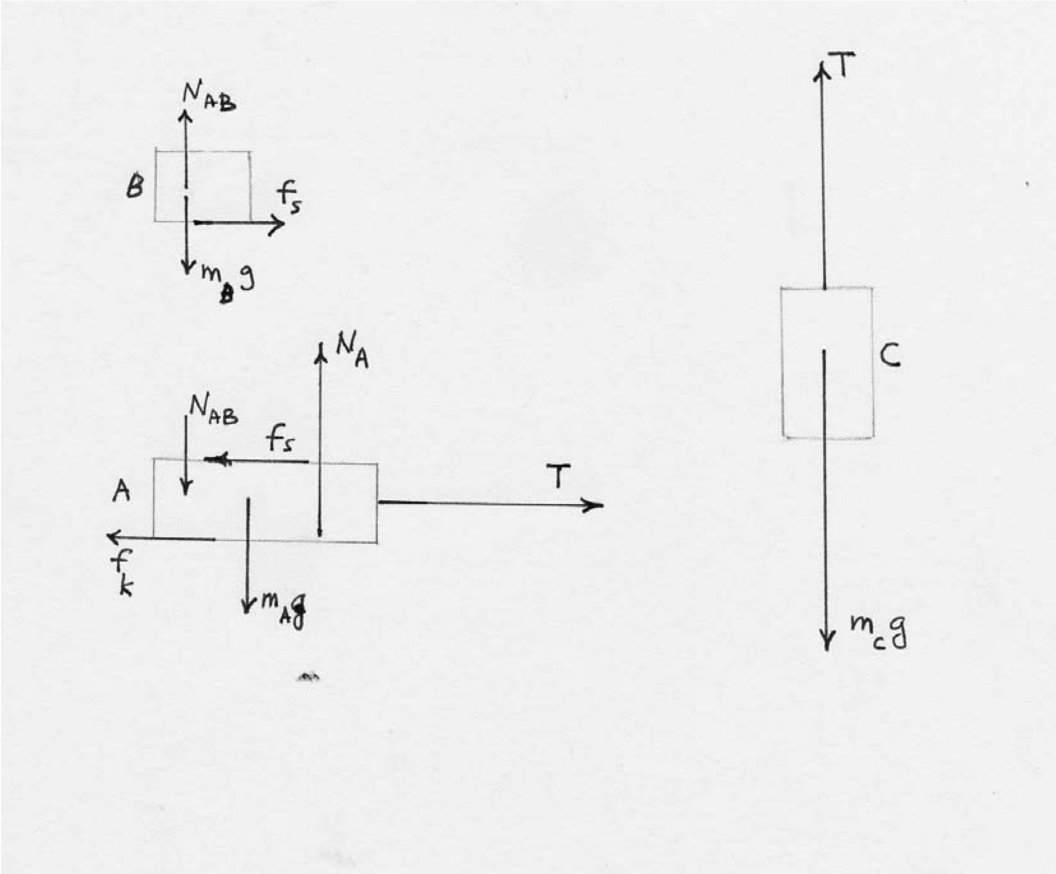

Figure 1: **5.86**

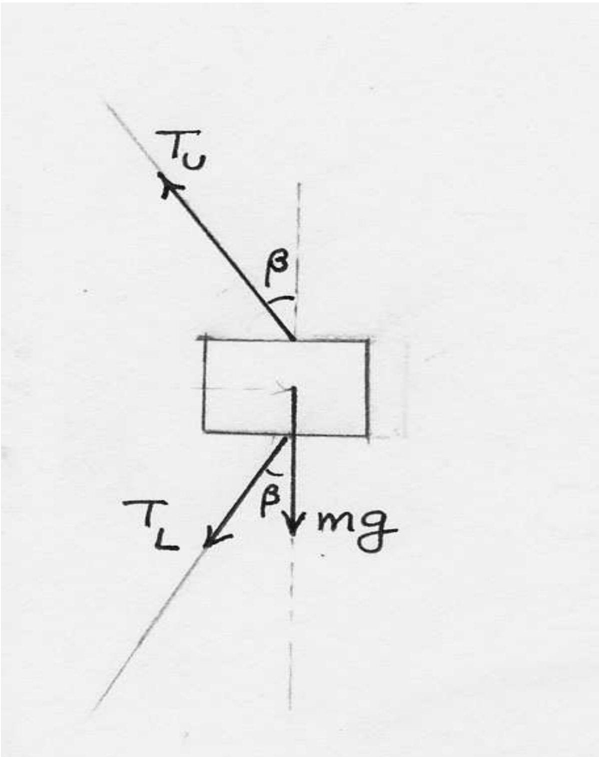

Figure 2: **5.104**

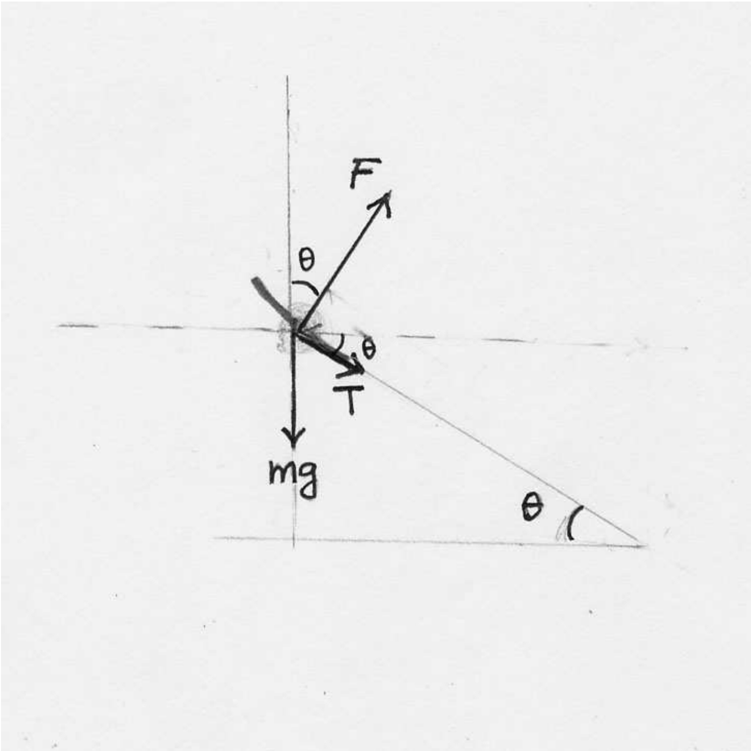

Figure 3: **U-Control Model Airplane**